# The Linkage between Large Currency Swings and Fundamentals

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**Abstract**

The distribution of foreign exchange returns has fat tails. Theoretical exchange rate models show that shocks which drive the fundamentals also move the exchange rate returns, but this link may be weak if the discount factor is high. To provide evidence for the connection between extreme exchange rate returns and fundamentals' large movements, we estimate the asymptotic dependence between the variables. The strongest links are found for Asian and Latin American currencies while the relations are weaker for European currencies. All the action stems from monetary and financial fundamentals; real income shocks appear disconnected.

**Keywords:** Exchange rates, fundamentals, asymptotic dependence.
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## 1 Introduction

It is well known that the distribution of foreign exchange (FX) returns exhibits heavy tails, to the extent that the probability of an extreme currency movement has a different order of magnitude than the normal distribution. See Westerfield (1977), Boothe and Glassman (1987), Akgiray, Booth and Seifert (1988), Koedijk, Schafgans and de Vries (1990), Koedijk, Stork and de Vries (1992), Koedijk and Kool (1994) and Susmel (2001).[1] Since in theoretical exchange rate models shocks that drive the fundamentals also drive the exchange rate, this heavy tail feature should appear on both sides of the equation. We ask whether this heavy tail feature can be attributed to the tail behavior of the macro fundamentals. This is similar to the volatility point made by Engel, Mark and West (2007), that at least the two sides should display comparable levels of volatility.

For a long time the literature on foreign exchange rate modelling has been haunted by the low explanatory power of macro fundamentals based specifications. Forecasts based on such models have not done well; shorter term no-change forecasts often produce a lower mean squared error. The recent literature on exchange rates

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[1] Moreover, it is well documented that at higher frequencies exchange rate returns exhibit volatility clustering, see Diebold (1988). This adds to the fat tail nature of the returns.

by Engel and West (2005) and Engel, Mark and West (2007) explains that this poor performance of the fundamentals based specifications is due to the non-stationarity of the drivers and the high discount factor. The non-stationarity has been recognized for some time. This has motivated model specifications in first differences of the fundamentals with exchange rate returns as the dependent variable. The importance of a high discount factor is more recent. Sarno and Sojli (2009) provide direct empirical evidence on the discount factor issue.

If a regression yields reliable coefficients, the typical conclusion is that the dependent and explanatory variable are connected. In a time series setting this implies predictive power which can then be tested by out of sample forecasting. For the fundamentals based exchange rate models, however, the regression coefficient for the composite fundamentals is close to zero due to a discount factor that is realistically near one. Small coefficients do not necessarily imply low economic relevance. But it so happens that this very value of the discount factor also magnifies the contribution of the non-observed fundamentals' shocks, dwarfing the contribution of the fundamentals.

Predictability and the size of the regression coefficient are not the only ways in which the standard models can be judged. Engel, Mark and West (2007), for example, compare the volatility of exchange rate returns with the volatility of a constructed measure of the discounted expected future fundamentals. They find that the ratio of the two volatilities hovers between 30% and 50%, concluding that fundamentals at least appear sufficiently volatile. The typical forward solution of the exchange rate models emphasizes that the exchange rate returns are driven by the discounted expectations about the future shocks to the fundamentals. Thus, an ex-post measure of these shocks should at least exhibit volatility similar in magnitude as the volatility of the exchange rate returns. In this paper we establish the direct linkage between the large shocks that are driving the observed fundamentals and the extreme movements of the exchange rate. In regression analysis this connection may not show up as the shocks average out. By focusing on the positive and negative shocks separately, we uncover the strong connection between the shocks and the large (positive and negative) exchange rate returns. Moreover, in this way we can easily allow for their asymmetric responses.

The presence of the same large shocks on both sides of the equation also implies a specific kind of dependency. The larger shocks imply that the exchange rate returns and fundamentals can be asymptotically dependent. Two random variables are said to be asymptotically dependent if the probability on a large realization of one variable, given a large realization of the other random variable, is non-zero, even in the limit. Otherwise the two random variables are asymptotically independent. Asymptotic independence does not imply independence, though. For example, consider two random variables that are either multivariate normally distributed or Student-t distributed with a correlation coefficient $\rho\epsilon(0,1)$. In the former case, the two random variables are asymptotically independent though correlated; in the latter case the dependence is preserved in the tail area.

If one finds no support for asymptotic dependence, this can be due to one of the following two explanations. One possibility is that the fundamentals' based exchange rate model does not apply, so that the noise is exogenous and is not related to the (macro fundamental) regressors. Alternatively, even if two random variables are (imperfectly) correlated, but follow e.g. a multivariate normal distribution, then all dependency vanishes asymptotically. Thus if we reject asymptotic dependence, there are two possible explanations. If we find that asymptotic dependence is not rejected, this at least suggests a strong linkage between the composite macro fundamentals and the exchange rates via the (larger) shocks that drive both variables.

To uncover the dependency between the exchange rate returns and the fundamentals in the tail area of their distributions, that is for the larger movements,

we rely on extreme value techniques from statistics. It is well documented that exchange rate returns exhibit heavy tails. Not much is known about the fundamentals in this respect, partly due to the low frequency nature of these data. We first argue theoretically that within a standard monetary macroeconomic model with Brainard type multiplicative uncertainty, the implied distributions of macroeconomic variables like the inflation rate and money stock can exhibit the heavy tail feature, even if the noise distributions have no tails at all (such as the uniform distribution). Then we use the additivity property of heavy tail distributions to show that this property carries over to the composite fundamentals. Subsequently, we show that if the fundamentals exhibit heavy tails, then standard type foreign exchange models induce these heavy tails onto the exchange rate returns. But not only that, those models also imply a specific type of dependency in the tail area, i.e. asymptotic dependence, through the linearity of the monetary model in the logarithm of the variables.

Financial and monetary variable shocks appear strongly connected with the larger movements in the exchange rates, but industrial production is not. This accords with theoretical model. The strength of interdependency in the tail area varies across regions, it is generally higher for non-OECD countries. The paper therefore lends further support to traditional exchange rate models in the vein of research initiated by Engel and West (2005) and Engel, Mark and West (2007).

The next section offers a theoretical explanation for the heavy tail nature of the data and the asymptotic dependency between the exchange rate returns and composite macro fundamentals. Section 3 provides the estimation methods. The empirical results are in section 4. Conclusions are presented in Section 5. Appendices give further details on theory and empirics.

# 2 Theory

Within a standard monetary macroeconomic model, we first show that multiplicative supply-side Brainard type noise with a bounded support induces heavy tails on the distribution of the macroeconomic aggregates, even in the setup that the noise itself does not have heavy tails. Subsequently, we provide a short review of the probabilistic properties of fat-tailed distributed random variables, their scaling properties and the strong linkage that this may imply between macro shocks and exchange rate returns.

## 2.1 Tail Events and Macroeconomic Fundamentals

One may wonder why macroeconomic fundamentals have distributions with heavy tails. An early statistically oriented explanation for inflation rates was offered by Engle (1982). Engle's ARCH model has random variables follow a martingale process with autoregressive behavior in the second moment causing clusters of high and low volatility. Even if the innovations are thin-tailed normally distributed, the unconditional distribution ends up having heavy tails like the Pareto distribution, see de Haan, Resnick, Rootzen and de Vries (1989). In the vein of Bollerslev (1987), Cumperayot (2002) shows that macroeconomic variables still exhibit heavy tails after filtering out the ARMA-GARCH components.

Here we develop an economic based explanation of how the distribution of a macroeconomic variable like the money stock or rate of inflation can exhibit the heavy tail feature. The idea is not to present a fully fledged theory, as this would be outside the scope of the paper, but to present a coherent argument for two of the macroeconomic variables involved. The next subsection then shows that the heavy tail feature is carried over to the exchange rate.

To this end consider the following standard comprehensive monetary macroeconomic model, as presented in Walsh (2003, p.440). The aggregate supply curve reads

$$Y_t = A_t(\Pi_t - E_{t-1}[\Pi_t]) + \varphi_t, \tag{1}$$

where $Y_t$ is the logarithmic level of output, $\Pi_t$ is an inflation rate and $E_{t-1}[\Pi_t]$ is the time $t-1$ expected inflation for time $t$, and $\varphi_t$ is a noise term. In the short run, deviations from the long-run output level are possible due to expectational errors. The elasticity of output with respect to inflation expectations' errors is $A_t$. Thus (1) is in a crude way the Lucas type supply curve. Aggregate demand depends on real interest rates, i.e. the nominal interest rate minus expected inflation $I_t - E_t[\Pi_{t+1}]$:

$$Y_t = -b(I_t - E_t[\Pi_{t+1}]) + \eta_t. \tag{2}$$

The reduced-form money market equation is based on the quantity equation

$$M_t = P_{t-1} + \Pi_t + \gamma Y_t - \lambda I_t + \nu_t, \tag{3}$$

where $M_t$ and $P_{t-1}$ stand for the logarithms of the quantity of money and price level, respectively. The demand and supply disturbances $(\varphi_t, \eta_t)$ are assumed to have mean zero i.i.d. noise with thin (exponential decline) or bounded tails (in case of bounded support). At this point we do not need to be specific about the shocks $\nu_t$ to the money market equation. The $\nu_t$ shocks are not required to be independent over time, i.e. they may follow a stochastic process.[2] The $\lambda$ is the semi-interest rate elasticity of money demand.

