GLWB Guarantees: Hedge Efficiency & Longevity Analysis

Etienne Marceau, Ph.D. A.S.A. (Full Prof. ULaval, Invited Prof. ISFA, Co-director Laboratoire ACT&RISK, LoLiTA)  Pierre-Alexandre Veilleux, FSA, FICA, (Industrielle Alliance, ULaval)

Longevity 11 2015
(Lyon, France)

U.Laval

September 7–9, 2015
1 Introduction
   - Context
   - Main objective

2 Risk management of GLWB guarantees
   - Valuation
   - Dynamic hedging
   - Assessment of hedge efficiency

3 Hedge efficiency empirical study
   - Modeling
   - Parameters
   - Results

4 Longevity analysis
   - Longevity risk impact
   - Risk allocation

5 Future work

6 References
Pierre-Alexandre Veilleux, BSc 2011, FSA 2013, is an actuary (full-time) at Industrial Alliance, Insurance and Financial Services.

Industrial Alliance, Insurance and Financial Services is an important insurance company in Canada.

Pierre-Alexandre Veilleux is working on segregated funds.

And he is a graduate student on a "Part-time" basis working under my supervision.

The results of our paper will be in his thesis.

Inspired from a real problem in practice.
Dear customer,

Unfortunately your flight LH494, 09.09.2015 has been cancelled. Please use the alternatives online under My Bookings:

→ My Bookings

We apologize for any inconvenience.

Would you like to receive timely flight information per text to your mobile phone? We offer to inform you, using the mobile phone number stored in your Miles & More profile. Please amend your Miles & More profile to take advantage of this service: miles-and-more.com

Best regards
GLWB (Guaranteed Lifetime Withdrawal Benefit) guarantees are a special case of variable annuity.

In Canada, GLWB guarantees are called segregated fund guarantees.

They have been very popular in recent years in Canada and the United States.

- Growing need for income at retirement
- Participation in equity markets

Insurers now have to adequately manage the risks associated with these guarantees.

The guarantee provides the client with a lifetime income with a participation in equity markets.

It offers a combination of growth and guaranteed income.

The company is at risk when the account value is exhausted.
Illustration: Initial deposit = 100
Introduction

Context

- GLWB guarantees: complex options
- Main risks:
  - Financial markets: account value level
  - Longevity: lifetime income
  - Interest rates:
    - risk-neutral projection
    - discounting
- Consequence: Risk management of GLWB guarantees is a main concern
- Quote from Silverman & Theodore (2014, Milliman):
  "Stochastic modeling of longevity risk can be a useful tool in the pricing and management of variable annuities with living benefits".

**Introduction**

**Main objective**

- Significant body of literature on segregated fund guarantees, variable annuity guarantees, and similar products

- Valuation and pricing of GLWB guarantees:
  - Shah and Bertsimas (2008)
  - Piscopo and Haberman (2011)
  - Holz, Kling, and Russ (2012)
  - Kling, Ruez, and Russ (2011)

- Risk management of GLWB guarantees:
  - Kling, Ruez, and Russ (2011):
    - Hedge efficiency
    - Impact of modeling on hedge efficiency
Introduction

Main objective

- Kling, Ruez, and Russ (2011) consider 1 risk (investment):
  - Stock markets: Heston Model (stochastic volatility)
  - Interest rate: deterministic
  - Mortality: deterministic
  - Segregate fund: stock only

- We consider 3 risks (investment, interest rate, longevity):
  - Stock markets: Regime switching model
  - Interest rate: stochastic (G2++)
  - Mortality: stochastic (Lee-Carter Model)
  - Segregate fund: stock & fixed income

- Analysis of the impact of stochastic mortality on hedge efficiency
Risk management of GLWB guarantees

Valuation

Let

\[ \Omega_T = \{ t_0, t_1, \ldots, t_{(\omega-x)/\Delta t} \} \]

be the times at which events can occur, where

- \( t_0 = 0 \) is the contract inception date
- \( x \) is the age at contract inception
- \( \omega \) is the maximum age
- \( t_{i+1} - t_i = \Delta t \quad \forall i \)