Frequently model estimates and new data lead to parameter revisions, see Sack (2000). We capture the model uncertainty via the Brainard (1967) effect and assume that the coefficient for the short-run Phillips effect $A_t$ is an i.i.d. random variable. Suppose $A_t$ has a beta distribution

$$P\{A \leq x\} = x^{\alpha}, \ \alpha > 2. \tag{4}$$

The support of this distribution is $[0, 1]$. Note that this distribution is clearly not fat tailed. The fact that zero is in the support reflects the possibility that the short-run supply curve may be vertical, i.e. coincides with the long-run supply curve. As will become clear shortly, in an expected sense the next period's supply curve is vertical as $E_{t-1}[Y_t] = 0$.

Suppose the goal of monetary policy is to stabilize the level of inflation around a target $\pi^*$. This reflects, e.g., the European Central Bank's single price stability objective, since the ECB does not have real income stabilization or employment as its prime objectives. Specifically, assume that the objective resembling the ECB's main task reads

$$\min_{I_t} E_{t-1}[(\Pi_t - \pi^*)^2]. \tag{5}$$

Based on information available at time $t-1$, the central bank determines the policy interest rate $I_t$ in order to minimize its expected loss from price instability.

To do this, the policy interest rate is set to ensure that the expected value of inflation equals the target level, i.e. $E_t[\Pi_{t+1}] = E_{t-1}[\Pi_t] = \pi^*$. Substituting out $Y_t$ from the first two equations (1) and (2) gives

$$\Pi_t = \frac{(b + A_t)\pi^* - bI_t + \eta_t - \varphi_t}{A_t}.$$

Since by assumption $\alpha > 2$ in (4), the $E[1/A_t]$ is bounded, see (6) below. Thus we can take expectations conditional on time $t-1$ information

$$E_{t-1}[\Pi_t] = b(\pi^* - I_t)E_{t-1}[1/A_t] + \pi^*,$$

[2] Walsh (2003) solves the model under the assumption that the shocks follow an AR(1) process.

and equate $E_{t-1}[\Pi_t] = \pi^*$. Hence, given its objective function (5), it is optimal for the central bank to set

$$I_t = \pi^*.$$

Solving for the inflation rate and log income, we find

$$\Pi_t = \pi^* + \frac{\eta_t - \varphi_t}{A_t}$$

and

$$Y_t = \eta_t.$$

For the money market equation (3) this implies that

$$M_t = P_{t-1} + \Pi_t + \gamma Y_t - \lambda\pi^* + \nu_t.$$

Use the first two equations (1) and (2) to substitute out $Y_t$ and $\Pi_t$, to get

$$M_t = P_{t-1} + (1-\lambda)\pi^* + (\gamma + \frac{1}{A_t})\eta_t - \frac{\varphi_t}{A_t} + \nu_t.$$

Now $\Pi_t$ and $M_t$ are heavy-tailed distributed since $(\eta_t - \varphi_t)/A_t$ is heavy-tailed distributed, but log income $Y$ is not! This follows from the fact that the random Phillips effect coefficient appears in the denominator of the expressions for $\Pi$ and $M$. Given the beta distribution assumption (4) regarding $A_t$, the distribution of the inverse is

$$\Pr\{\frac{1}{A} \leq x\} = 1 - P\{A \leq \frac{1}{x}\} = 1 - \frac{1}{x^\alpha}, \tag{6}$$

with support $x\epsilon[1,\infty)$. Thus the inverse of $A$ has a heavy-tailed Pareto distribution (conditional on the distribution of $\eta_t$ and $\varphi_t$) and has moments $k$ only up to $\alpha$. This can be easily seen from

$$E[(1/A)^k] = \int_1^\infty x^{k-\alpha-1}dx = \frac{1}{k-\alpha}x^{k-\alpha}|_1^\infty.$$

The power decline of the Pareto density implies that not all moments exist. This is the defining characteristic of heavy-tailed distributions.[3]

As a result, the unconditional distributions of $\Pi_t$ and $M_t$ are also heavy-tailed. To see this, let $Q = \eta - \varphi$ and consider the distribution of $Q/A$. Suppose the distribution of $Q$ does not have heavy tails, in the sense that all moments are bounded; in particular $E_Q[Q^\alpha] < \infty$. Using the conditioning argument of Breiman and (6) then shows that

$$\begin{aligned}
\Pr\{\frac{Q}{A} &> x\} = E_Q[P\{\frac{q}{A} > x|Q = q\}] \\
&= E_Q[P\{\frac{1}{A} > \frac{x}{q}|Q = q\}] \\
&= E_Q[\left(\frac{Q}{x}\right)^\alpha] \\
&= E_Q[Q^\alpha]x^{-\alpha}. \qquad (7)
\end{aligned}$$

Therefore, due to the random Phillips curve coefficient, the unconditional distributions of $\Pi$ and $M$ are also heavy tailed. But income $Y$ is not heavy tailed, unless the distribution of $\eta$ is heavy tailed itself. These predictions lend themselves to empirical investigation. Next, we show how the heavy tail feature can be carried over to the exchange rate return distribution.

[3] The result may appear specifically due to the assumption of the beta distribution in (4). What is crucial is that zero is in the support of the distribution. The result therefore also follows if we had assumed an exponential distribution, say.

### 2.1.1 Linearization

Why do typical macro models not display heavy tails in the solution of the model? Given the complexity of a typical micro based macro model, the solution often entails calibration after linearizing the model. Consider the effects of linearization for two different cases, one involving a product of two random variables, $SR$ say, and one involving a ratio $Q/A$. For the product case, a second order Taylor approximation does a perfect job, since at $S = s$, $R = r$

$$SR|_{s,r} = sr + r(S - s) + s(R - r) + (S - s)(R - r) = SR.$$

Next, consider the Taylor expansion for the ratio around $Q = q$ and $A = a$

$$\begin{aligned} \frac{Q}{A}|_{q,a} &= \frac{q}{a} + \frac{1}{a}(Q - q) - \frac{q}{a^2}(A - a) + \frac{1}{2}2\frac{q}{a^3}(A - a)^2 - \frac{1}{a^2}(Q - q)(A - a) \\ &= \frac{q}{a} + 2\frac{1}{a}Q - 2\frac{q}{a^2}A + \frac{q}{a^3}A^2 - \frac{1}{a^2}QA. \end{aligned}$$

The random ratio never shows up again, no matter how many terms in the Taylor approximation are used. Around a particular point the ratio can be approximated as precise as is desired, but the global property of the heavy tail implication stemming from a ratio is missed.

## 2.2 The Canonical Exchange Rate Model

The literature on foreign exchange rates was riddled by the Meese and Rogoff result regarding the low forecasting power of the fundamentals model in comparison to the simple random walk. Traditional regression analysis does not deliver very impressive coefficients on the composite fundamental, though the panel approach works better, see Mark and Sul (2001). Engel and West (2005) recently offered an explanation for this. We briefly recap their approach and subsequently combine it with the above macro model. From (3) we have

$$M_t = P_t + \gamma Y_t - \lambda I_t + \nu_t.$$

A similar relation holds abroad. Taking the difference of the two expressions yields the expression relative to a base country

$$m_t = p_t + \gamma y_t - \lambda i_t + u_t,$$

using lower case letters to denote relative country variables (where $u$ is the differential shock). The real exchange rate $z$ equals the nominal exchange rate minus the relative prices, i.e. $z = s - p$. Substituting this into the above yields

$$s_t = m_t + z_t - \gamma y_t + \lambda i_t - u_t.$$

Furthermore, let $\rho$ be the deviation from uncovered interest parity (UIP), so that

$$i_t = E_t[s_{t+1}] - s_t + \rho_t.$$

Using the UIP relation in the expression for the exchange rate gives monetary approach based Cagan style exchange rate expression

$$s_t = \frac{1}{1+\lambda}[m_t + z_t - \gamma y_t] + \frac{\lambda}{1+\lambda}\rho_t + \frac{\lambda}{1+\lambda}E_t[s_{t+1}]. \tag{8}$$

Forward iteration gives the standard no-bubbles solution to (8)

$$s_t = \frac{1}{1+\lambda}\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^j E_t[m_{t+j} + z_{t+j} - \gamma y_{t+j}] + \frac{\lambda}{1+\lambda}\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^j E_t[\rho_{t+j}]. \tag{9}$$

To fix ideas it helps to consider a specific stochastic processes for the fundamentals and the deviation from UIP. Suppose as in Engel, Mark and West (2007) that the fundamentals have a unit root and that the changes in the fundamental follow a stationary AR(1) process[4]

$$\Delta x_t = \phi \Delta x_{t-1} + \varepsilon_t,\ E_t\left[\varepsilon_{t+1}\right] = 0,\ \phi\epsilon(0,1). \tag{10}$$

Here $\Delta$ is the difference operator, and $\varepsilon$ represents i.i.d. composite shocks that drive the composite fundamental $x = m + z - \gamma y$.

Regarding the UIP relation, we assume an $AR(1)$ process driven by shocks and fundamentals

$$\rho_t = a\rho_{t-1} + \tau \Delta x_t + u_t,\ E_t\left[u_{t+1}\right] = 0,\ a\epsilon(0,1] \tag{11}$$

where the shocks $u$ are i.i.d. The UIP deviations depend on shocks and fundamentals as in Engel and West (2005). The fundamentals are part of this expression to signify that the risk premium is fundamentally based, see Burnside, Eichenbaum, Kleshchelski and Rebelo (2011) and Sarno, Schneider and Wagner (2012). We use the same vector of fundamentals as suggested by the money demand equation, but this is not necessary. The coefficient $\tau$ signifies the importance of the fundamental part in driving the UIP deviations and may vary across countries.

The solution to (9) under these assumptions yields the following expression for the exchange rate returns

$$\Delta s_t = (1-b)\frac{(1-ab)\,\phi + \tau b\phi}{(1-ab)\,(1-b\phi)}\Delta x_{t-1} + \frac{1+b\,(\tau - a)}{(1-ab)\,(1-b\phi)}\varepsilon_t - \frac{(1-a)\,b}{1-ab}\rho_{t-1} + \frac{b}{1-ab}u_t \tag{12}$$

and where the discount factor $b$ is shorthand for

$$b = \frac{\lambda}{1+\lambda}.$$

Engel and West (2005) and Engel, Mark and West (2007) argue and demonstrate that typical values of the discount factor $b$ are close to 1. By (12) this implies that the changes in the fundamentals become unimportant relative to their shocks $\varepsilon_t$, the lagged deviation from UIP and the exogenous UIP shocks. As a result the exchange rate can have the appearance of a random walk.

Whether the UIP deviations also affect the exchange rate returns via the fundamentals depends on the unobserved intercorrelation with the fundamentals. This is captured by the coefficient $\tau$. If $\tau$ is small or zero, the exchange rate returns simplify

$$\Delta s_t = (1-b)\frac{\phi}{1-b\phi}\Delta x_{t-1} + \frac{1}{1-b\phi}\varepsilon_t - \frac{(1-a)\,b}{1-ab}\rho_{t-1} + \frac{b}{1-ab}u_t.$$

This is one of the specific specifications discussed in Engel, Mark and West (2007). It clearly exposes the relevance of the height of the discount factor. If $b = 1$, then only time $t-1$ unobservable drive the time $t$ exchange rate returns.

The $AR(1)$ coefficient in the UIP process is, in a way, not essential. To see this consider the unit root case $a = 1$. In that case (12) simplifies to

$$\Delta s_t = \frac{(1-b)\,\phi + \tau b\phi}{1-b\phi}\Delta x_{t-1} + \frac{1+b\,(\tau - 1)}{(1-b)\,(1-b\phi)}\varepsilon_t + \frac{b}{1-b}u_t. \tag{13}$$

The exchange rate returns are still determined by the lagged fundamentals and the contemporaneous shocks. But in (13) lagged shocks only enter through the lagged

[4] For this assumption to be consistent with the macro model requires that the first differences in the monetary shocks $\Delta\nu_t$ also follow this $AR(1)$ process with parameter $\phi$.

change in the fundamentals, whereas in (12) these also enter through $\rho_{t-1}$. Whether or not $\Delta x_{t-1}$ is important, still depends on whether the discount factor $b$ is close to unity and the extent to which the fundamentals contribute to the UIP deviations, as measured by the coefficient $\tau$. Empirically, evidence in Sarno and Sojli (2009) indicates that the discount factor is near unity, while Sarno et al. (2012) show the significant relationship between risk premiums and traditional exchange rate fundamentals. Since (13) captures all the ingredients, we focus the subsequent analysis on this specification.

### 2.2.1 Regression Analysis

Suppose a regression analysis is applied to (13). What would this give? Let $\beta$ be shorthand for the coefficient on $\Delta x_{t-1}$ in (13). Suppose that $u_t$ and $\varepsilon_t$ are uncorrelated. A simple OLS regression of $\Delta s$ on the composite fundamentals $\Delta x_{-1}$ yields

$$\widehat{\beta} = \beta + \frac{\sum_i^n \Delta x_{i-1} \left( \frac{1+b(\tau-1)}{(1-b)(1-b\phi)} \varepsilon_t + \frac{b}{1-b} u_t \right)}{\sum_i^n \left( \Delta x_{i-1} \right)^2}.$$

Given that $\Delta x_{i-1}$ is independent from the $\varepsilon_i$ and $u_i$

$$p\lim \left[ \widehat{\beta} \right] = \frac{(1-b)\,\phi + \tau b \phi}{1 - b\phi}.$$

Hence, if the discount factor $b$ is close to 1, $\widehat{\beta}$ may be near zero if $\tau$ is small. In this case, the lagged fundamentals are not informative with regard to the current exchange rate return.[5] If $\widehat{\beta}$ is biased towards zero, this may increase the standard error of the estimate $\widehat{\beta}$ (see West, 2012). If the changes in the deviations from UIP $\Delta\rho$, however, are strongly correlated with the changes in the fundamentals $\Delta x$, or if $b$ is not so large, then $\widehat{\beta}$ can be significantly different from zero.

### 2.2.2 Limit Copula

From regression analysis one obtains the average response of the dependent variable to the average of the explanatory variable. Moreover, the analysis is dominated by observations from the center of the distribution. One interpretation of regression coefficients is in terms of moments:

$$\frac{E\left[XY\right]}{E\left[X^2\right]} = \frac{\int\int xyf(x,y)dxdy}{\int x^2 f(x)dx}.$$

Given the very particulars of the exchange rate model, this average response is small. Nevertheless large shocks $\varepsilon_t$ can greatly influence the exchange rate returns, especially if $b$ is close to unity, since in that case $\partial\Delta s_t/\partial\varepsilon_t \simeq \frac{1+b(\tau-1)}{(1-b)(1-b\phi)}$ by (13). On average positive and negative shocks net out and regression analysis may overlook their influence. How to uncover the influence of the larger movements in $\Delta x_{t-1}$, $\varepsilon_t$ and $u_t$ on $\Delta s_t$ and the signs of these movements?

Instead of using the first and second cross moment to elicit the dependency, one may try to use probabilities, which are the corresponding zero moments:

$$\frac{\int_q \int_q x^0 y^0 f(x,y)dxdy}{\int_q y^0 f(y)dy} = \frac{\Pr\left\{X > q, Y > q\right\}}{\Pr\left\{Y > q\right\}}.$$

If the large shocks are the primary drivers of the exchange rate changes, we can zoom in on the tail. This gives the so called the limit copula

[5] Note that this exercise is still predicated on having precise knowledge of the income elasticity $\gamma$.

$$\lim_{t\to\infty} \frac{\Pr\{X > q, Y > q\}}{\Pr\{Y > q\}} = \lim_{t\to\infty} \frac{\int_q \int_q x^0 y^0 f(x,y)dxdy}{\int_q y^0 f(y)dy},$$

see McNeil Frey and Embrechts (2005, Ch. 5.2). Note that this measure zooms in on one of the tails and therefore shocks in $x$ and $y$ are not averaged out. By changing the signs of the random variables in this conditional probability the other tail areas can be investigated as easily.

This suggests an alternative to regression analysis. The limit conditional probability measures the linkage through the most prominent shocks in $\varepsilon$. More precisely, we propose to measure the amount of asymptotic dependency between $\Delta s_t$ and $\Delta x_{t-1}$. Two random variables are asymptotically dependent if the probability on a large realization of one variable given a large realization of the other random variable is non-zero, even in the limit. Otherwise the two random variables are said to be asymptotically independent.

If we find that $\Delta s_t$ and $\Delta x_{t-1}$ are not asymptotically dependent, then this can be either due to the fact that the above model is incorrect and the shocks do not appear on both sides of the equation. Or that the dependency is lighter in nature, such as is the case if $\Delta s_t$ and $\Delta x_{t-1}$ are bivariate normally distributed with correlation $|\rho(\Delta s, \Delta x_{-1})|\epsilon(0,1)$. If random variables are multivariate normally distributed, the dependency vanishes asymptotically. So the alternative hypothesis is a composite hypothesis. The null hypothesis, though, implies strong dependency between the $\Delta s_t$ and $\Delta x_{t-1}$ in the tail area.