- **Financial market:**
  - Stock market: \( S = \{ S_{t_i}, t_i \in \Omega_T \} \)
  - Bond market: \( P = \{ P_{t_i}, t_i \in \Omega_T \} \)
Segregated fund:

- $F = \{F_{t_i}, t_i \in \Omega_T\}$
- Diversified fund
  - Proportion $\omega_{t_i}$ in the stock market index ($S_{t_i}$)
  - Proportion $1 - \omega_{t_i}$ in the bond market index ($P_{t_i}$)

Dynamic of $F$:

$$F_{t_i} = F_{t_{i-1}} \left( \omega_{t_{i-1}} \frac{S_{t_i}}{S_{t_{i-1}}} + (1 - \omega_{t_{i-1}}) \frac{P_{t_i}}{P_{t_{i-1}}} \right) e^{-m_A \cdot (t_i - t_{i-1})}, \quad t_i \in \Omega_T,$$

where $F_{t_0} = F_0$ and $m_A$ is the fund fee.
Risk management of GLWB guarantees

Valuation

- Account value: \( A = \{A_{t_i}, t_i \in \Omega_T\} \)

\[
A_{t_i} = \begin{cases} 
\max \left( \frac{F_{t_i}}{F_{t_{i-1}}} e^{-g_A (t_i-t_{i-1})} - \frac{1}{n} L_{t_{i-1}}; 0 \right), & \text{if withdrawal} \\
A_{t_{i-1}} \frac{F_{t_i}}{F_{t_{i-1}}} e^{-g_A (t_i-t_{i-1})}, & \text{otherwise}
\end{cases}
\]

where

- \( L_{t_i}, t_i \in \Omega_T \) is the annual withdrawal amount at time \( t_i \),
- \( g_A \) is the guarantee fee
- \( n \) is the withdrawal frequency.
Let $V = \{ V_{t_i}, t_i \in \Omega_T \}$ be the guarantee liability process:

$$V_{t_i} = \text{expected PV of the benefits} - \text{expected PV of the "premiums"}$$
Risk management of GLWB guarantees

It means

\[ V_{t_i} = E_\mu \left[ E^Q \left( \sum_{j=\max(k^*,i)+1}^{(\omega-x)/\Delta t} \frac{t_j}{\Delta t} p_x^{(\mu)} e^{-\int_{t_i}^{t_j} r_s \, ds} \frac{1}{n} L_{t_{j-1}} 1\{nt_j \in \mathbb{N}\} \right) \right] \]

\[ - E_\mu \left[ E^Q \left( \sum_{j=i}^{(k^*-1)} t_j p_x^{(\mu)} A_{t_j} \left( 1 - e^{-g_A(t_j+1-t_j)} \right) e^{-\int_{t_i}^{t_j} r_s \, ds} \right) \right], \]

where

- \( \mathcal{F}_{t_i} \) and \( \mathcal{G}_{t_i} \) are the \( \sigma \)-algebras containing all financial and mortality information respectively.
- \( \mu = \{\mu_{t_0}, \mu_{t_1}, \ldots, \mu_{(\omega-x)/\Delta t - 1}\} \) is the force of mortality vector.
- \( t_{k^*} \) is the time at which the account value is exhausted.
- \( B_{t_{k^*}} = A_{t_{k^*-1}} \frac{F_{t_i}}{F_{t_{k^*-1}}} e^{-g_A(t_{k^*-1} - t_{k^*})} - \frac{1}{n} L_{t_{k^*-1}} \)
A common strategy in the insurance industry is dynamic hedging:

- Liquid asset portfolio
- Frequent rebalancing

This strategy consists in compensating our guarantee liability sensitivity to various risk factors:

- Stock market (delta)
- Bond market (delta)
- Interest rates (rho)

Sensitivities are valued using finite difference techniques for all risk factors:

- Stock market index $S$
- Bond market index $P$
- Interest rate curve sections
Risk management of GLWB guarantees

Dynamic hedging

- Let $V_t \equiv V_t(\theta_1, \ldots, \theta_i, \ldots, \theta_m)$ be the guarantee liability and $\theta_i, i = 1, \ldots, m$ the risk factors that affect its value.