It is well known that the distribution of exchange rate returns exhibits heavy tails, to the extent that the probability of an extreme currency movement has a different order of magnitude than the normal distribution. We do not know whether this is also the case for the fundamentals. The tail property of the distribution of the composite fundamentals is somewhat difficult to measure due to the low frequency at which the fundamentals are available. Nevertheless, the fact that within the theory as presented above the shocks that drive the fundamentals also drive the exchange rate, the heavy tail feature should be on both sides of the equation.[6] We exploit this idea further below.

## 2.3 Asymptotic Dependence and Independence

Above we demonstrated that the distribution of individual macro variables can exhibit a power law, i.e.

$$\Pr\{X > q\} = 1 - F(q) \sim Aq^{-\alpha}, \text{ as } q \to \infty. \tag{14}$$

How does this translate to the composite fundamental $x$? According to Feller's Convolution Theorem (1971, VIII.8), if $X_1$ and $X_2$ are independent with common c.d.f. $F(q)$ from (14), then

$$\Pr\{X_1 + X_2 > q\} \sim 2Aq^{-\alpha}, \text{ as } q \to \infty. \tag{15}$$

Thus, to a first order at large thresholds $s$ the probability of the sum equals the sum of the marginal probabilities. In Appendix A, we offer an intuitive proof for this result. The main takeaway is that to a first order at large quantile levels $q$, only the univariate probability mass along the axes in the $(X_1, X_2)$ contributes to the probability of a large realization.

[6] Lux and Sornette (2002) argue that the fat tails of the exchange rate returns are driven by rational expectations bubbles, proposed by Blanchard and Watson (1982). However, the prediction resulting from their exogenous bubbles model is excessive compared to the conventional empirical findings.

As is further explained in the appendix, this result also holds if the random variables are not independent but exhibit multivariate regular variation (which is the multivariate analogue of the above power law assumption). A further result is that if the tail indices differ, the random variable with the thickest tail (smallest $\alpha$) dominates the sum that defines the composite fundamental. In short, if individual macro variables exhibit power law behavior, so does the distribution of the sum of these random variables. The monetary model is linear in the fundamentals and hence the convolution theorem applies if (some of) the fundamental variables do have a distribution with a heavy tail.

Specifically, suppose that in the exchange rate specification (13) the $\varepsilon$ and $u$ are independent Student-t distributed random variables with the same $\alpha > 2$ degrees of freedom (so that means and variances do exist). We allow for different scales (variances); it suffices to assume that $u$ has a scale $w$, while we keep the scale of $\varepsilon$ equal to the scale of a standard Student-t distributed random variable. Then for large $q$

$$\Pr\{|\varepsilon| > q\} = \Pr\{|u|/w^{1/\alpha} > q\} \sim 2cq^{-\alpha}, \text{ where } c = \Gamma\left(\left(\alpha+1\right)/2\right)\alpha^{(\alpha-1)/2}/\Gamma\left(\alpha/2\right)\sqrt{\alpha\pi}.$$

This implies that

$$\Pr\{|\frac{1+b\left(\tau-1\right)}{\left(1-b\right)\left(1-b\phi\right)}\varepsilon| > q\} \sim 2c\left(\frac{1+b\left(\tau-1\right)}{\left(1-b\right)\left(1-b\phi\right)}\right)^{\alpha} q^{-\alpha} \tag{16}$$

and

$$\Pr\{|\frac{b}{1-b}u| > q\} \sim 2cw\left(\frac{b}{1-b}\right)^{\alpha} q^{-\alpha}. \tag{17}$$

By Feller's convolution theorem

$$\Pr\{\Delta s_t \leq -q|\Delta x_{t-1}\} = \Pr\{\Delta s_t > q|\Delta x_{t-1}\} \sim c\left[\left(\frac{1+b\left(\tau-1\right)}{\left(1-b\right)\left(1-b\phi\right)}\right)^{\alpha} + w\left(\frac{b}{1-b}\right)^{\alpha}\right] q^{-\alpha}. \tag{18}$$

Alternatively, if the $\varepsilon$ and $u$ are independent zero mean normal shocks, then the weighted sum in (13) is also normally distributed (the $\Delta x_{-1}$ is a weighted average of past shocks $\varepsilon$ and is therefore also normally distributed). Laplace's classical expansion for the tail probabilities is the density to the quantile, which immediately shows that the tail probabilities are of exponential nature. Briefly, if say both $\varepsilon$ and $u$ are independent standard normally distributed, then for large $q$

$$\Pr\{\varepsilon > q\} = \Pr\{u > q\} \sim \frac{1}{q}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}},$$

so that

$$\Pr\{\varepsilon + u > q\} \sim \frac{2}{q}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{4}}, \text{ as } q \to \infty.$$

The important difference between adding the Student-t random variables and the normal case is that in the former case the power $\alpha$ remains as it is, but in the normal case the power changes from $-1/2$ in the exponent to $-1/4$. This explains why the dependency between normally distributed random variables eventually vanishes. For more details, see e.g. De Vries (2005).

Thus we can expect heavy tails in the exchange rate distribution if the fundamentals have distributions that are heavy tailed. Moreover, this induces asymptotic dependence between the right-hand-side and left-hand-side variables of (13). In general, it is not the case if the marginal distributions have heavy tails, that the random variables are asymptotically dependent (for example, Student-t distribute random

variables combined with a Gaussian copula, are correlated but asymptotically independent). But the linearity of the above model (13) combined with the marginal heavy tail feature does induce the asymptotic dependency between $\Delta s$ on the one hand and the $\Delta x_{t-1}$ components and their contemporaneous shocks on the other hand.

One way in which the asymptotic dependency between two random variables $Y$ and $X$ can be expressed is through the conditional tail probability

$$\lim_{q\to\infty} \frac{\Pr\{X > q, Y > q\}}{\Pr\{Y > q\}} > 0. \tag{19}$$

If instead $X$ and $Y$ are dependent, but the limit is zero, one speaks of asymptotic independence. This occurs for example if the random variables are bivariate standard normally distributed and where the correlation coefficient is not equal to 0, $-1$ or 1. The measure (19) was first proposed by in Huang (1992), is extensively discussed in McNeil, Frey and Embrechts (2005). It is applied in Poon, Rockinger and Tawn (2004) and Hartmann, Straetmans and de Vries (2010). At a finite quantile level $q$ the measure gives the probability on a joint excess, given that one of the two random variables exceeds $q$. One might ask what the relevance is of evaluating this probability in the limit. One contribution of statistical extreme value theory is that it shows that the limit conditional probability is a good approximation for the values at finite levels of $q$ in the range of the tail area of the joint distribution.

In (19) the dependency is measured along the diagonal, but one can measure along different rays by scaling the quantiles $q$ for $X$ and $Y$ differently. A popular alternative in applied work is to use the same probability level $p$ on an excess for the marginals instead of the same quantile level $q$. Define the quantiles $q_x$ and $q_y$ as follows

$$\Pr\{X > q_x\} = \Pr\{Y > q_y\} = p$$

and to evaluate

$$\lim_{p\to 0} \frac{\Pr\{X > q_x(p), Y > q_y(p)\}}{\Pr\{Y > q_y(p)\}} = \lim_{p\to 0} \frac{\Pr\{X > q_x(p), Y > q_y(p)\}}{p}. \tag{20}$$

Note that $q_x(p)$ and $q_y(p)$ do generally differ. If the scale of the random variables differs considerably, the measure (20) is smaller than (19). In the empirical work, we mainly rely on the latter (more conservative) measure conditioning on the same probability level, but results using the same quantiles are also available upon request. Similar measures exist for the other quadrants (simply switch the signs of $X$ and, or $Y$).

The idea is to demonstrate the asymptotic linkage between the exchange rate returns and the fundamentals, if at all present. To this end, suppose that the shock distributions do satisfy (16) and (17). We need the joint probability $\Pr\{\Delta s_t > q, \Delta x_{t-1} > q\}$ for the numerator of (19). Inserting (13) gives

$$\Pr\{\frac{(1-b)\,\phi + \tau b\phi}{1-b\phi}\Delta x_{t-1} + \frac{1+b\,(\tau-1)}{(1-b)\,(1-b\phi)}\varepsilon_t + \frac{b}{1-b}u_t > q, \Delta x_{t-1} > q\}.$$

This probability can be simplified for large quantiles $q$. Clearly, the time $t$ shocks $\varepsilon_t$ and $u_t$ are independent from the fundamentals $\Delta x_{t-1}$ at time $t-1$. Hence, we can invoke the Feller convolution theorem in the case that $\Delta x_{t-1}$, $\varepsilon_t$ and $u_t$ are heavy tailed. As (15) expresses and is further explained in the Appendix A, for large $q$ only the univariate probability mass along the three axes in the $(\Delta x_{t-1}, \varepsilon_t, u_t)$ space contributes to the joint probability $\Pr\{\Delta s_t > q, \Delta x_{t-1} > q\}$. But since the joint probability requires that not only must $\Delta s_t > q$ hold, but also at the same time $\Delta x_{t-1} > q$ must be satisfied. Along the three axes, this can only be guaranteed if

both conditions are satisfied along the $\Delta x_{t-1}$dimension. This implies that for large $q$

$$\Pr\{\Delta s_t > q, \Delta x_{t-1} > q\} = \Pr\{\frac{(1-b)\,\phi + \tau b\phi}{1-b\phi}\Delta x_{t-1} > q, \Delta x_{t-1} > q\}.$$

The joint probability becomes

$$\Pr\{\Delta s_t > q, \Delta x_{t-1} > q\} = \begin{cases} \Pr\{\frac{(1-b)\phi+\tau b\phi}{1-b\phi}\Delta x_{t-1} > q\} & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} < 1 \\ \Pr\{\Delta x_{t-1} > q\} & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} \geq 1 \end{cases}.$$

For the denominator and the numerator, we also need to determine the asymptotic expression for $\Pr\{\Delta x_{t-1} > q\}$. First note that since

$$\Delta x_{t-1} = \sum_{i=0}^{\infty} \phi^i \varepsilon_{t-1-i}.$$

By iterated application of the convolution theorem,

$$\Pr\{\Delta x_{t-1} > q\} \sim \frac{1}{1-\phi^\alpha} c q^{-\alpha}. \tag{21}$$

We can now obtain the limit conditional probability. Combining the above expressions gives for the numerator

$$\Pr\{\Delta s_t > q, \Delta x_{t-1} > q\} \sim \begin{cases} c\left(\frac{(1-b)\phi+\tau b\phi}{1-b\phi}\right)^\alpha q^{-\alpha} & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} < 1 \\ cq^{-\alpha} & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} \geq 1 \end{cases}.$$

We already have the probability for the denominator in (19) from (21). The conditional probability that there is jointly a large movement in the fundamentals $\Delta x$ and the exchange rate returns $\Delta s$ is asymptotically

$$\lim_{q\to\infty} \frac{\Pr\{\Delta s_t > q, \Delta x_{t-1} > q\}}{\Pr\{\Delta x_{t-1} > q\}} = \begin{cases} \left(\frac{(1-b)\phi+\tau b\phi}{1-b\phi}\right)^\alpha & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} < 1 \\ 1 & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} \geq 1 \end{cases}. \tag{22}$$

Consider the case that the discount factor $b$ is close to one. For example, as in Engel and West (2005) and Engel, Mark and West (2007) consider the parameter configuration $\tau = 0$, $\lambda = 10$, so that $b \simeq 0.9$, and autocorrelation $\phi = 0.5$. In this case the probability limit of the OLS coefficient estimate $\widehat{\beta}$ in the regression of $\Delta s_t$ on $\Delta x_{t-1}$ is only 1/11 or 0.09. A typical value of $\alpha$ as reported in the literature is 3.0. So the asymptotic dependence would be less than 0.001. But with a non-zero contribution of the fundamentals to the UIP risk premium or a somewhat lower value for the velocity of money parameter $\lambda$, these values can be quite different. For example, take $b = 1/2$, $\tau = 1/2$ and $\phi = 1/2$, then $p\lim\left[\widehat{\beta}\right] = 1/2$ and the asymptotic dependence becomes 1/8.

A problem with using $x$ is that this is a composite fundamental that relies on knowing the coefficients like $\gamma$ on the individual fundamentals. Measuring these coefficients is difficult, however, due to the low correlation between the fundamentals and the exchange rate returns. The above approach can also be applied to the individual fundamentals separately. This circumvents the need to measure the contribution of the specific fundamental to the composite. Moreover, since different theories suggest different sets of fundamentals, we can identify the relevance of each separately or jointly (to take care of their interaction[7]).

[7] For example, the joint contribution of monetary shocks and output shocks can be evaluated by means of the conditional probability

$$\frac{\Pr\{\Delta s_t > q\} + \Pr\{\Delta m_t > q\} + \Pr\{\Delta y_t > q\}}{1 - \Pr\{\Delta s_t \leq q, \Delta m_t \leq q, \Delta y_t \leq q\}}.$$

For example, suppose that the income elasticity of money demand is negligible ($\gamma = 0$) and that the real exchange rate $z$ is unobserved. Then, one shows that nevertheless

$$\lim_{q\to\infty} \frac{\Pr\{\Delta s_t > q, \Delta m_{t-1} > q\}}{\Pr\{\Delta m_{t-1} > q\}} = \begin{cases} \left(\frac{(1-b)\phi+\tau b\phi}{1-b\phi}\right)^{\alpha} & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} < 1 \\ 1 & \text{if } \frac{(1-b)\phi+\tau b\phi}{1-b\phi} \geq 1 \end{cases}$$

like in (22). Thus, one can also rely on a subset of the fundamentals to examine the extreme linkage between exchange rate returns and economic fundamentals.

This demonstrates the force of exploiting the tail properties of heavy tailed random variables. For example, if we replace $\Delta x$ in the above by $\Delta m$, and given the assumptions made before, we find the same limit probability. In contrast, if we would use $\Delta y$, the outcome may be zero. In the macro model we give reasons for $\Delta m$ and inflation to be heavy tail distributed, but not for $\Delta y$. If we find essentially zero dependency when we use $\Delta y$, this can be either due to the fact that output growth rates do not exhibit fat tails, or play no role in the determination of exchange rate returns. If, per contrast, for any of the specific fundamentals we do find a non-zero value for the asymptotic dependence, then we know that the variable is heavy tailed distributed and bears upon the exchange rate returns.

For the empirical analysis, we subsequently estimate the asymptotic dependence measure non-parametrically. We do not adopt the route of estimating a specific copula, for two reasons. The first is that we do not know what the correct copula is. Secondly, copula estimation captures the dependency globally and hence is primarily driven by the realizations in the center of the distributions, while the theory above has only something to say about the larger realizations. Thus this establishes only the association between the larger fundamental shocks and the exchange rates. But this is exactly when the interdependency counts the most. Only when we measure the tail shape by means of the tail index $\alpha$, we proceed on a semi-parametric basis.

# 3 Estimation

In this section, we explain the non-parametric estimation of the extreme linkage measure. To estimate the dependence measure (19), we can rewrite the linkage measure (19) at a finite $q$ as follows

$$L(q) = \frac{\Pr\{X > q, Y > q\}}{\Pr\{Y > q\}} = \frac{\Pr\{\min[X, Y] > q\}}{\Pr\{Y > q\}}. \tag{23}$$

This suggests a simple count estimator for the linkage measure

$$\widehat{L(q)} = \frac{\#\{\min[X, Y] > q\}}{\#\{Y > q\}}, \tag{24}$$

where we count (indicated by #) the number of pairs that satisfy the criterion for threshold $q$. The count procedure to estimate the (limit-) copula expresses the non-parametric approach that we take. Again, one needs to select a sufficiently high threshold $q$ to do the counting. In practice we use the ordered observations from both series as the thresholds and then use the graphical selection procedure by eyeballing the plot of $\widehat{L(q)}$ against $q$. In the end we use the cutoff percentages of 2.5% and 5%. The difference between asymptotically independent and dependent data is that in the former case the plot first lingers in the neighborhood of 0 (due to asymptotic independence) before slowly rising towards 1. While in the case of asymptotic dependence, not far from the origin the $\widehat{L(q)}$ graph jumps to a stable

plateau that indicates the level of asymptotic dependence, see Slijkerman et al. (2013) for further details on this methodology.

Alternatively, we condition on the same probability level, instead of the same quantile level. That is to evaluate

$$\lim_{p\to 0} \frac{\Pr\{X > q_x(p), Y > q_y(p)\}}{p}. \tag{25}$$

In the non-parametric setup that exclusively focusses on the tail area, let $n$ be the sample size and let $k$ be a sequence of numbers such that $k(n) \to \infty$ as $n \to \infty$, but $k(n)/n \to 0$. The probability $p$ is proxied by $k/n$. For the numerator, let $X_{(i)}$ and $Y_{(i)}$ denote the descending order statistics of $X_i$ and $Y_i$. The corresponding empirical distribution functions are respectively $F_n(x)$ and $G_n(y)$, say. The empirical counterpart of (25) then reads

$$1 - F_n(X_{(k)}) = 1 - G_n(Y_{(k)}) = \frac{k}{n+1}.$$

This suggests the following count estimator for (25)

$$\frac{\frac{1}{n+1}\sum_{i=1}^{n} 1_{\{X_i > X_{(k)}, Y_i > Y_{(k)}\}}}{k/(n+1)} = \frac{1}{k}\sum_{i=1}^{n} 1_{\{X_i > X_{(k)}, Y_i > Y_{(k)}\}}, \tag{26}$$

where $1_{\{\}}$ is the indicator function, see Ferreira and De Haan (2006, ch.7).