- We have

$$\frac{\partial V_t}{\partial \theta_i} \approx \frac{V_t(\theta_1, \ldots, \theta_i, \ldots, \theta_m) - V_t(\theta_1, \ldots, \theta_i - h, \ldots, \theta_m)}{h}.$$ 

- The asset portfolio, $H_t$, is then built such that

$$\frac{\partial H_t}{\partial \theta_i} = \frac{\partial V_t}{\partial \theta_i}$$

using simple financial instruments:

- Short positions on $S_t$ and $P_t$
- Long positions in zero-coupon bonds
Risk management of GLWB guarantees
Assessment of hedge efficiency

Goal: Assess how modeling of the guarantee liability impacts hedge efficiency

We are now working under two perspectives:
- Projection under the real-world measure $\mathbb{P}$
- Valuation under the risk-neutral measure $\mathbb{Q}$

Steps:
1. Simulation of a real-world scenario ($\mathbb{P}$)
2. For all $t_i \in \Omega_T$ in the real-world scenario,
   1. calculate the guarantee liability ($\mathbb{Q}$)
   2. calculate deltas and rhos ($\mathbb{Q}$)
   3. determine the asset portfolio ($\mathbb{Q}$)
   4. calculate the hedge gains and losses ($\mathbb{P}$)
3. Discount the hedge gains and losses ($\mathbb{P}$)
Risk management of GLWB guarantees
Assessment of hedge efficiency

- We have the tools required for steps 2(a) - 2(c)
- But we must make an appropriate link between
  - the scenario under the $\mathbb{P}$ measure
  - the valuation of guarantee liability under the $\mathbb{Q}$ measure
- Then, we complete steps 2(d) and 3
Risk management of GLWB guarantees
Assessment of hedge efficiency

- Let $GP = \{GP_{t_i}, t_i \in \Omega_T\}$ be the process of hedge gains with

$$GP_{t_i} = H_{t_i}^- - H_{t_{i-1}}^- + t_i p_{x}^{(\mu)} (R_{t_i} - C_{t_i}) - (V_{t_i} - V_{t_{i-1}}),$$

where

- $R_{t_i}$: revenue from the guarantee fee at time $t_i$
- $C_{t_i}$: claim payment made by the company at time $t_i$
- $H_{t_i}^-$: asset portfolio value before rebalancing at time $t_i$
- $H_{t_{i-1}}^-$: asset portfolio value after rebalancing at time $t_{i-1}$

- Let $PVGP$ be the present value of gains and losses under the $\mathbb{P}$ measure:

$$PVGP = \sum_{i=1}^{\omega-x/\Delta t} GP_{t_i} \prod_{j=0}^{i-1} ZC(t_j, t_{j+1}),$$

where $ZC(t_j, t_{j+1})$ is the zero-coupon bond of maturity $t_{j+1} - t_j$ at time $t_j$ in our real-world scenario.
The discount function implies an investment in the money market account.

Assessing hedging efficiency is a stochastic-on-stochastic calculation:
- Outer loop: scenarios under the $\mathbb{P}$ measure
- Inner loop: liability valuation and greeks under the $\mathbb{Q}$ measure

The computation time involved is substantial.
Stock market:

- Lognormal model (LN): \( dS_t = \mu^S S_t dt + \sigma^S S_t dW^S_t \)
- Regime-switching lognormal model (RSLN):

\[
dS_t = \mu^S \rho_t S_t dt + \sigma^S \rho_t S_t dW^S_t,
\]

where \( \rho_t \) is a two-state continuous-time Markov process

Stochastic mortality:

- Let \( \mu_{x,t_i} = e^{\alpha_x + \beta_x \kappa_{t_i}} \), where \( \mu_{x,t_i} \) is the force of mortality for age \( x \) between \( t_i \) and \( t_{i+1} \).
- Constant mortality improvement (Cst): \( \kappa_{t_i} = \kappa_{t_{i-1}} + \theta(t_i - t_{i-1}) \)
- Lee-Carter model in discrete time (LC):

\[
\kappa_{t_i} = \kappa_{t_{i-1}} + \theta(t_i - t_{i-1}) + \sigma^\mu \sqrt{\Delta t} \epsilon_{t_i}^\mu, \quad \epsilon_{t_i}^\mu \sim N(0, 1), \quad t_i \in \Omega_T
\]
Interest rates:
- Let $s(t, t + T)$ be the continuously compounded $T$-year spot rate at time $t$
- Constant curve (Cst):
  \[ s(t, t + T) = \frac{1}{T} [(T + 1)s(t, t + T + 1) - s(t, t + 1)] \]
- G2++ model (G2):
  \[ s(t, t + T) = \frac{-1}{T} \left[ \ln \left( \frac{P^M(0, t + T)}{P^M(0, t)} \right) - \frac{1}{2} (V(t, T) + V(0, t) - V(0, T)) \right. \\
  \left. + x(t)B(a, T - t) + y(t)B(b, T - t) \right] \]
  \[ dx(t) = a(\lambda_1 - x(t))dt + \sigma dW_1^r(t) \quad x(0) = 0 \]
  \[ dy(t) = b(\lambda_2 - y(t))dt + \eta dW_2^r(t) \quad y(0) = 0 \]
  \[ dW_1^r(t) dW_2^r(t) = \rho dt \]
Parameters

- **Contract holder:**
  - 65-year-old male
  - $100,000 single premium
  - Withdrawals deferred for 5 years (at age 70)

- **Contractual parameters:**
  - \( n = 4 \) (withdrawal frequency)
  - \( g_A = 1.5\% \) (guarantee fee)
  - \( m_A = 3.0\% \) (fund fee)
  - \( l_{70} = 5.5\% \) (withdrawal rate)
Hedge efficiency empirical study

Parameters

- Projection: Monthly \((\Delta t = \frac{1}{12})\)
  - Financial variables
  - Mortality
  - Hedge portfolio rebalancing
- Stock market: Canadian stock market (TSX TR)
- Interest rates:
  - Canadian swap curve as of December 31, 2014
  - G2++: Babbs and Nowman (1999)
- Longevity: Canadian males
Hedge efficiency empirical study

Results

- $PVGP$ is the present value of hedge gains and losses
  - $PVGP > 0 \Rightarrow$ gain
  - $PVGP < 0 \Rightarrow$ loss

- To quantify risk in the left tail, we use
  \[ E_{\alpha}^{PVGP} = E \left[ PVGP | PVGP < \text{VaR}_{1-\alpha}(PVGP) \right], \]
  the TVaR of hedge losses.

- Modeling under the $\mathbb{P}$ measure:
  - Stock market: RSLN2
  - Interest rates: Two-factor Gaussian model
  - Longevity: Lee-Carter
Results

Our results:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Interest</th>
<th>Longevity</th>
<th>$E_{PVGP}^{\alpha}$ as % of $A_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td>LN</td>
<td>Cst</td>
<td>Cst</td>
<td>-1.65</td>
</tr>
<tr>
<td>RSLN</td>
<td>Cst</td>
<td>Cst</td>
<td>-1.80</td>
</tr>
<tr>
<td>RSLN</td>
<td>G2</td>
<td>Cst</td>
<td>-1.26</td>
</tr>
<tr>
<td>RSLN</td>
<td>G2</td>
<td>LC</td>
<td>-1.26</td>
</tr>
</tbody>
</table>

Results from Kling, Ruez, and Russ (2011):

<table>
<thead>
<tr>
<th>Stock</th>
<th>$E_{PVGP}^{\alpha}$ as % of $A_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9</td>
</tr>
<tr>
<td>LN</td>
<td>-4.3</td>
</tr>
<tr>
<td>Heston</td>
<td>-4.2</td>
</tr>
</tbody>
</table>