In the Appendix B, we also execute a formal statistical test to test against asymptotic dependence. Some simulated plots of the count estimator for the multivariate normal and Student distribution are also given. To deal with the small number of observations resulting from low frequency macroeconomic variables, we pool the data by region: combining European, Asian and Latin American countries in three groups. Observations used to estimate and test the extreme linkage between exchange rate returns and economic fundamentals, then range from 2630 to 3653. The panel analysis of the joint tail events is partly justified by the similarity of tail indices across countries.[8]

# 4 Empirical Results

## 4.1 Data

The data are monthly observations on the exchange rate, money supply (M2), production index, interest rate and consumer price index from the IMF International Financial Statistics (IFS). For most countries, variables are relative to the US and the data ranges from February 1974 to December 2007. For the European countries, variables are relative to Germany and the data ends in December 1998.

For most countries, the US dollar exchange rates from IFS are coded AE. The monetary aggregate is a sum of IFS codes 34A and B. Industrial or manufacturing production index, code 66 or 66Y, is used as a proxy for real income. For the interest rate, we use the money market rate or deposit rate (code 60B or 60L). The consumer price index (code 64) is used for the price level.

A list of 27 countries used in our study consists of Argentina, Austria, Bolivia, Brazil, Chile, Columbia, Denmark, Ecuador, Finland, France, India, Indonesia, Israel, Japan, Jordan, Korea, Malaysia, Mexico, Netherlands, Norway, Pakistan, Peru, Philippines, Spain, Sweden, UK and Venezuela.

[8] To save space, estimated tail indices for individual countries are available upon request.

## 4.2 Descriptive Statistics and Tail Indices

Table 1 provides descriptive statistics for the exchange rate returns $\Delta s$, relative money supply growth $\Delta m$, relative real income growth $\Delta y$, relative inflation $\Delta p$ and changes in the interest rate differential $\Delta i$. The Jarque-Bera (J-B) normality test rejects the null hypothesis of a normal population distribution at the 1% significance level (*p-values* equal zero) in all cases, while skewness and kurtosis describe characteristics of the non-normal distributions of the variables.

Table 2 reports estimated tail indices with asymptotic 95% confidence intervals using the DEdH estimator, see Dekkers, Einmahl and De Haan (1989). This estimator only uses the observations from the tail and applies both in the case of fat tails and in the case of exponentially thin tails or distributions with an endpoint. Furthermore, the estimator is asymptotically normally distributed, so that one can test for the type of tail.

Let $X_{(i)}$ be the descending order $X_{(1)} \geq X_{(2)} \geq ... \geq X_{(n)}$ from the sample of size $n$. Consider the upper tail, we then define the first two conditional log-moments empirical moments

$$H = \frac{1}{M} \sum_{i}^{M} \log \frac{X_{(i)}}{X_{(m)}}$$

and

$$K = \frac{1}{M} \sum_{i}^{M} (\log \frac{X_{(i)}}{X_{(m)}})^2,$$

where $X_{(m)}$ is a suitable threshold and there are $M$ observations above the threshold.

Note that $H$ is the familiar Hill (1975) estimator which is predicated on heavy tails. The DEdH estimator for the inverse of the tail index ($\gamma = 1/\alpha$) reads

$$\widehat{\gamma} = 1 + H + \frac{1}{2} \frac{\frac{K}{H}}{H - \frac{K}{H}}$$

and

$$\sqrt{M} \left( \widehat{\gamma} - \gamma \right)$$

is asymptotically normally distributed with variance $1 + \gamma^2$ (as long as $\gamma \geq 0$, i.e. as long as the support of the distribution is unbounded).

In Table 2, we show the estimation results using two typical tail sizes (5% and 2.5% of the overall sample size). Evidence indicates that not only are the exchange rate returns heavy-tailed distributed, but the fundamental variables also exhibit heavy tails. However, tail behaviors of the variables differ across regions. For Europe, with 95% confidence the null hypothesis of a thin-tailed distribution can be rejected for the depreciation side of the FX returns, inflation differential and interest rate differential. In the case of Asia, the null hypothesis can be rejected for all, but the output growth. For Latin America, the null hypothesis can be rejected for all variables.

For the case that the null hypothesis can be rejected, almost all the lower bounds of the 95% confidence interval are higher than 0.1. It indicates that with 95% confidence the tail index $\alpha$ ($\alpha = 1/\gamma$) is in the single digits for all these cases. For majority of the cases, the lower bounds of the 95% confidence interval are higher or marginally lower than 0.25. The tail index $\alpha$ is then likely to be below or marginally above 4 which means that the fourth and higher moments are infinite. Moreover, for the cases that the lower bounds of the 95% confidence interval are above 1/3, 1/2 or 1 there is the lack of convergence of the third, second or first moment (and higher moments), respectively.

### 4.3 Extreme Linkages

The distributions of exchange rate returns and macro fundamentals exhibit heavy tails. The linearity of log exchange rate models implies the asymptotic dependence between the exchange rate returns and macro fundamentals as discussed earlier. In this subsection, we examine whether large swings in exchange rates are associated with the heavy-tailed macro fundamentals by estimating the extreme linkage between the exchange rate returns $\Delta s$ and lagged economic fundamentals $\Delta x_{-1}$ using the linkage measure (25).

Instead of fixing the quantile level, here we show the alternative linkage measure conditioning on the same probability level (25).[9] We pool the data across countries and offer the estimation results for subsets of countries: European, Asian and Latin American countries. The low frequency of the macro data severely constrains the possibility to reliably estimate the extreme linkages. Note also that pooling across countries errs on the cautious side. If there is one country for which the series are asymptotically dependent and another country for which the variables are asymptotically independent, but where the scale of these variables is much larger, it follows that for the pooled data one likelily finds that the variables are asymptotically independent. But if both countries' variables have similar scale and are asymptotically dependent, then pooling does help, as there will be more observations in the tail area.

In Figure 1, we show the plots of the conditional probability (25) as $p \to 0$, for the depreciation side of the domestic currency. In the plots the y-axis shows the conditional probability (25) and the x-axis is $1-p$. The first row indicates whether we consider the relation of exchange rate returns $\Delta s$ (depreciation side) with increases $(+)$ or decreases $(-)$ in the lagged macro fundamental $\Delta x_{-1}$ under investigation. For the interest rate changes, we investigate both positive and negative relations as suggested by the exchange rate theories.

The plots in Figure 1 simply illustrate the differences between asymptotic dependence and asymptotic independence. For the cases of Europe and the lagged output growth the conditional probability approaches zero as we move deeper into the tail, i.e. $p \to 0$. According to the definition of asymptotic dependence the FX returns and lagged economic fundamentals are asymptotically independent. If the variables are asymptotically dependent the plot of conditional probabilities lingers around some positive number. The probability of joint extremes of $\Delta s$ and $\Delta x_{-1}$ is non-zero in limit. This occurs in the case of the relation between FX returns and increases in lagged relative money supply growth $\Delta m$, relative inflation $\Delta p$ and the lagged interest rate differential $\Delta i$ in Asia and Latin America.

Table 3 shows the linkage measure, i.e. the conditional probability (25), for the 5% and 2.5% tail probabilities for both depreciation and appreciation of the domestic currency, with the first column indicating the relation between the FX returns and lagged macro fundamentals. The results in the upper part of Table 3 support what we have concluded from Figure 1. Large depreciations of the domestic currency are asymptotically dependent with large increases in $\Delta m_{-1}$, $\Delta p_{-1}$ and $\Delta i_{-1}$ but not $\Delta y_{-1}$, and the links are exclusive for Asian and Latin American currencies. For the appreciation side in the lower panel, the limit condition probability is close to zero, indicating the case of asymptotic independence.

In this subsection, we find a strong connection between large swings in currencies and macro fundamentals. We uncover the asymmetric responses of the exchange rate returns to large changes in macro fundamentals. Furthermore, we find that the relation tends to vary from one region to another. The monetary variable

[9] Fixing the probability level has an advantage of avoiding problems of comparing variables with different scales. However, the results based on the linkage measure $L(q)$ from (23) applied to both raw data and unit Pareto transferred data are available upon request.

displays asymptotic dependency with the exchange rate with the limit conditional probability above 1/3 in most cases. Therefore, the paper lends further support to traditional exchange rate models in the vein of research initiated by Engel and West (2005) and Engel, Mark and West (2007).

# 5 Conclusion

The recent literature on exchange rates explains why macro fundamentals based regressions and shorter term forecasts perform poorly. This results from the non-stationarity of the drivers and the high discount factor. In standard exchange rate models the shocks to fundamentals drive both the explanatory variables and the dependent variable. The larger shocks nevertheless imply a strong connection between the larger movements in the exchange rate returns and the fundamentals. To uncover this linkage we estimate the asymptotic dependency between the variables.

Exchange rate returns exhibit distributions with fat tails. We first ask whether this also applies to the fundamentals that are supposedly driving the exchange rate returns. We offer some theoretical considerations from macroeconomics that might explain this data feature. Specifically, we consider a random Phillips curve coefficient in the spirit of Brainard (1967) in an otherwise standard monetary macroeconomic model. The unconditional distributions of macroeconomic variables like money growth and inflation are heavy tailed, even if the shocks are not. Since standard models of the exchange rate imply that the exchange rate returns are driven by the growth rates of the macro fundamentals, the exchange rate returns are heavy-tailed distributed by implication. We show that the heavy tails induce asymptotic dependence between the exchange rate returns and the macro shocks. Since these shocks drive both the exchange rate returns and the fundamentals, this provides a direct link between the exchange rate and the fundamentals.

In our empirical investigation, we use a data set consisting of monthly observations from 27 countries over the period 1974-2007. To deal with the small number of observations resulting from the low frequency of macroeconomic variables, we pool the data by region: combining European, Asian and Latin American countries in three groups. Based on the DEdH estimator of the tail shape parameter, we demonstrate that both exchange rate returns and fundamentals from the traditional exchange rate models are heavy tailed. The FX returns and lagged macro fundamentals are asymptotically dependent. Large downward swings in currency prices are strongly linked to larger movements in monetary variables, in particular for Asian and Latin American currencies.

We uncover that the strong connection between large swings in currencies and macro fundamentals is asymmetric, only appearing on the depreciation side. Further, the relation varies from one region to another. The monetary variable displays asymptotic dependence with the exchange rate with limit conditional probability above 1/3 in most cases. We conclude that the heavy tail feature of the FX returns can be, at least partially, attributed to the tail behavior of the macro fundamentals. The paper, furthermore, lends support to traditional exchange rate models in the vein of research initiated by Engel and West (2005) and Engel, Mark and West (2007).

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# Appendix A: Regular Variation and Tail Additivity

The monetary-approach exchange rate model is linear in the macro fundamental variables. Suppose that the distributions of the macroeconomic variables exhibit heavy tails. We first show that if the macroeconomic variables are i.i.d., then the exchange rate also has a distribution with heavy tails. Subsequently, we argue that this result still follows if the macroeconomic variables are (cross sectionally) dependent. From an economic point of view the independence case is, in a way, the hardest case to treat. Since if, say, the macroeconomic fundamentals are driven by a common component that is heavy-tailed distributed, then it is almost immediate that this property is transferred to the distribution of the exchange rate.

We adopt the following general notion of heavy tails. A distribution function $F(x)$ is said to exhibit heavy tails if its tails vary regularly at infinity. The upper tail varies regularly at infinity with tail index $\alpha$ if [10]

$$\lim_{t\to\infty} \frac{1-F(tx)}{1-F(t)} = x^{-\alpha},\ x>0 \text{ and } \alpha>0. \tag{27}$$

Regular variation implies that the tail of the distribution changes at a power rate. This contrasts with, e.g., the normal distribution that has tail probabilities that decline at an exponential rate. The number of bounded moments of $F(.)$ is finite and equals the integer value of $\alpha$, i.e. the $\alpha$-moment.[11] One checks that the Student-t distribution satisfies (27) by using L'Hôpital's rule and the expression for the density, for the Pareto distribution this is trivial.

Random variables with regularly varying distributions satisfy an important additivity property in the tail area. Suppose a distribution has heavy tails, so that

$$\Pr\{X>x\} = 1-F(x) \sim Ax^{-\alpha},\ as\ x\to\infty. \tag{28}$$

According to Feller's Convolution Theorem (1971, VIII.8), if $X_1$ and $X_2$ are i.i.d. with c.d.f. $F(x)$ which has regularly varying tails as in (28), then

$$\Pr\{X_1+X_2>s\} \sim 2As^{-\alpha},\ as\ s\to\infty. \tag{29}$$

This result says that in the tail area the probability of the sum of random variables is equal to the sum of the marginal probabilities. If $X$ and $Y$ are i.i.d. and if $X$ has a tail index of $\alpha$ and $Y$ has a lighter tail (e.g. has a hyperbolic tail with a higher power than $\alpha$ or even has an exponential type tail), then analogous to the proof of (7) one shows that

$$\Pr\{X+Y>s\} \sim As^{-\alpha}. \tag{30}$$

In this case the convolution is dominated by marginal distribution of the heavier tail.

Some intuition for the Feller theorem is as follows. Let $X$ be i.i.d. Pareto distributed with scale $A=1$. Then for large $s$

$$1-\Pr\{X_1\le s, X_2\le s\} = 1-\left(1-s^{-\alpha}\right)^2 \approx 2s^{-\alpha}$$

since the term $s^{-2\alpha}$ is of smaller order. This is why only the (univariate) probability mass along the axes counts. To a first order, the probability mass above the line $X_1+X_2=s$ is also determined by how much probability mass is aligned along the

[10] For the lower tail, $\lim_{t\to\infty} F(-tx)/F(-t) = x^{-\alpha}$, $x>0$ and $\alpha>0$.

[11] For instance, the Pareto distribution satisfies the Power law and has a number of bounded moments equal to an integer of $\alpha$. The Student-t distribution has moments equal to its degree of freedom. Per contrast, the thin-tailed normal distribution has all moments bounded.

axes above this line, i.e. $2s^{-\alpha}$. The probability mass above the line away from the axes is of smaller order. To see this, note that

$$\Pr\{\lambda X_1 > s, (1-\lambda) X_2 > s\} = \Pr\{\lambda X_1 > s\} \Pr\{(1-\lambda) X_2 > s\} = O\left(s^{-2\alpha}\right).$$

The convolution result (29) and (30) are very powerful. To give an illustrative example, consider the quasi-reduced-form specification of the exchange rate models in the logarithmic form

$$\Delta s = \varphi_1 \Delta m_{-1} + \varphi_2 \Delta y_{-1} + w, \tag{31}$$

that is behind (12) from the main text. The $w$ is the composite of the shocks.

The convolution theorem holds that if the distributions of $\Delta m_{-1}$, $\Delta y_{-1}$ and $w$ adhere to (27), are independent and

$$\alpha_s = \alpha_m = \alpha_y = \alpha_w, \tag{32}$$

then

$$\begin{aligned} \Pr\{\Delta s > t\} &= P\{\varphi_1 \Delta m_{-1} + \varphi_2 \Delta y_{-1} + w > t\} \\ &\sim (\varphi_1^\alpha + \varphi_2^\alpha + 1)\, t^{-\alpha}. \end{aligned} \tag{33}$$

If, however, for example

$$\alpha_s = \alpha_m = \alpha_w < \alpha_y$$

then

$$\alpha_s = \min(\alpha_m, \alpha_y, \alpha_w) = \alpha_m = \alpha_w$$

We find that the tail shape of the exchange rate returns $s$ is governed by the tail shape of the fundamentals and the noise distributions with the heaviest tails.

The convolution results (29) and (33) assume that the macroeconomic variables from (31) are independent random variables. This is often not the case due to endogeneity. Consider therefore the multivariate extension of (27). Suppose that the vector $\mathbf{x}$ of fundamental variables is multivariate regularly varying in the sense that

$$\lim_{t\to\infty} \frac{1 - F(t\mathbf{x})}{1 - F(t\mathbf{1})} = W(\mathbf{x}),\ \mathbf{x} > 0,$$

where $W(.)$ is a function such that $W(\lambda \mathbf{x}) = \lambda^{-\alpha} W(\mathbf{x})$, $\alpha > 0$, $\lambda > 0$ and $\mathbf{1}$ is the unit vector. Suppose the marginal distributions are as in (28) so that the scales are of the same order, and all the marginal distributions have the same tail index $\alpha$. Then for any non-zero weight vector $\mathbf{w}$, $P\{\mathbf{w}^T \mathbf{x} > s\} \sim C s^{-\alpha}$, as $s \to \infty$. Here the scale constant $C$ depends on the type of dependence and can no longer be determined as in (33), i.e. it requires specific knowledge of the copula. Nevertheless, the weighted sum of macroeconomic variables that determines the distribution of the exchange rate still has a Pareto like upper tail with the tail index $\alpha$. Moreover, it is still the case that if the marginal distributions have different tail indices, the fundamental with the heaviest tail determines the tail index of the exchange rate returns.

In addition, we like to note that the a-temporal convolution result still holds when the economic variables are stationary time series. This is so since the convolution is a 'cross-section' like aggregation at a specific point in time. As the exchange rate and macroeconomic variables display bouts of quiescence and turbulence, changes in the economic variables are often captured by ARMA-GARCH type of models. From the convolution result (29), one can show that when time series are not i.i.d. but serially dependent, the occurrence of extremes may affect

the distribution of order statistics, but not the tail index $\alpha$. That is the exchange rate return distribution still has hyperbolic tails.