Computation involves simulations with simulations:
- Outer loop: 1000 scenarios (under $P$)
- Inner loop: 1000 scenarios at each month to compute the value of the
Hedge efficiency empirical study

Results

- Observations and conclusions:
  - Substantial computation time
    - Optimized programming in C++ and R
    - Total computation time: 5-7 days on 8 cores in parallel
  - Adding stochastic volatility alone does not improve the hedge efficiency
    - Conclusion similar to the one of Kling, Ruez, and Russ (2011)
    - Impact is more pronounced for the RLSN model
  - Including the G2++ model materially improves the hedge efficiency
  - Stochastic longevity has a negligible effect on results:
    - The guarantee valuation looks at the average of scenarios
    - The mean and median of the Lee-Carter model are fairly close
  - Calculations at age 50 lead to similar conclusions:
    - Substantial reduction of risk when interest rate volatility is introduced
    - Relatively small longevity impact
There are two levels in the hedge efficiency analysis:

- Projection under the real-world measure $\mathbb{P}$
- Valuation under the risk-neutral measure $\mathbb{Q}$

Stochastic mortality in the guarantee liability valuation has little impact on hedge efficiency.

What about in the real-world projection?
We wish to allocate the risk between financial and longevity risks.

Euler’s capital allocation method:
- Let $S = X_1 + X_2$
- Contribution of risk $X_i$ in $TVaR_\kappa(S)$:
  $$C^TVaR_\kappa(X_i) = \frac{1}{1 - \kappa} \int_{\text{VaR}_{\kappa}(X_1 + X_2)}^{\infty} E[X_i \times 1\{S=y\}] \, dy.$$

Anecdote: Did you know? Euler (1707-1783) also made contribution on the computation of premiums for life annuities!
Longevity analysis
Risk allocation

- Let \( S = \varphi (X_1, \ldots, X_n) \)
- They are two possible cases for \( \varphi \)
  - \( \varphi = \) linear function of the components of \( (X_1, \ldots, X_n) \)
    - We can directly apply Euler’s capital allocation method
    - \( X_1, \ldots, X_n \): insurance contracts, annuity contracts, lines of business, assets, loans, etc.
  - \( \varphi = \) nonlinear function of the components of \( (X_1, \ldots, X_n) \)
    - We cannot directly apply Euler’s capital allocation method
    - \( X_1, \ldots, X_n \): risk factors such as interest rate, mortality index, inflation, etc.
    - We need to decompose \( S \) in linear components in order to apply Euler’s allocation method

- Some possible decomposition methods:
  - Hoeffding decomposition (Rosen & Saunders (2010))
  - Taylor expansion (Karadey et al. (2014))

- We conclude with an illustration of the method in the next part
We use Hoeffding’s decomposition:

\[ PVGP = g(Z_1, Z_2) \]

\[ = E[PVGP|Z_1] + (PVGP - E[PVGP|Z_1]) \]

where

- \( Z_1 \) represents the mortality path
- \( Z_2 \) represents financial variables
- \( PVGP \) be the present value of hedged gains with both financial and longevity risks

Interpretation:

- \( E[PVGP|Z_1] \): Average \( PVGP \) over all mortality paths
- \( PVGP - E[PVGP|Z_1] \): Additional risk caused by stochastic mortality

<table>
<thead>
<tr>
<th>Allocation</th>
<th>( E_\alpha^{PVGP} ) as % of ( A_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>0.6</td>
</tr>
<tr>
<td>Financial Risks</td>
<td>-0.99</td>
</tr>
<tr>
<td>Longevity Risks</td>
<td>-0.27</td>
</tr>
</tbody>
</table>

Conclusions and observations:

- Longevity risk has a strong relative importance
- Allocation is fairly constant across confidence levels
Future work

- Hedge efficiency analysis
  - Sensitivity to parameters
  - Impact of a one-factor interest rate model on hedge efficiency

- Longevity analysis
  - Comparison with actuarial margins for adverse deviations
References