The convolution theorem can nevertheless also be used to study the aggregation of time series over time. Suppose for example that $m$ follows the following $MA(1)$ process

$$m_t = \varepsilon_t + \gamma\varepsilon_{t-1}, \text{ and } \gamma > 0,$$

and where the innovations $\varepsilon$ are i.i.d. with distribution function as in (28). Then, by Feller's Convolution Theorem

$$\Pr\{m > x\} \sim A\,(1+\gamma^{\alpha})\,x^{-\alpha}, \ as\ x \to \infty.$$

Furthermore, $P\{m_t + m_{t-1} > x\} \sim A\,[1 + (1+\gamma)^{\alpha} + \gamma^{\alpha}]\,x^{-\alpha}$, $as\ x \to \infty$. Note that the convolution results show that the scales of the random variables change due to the moving average process, but not the tail index $\alpha$.

More complicated time series models can also be handled. For instance, Engle's (1982) original contribution modeled the inflation rate by the ARCH process. De Haan et al. (1989) showed that the tail of the stationary distribution of the ARCH process is regularly varying. Basrak et al. (2002) discuss the convolution of GARCH processes.

Table 1: Descriptive statistics of FX returns and fundamentals

| | | | | | |
|---|---|---|---|---|---|
| | | | Europe | | |
| | $\Delta s$ | $\Delta m$ | $\Delta y$ | $\Delta p$ | $\Delta i$ |
| Mean | 0.0022 | 0.0013 | 0.0007 | 0.0025 | 5.94E-06 |
| Median | 0.0005 | 0.0002 | -0.0007 | 0.0019 | -1.67E-05 |
| Max | 0.2208 | 0.1653 | 1.0209 | 0.0423 | 0.0581 |
| Min | -0.0778 | -0.1455 | -1.1087 | -0.0282 | -0.0545 |
| Std. Dev. | 0.0171 | 0.0230 | 0.1639 | 0.0058 | 0.0021 |
| Skewness | 2.3743 | 0.3072 | 0.1498 | 1.0496 | 1.2346 |
| Kurtosis | 24.3787 | 7.0497 | 14.0916 | 8.0280 | 424.7925 |
| J-B test | 53774.99 | 1838.54 | 13803.97 | 3328.72 | 19570649 |
| Obs. | 2691 | 2630 | 2691 | 2691 | 2640 |
| | | | Asia | | |
| | $\Delta s$ | $\Delta m$ | $\Delta y$ | $\Delta p$ | $\Delta i$ |
| Mean | 0.0052 | 0.0092 | 0.0032 | 0.0043 | -1.36E-05 |
| Median | 0.0000 | 0.0061 | 0.0047 | 0.0015 | -1.67E-05 |
| Max | 0.8025 | 1.4456 | 0.6695 | 0.2414 | 0.2934 |
| Min | -0.3487 | -0.4870 | -0.6170 | -0.0686 | -0.3972 |
| Std. Dev. | 0.0359 | 0.0382 | 0.0705 | 0.0172 | 0.0139 |
| Skewness | 5.8698 | 15.1343 | -0.0437 | 5.1946 | -4.9130 |
| Kurtosis | 102.0137 | 590.2759 | 9.1150 | 47.3121 | 401.2999 |
| J-B test | 1482118 | 51222990 | 5270.38 | 306753.4 | 22739418 |
| Obs. | 3578 | 3555 | 3382 | 3554 | 3438 |
| | | | Latin America | | |
| | $\Delta s$ | $\Delta m$ | $\Delta y$ | $\Delta p$ | $\Delta i$ |
| Mean | 0.0312 | 0.0338 | -0.0007 | 0.0310 | -3.28E-05 |
| Median | 0.0051 | 0.0180 | -0.0026 | 0.0098 | -1.67E-05 |
| Max | 2.7221 | 1.1846 | 1.8877 | 1.5942 | 69.4263 |
| Min | -0.2480 | -0.8963 | -1.6044 | -0.1062 | -116.4456 |
| Std. Dev. | 0.1172 | 0.0802 | 0.0884 | 0.0737 | 3.2854 |
| Skewness | 10.4076 | 2.9762 | 1.3663 | 7.2883 | -16.6156 |
| Kurtosis | 164.5514 | 42.5933 | 112.6683 | 96.8776 | 898.2192 |
| J-B test | 4038409 | 237586.3 | 1546446 | 1350814 | 1.16E+08 |
| Obs. | 3653 | 3557 | 3084 | 3592 | 3469 |

Table 2: Tail index estimates of FX returns and fundamentals

| | | Europe | | Asia | | Latin America | |
|---|---|---|---|---|---|---|---|
| | | 5 percent Tail | 2.5 percent Tail | 5 percent Tail | 2.5 percent Tail | 5 percent Tail | 2.5 percent Tail |
| $\Delta s$ | left | -0.0756 | -0.0545 | 0.3637 | 0.4147 | 0.3891 | 0.3589 |
| | | (-0.2448, 0.0935) | (-0.2926, 0.1835) | (0.2078, 0.5196) | (0.1910, 0.6384) | (0.2336, 0.5446) | (0.1418, 0.5760) |
| | right | 0.3743 | 0.3193 | 0.4668 | 0.4077 | 0.4724 | 0.5278 |
| | | (0.1942, 0.5545) | (0.0698, 0.5688) | (0.3052, 0.6285) | (0.1846, 0.6308) | (0.3121, 0.6326) | (0.2967, 0.7589) |
| $\Delta m$ | left | 0.1047 | -0.1426 | 0.5966 | 0.6383 | 0.6913 | 0.6704 |
| | | (-0.0669, 0.2762) | (-0.3863, 0.1011) | (0.4255, 0.7677) | (0.3918, 0.8848) | (0.5127, 0.8699) | (0.4203, 0.9206) |
| | right | -0.1019 | -0.0398 | 0.5254 | 0.4564 | 0.1965 | 0.1720 |
| | | (-0.2734, 0.0696) | (-0.2812, 0.2017) | (0.3594, 0.6913) | (0.2280, 0.6848) | (0.0468, 0.3462) | (-0.0388, 0.3828) |
| $\Delta y$ | left | -0.1839 | 0.1323 | 0.1329 | 0.1285 | 0.4512 | 0.5087 |
| | | (-0.3555, -0.0124) | (-0.1074, 0.3721) | (-0.0188, 0.2845) | (-0.0859, 0.3428) | (0.2791, 0.6234) | (0.2597, 0.7576) |
| | right | -0.3434 | 0.1241 | 0.1443 | 0.2095 | 0.4721 | 0.3902 |
| | | (-0.5218, -0.1650) | (-0.1154, 0.3637) | (-0.0076, 0.2961) | (-0.0077, 0.4267) | (0.2986, 0.6457) | (0.1519, 0.6284) |
| $\Delta p$ | left | 0.2649 | 0.2544 | 0.2528 | 0.1345 | 0.3818 | 0.3884 |
| | | (0.0904, 0.4394) | (0.0091, 0.4996) | (0.1016, 0.4039) | (-0.0751, 0.3441) | (0.2254, 0.5382) | (0.1667, 0.6100) |
| | right | 0.1792 | 0.0834 | 0.2874 | 0.1823 | 0.2560 | 0.3179 |
| | | (0.0078, 0.3506) | (-0.1551, 0.3219) | (0.1350, 0.4398) | (-0.0289, 0.3935) | (0.1052, 0.4068) | (0.1011, 0.5347) |
| $\Delta i$ | left | 0.4986 | 0.5333 | 0.2529 | 0.1382 | 1.7535 | 1.7648 |
| | | (0.3080, 0.6892) | (0.2599, 0.8067) | (0.0987, 0.4070) | (-0.0751, 0.3516) | (1.4536, 2.0535) | (1.3386, 2.1911) |
| | right | 0.3297 | 0.4591 | 0.3937 | 0.0690 | 1.8741 | 1.6113 |
| | | (0.1501, 0.5093) | (0.1936, 0.7246) | (0.2330, 0.5543) | (-0.1428, 0.2809) | (1.5585, 2.1897) | (1.2128, 2.0098) |

Table 3: Linkage measure between FX returns and lagged fundamentals

| **Depreciation** | | | | | | |
|---|---|---|---|---|---|---|
| | Europe | | Asia | | Latin America | |
| | 5 percent Tail | 2.5 percent Tail | 5 percent Tail | 2.5 percent Tail | 5 percent Tail | 2.5 percent Tail |
| $(\Delta s, \Delta m_{-1}, +)$ | 0.05 | 0.00 | 0.34 | 0.32 | 0.53 | 0.38 |
| $(\Delta s, \Delta y_{-1}, -)$ | 0.00 | 0.00 | 0.03 | 0.04 | 0.00 | 0.00 |
| $(\Delta s, \Delta p_{-1}, +)$ | 0.08 | 0.03 | 0.42 | 0.36 | 0.58 | 0.41 |
| $(\Delta s, \Delta i_{-1}, +)$ | 0.08 | 0.05 | 0.19 | 0.24 | 0.43 | 0.38 |
| $(\Delta s, \Delta i_{-1}, -)$ | 0.08 | 0.05 | 0.15 | 0.13 | 0.14 | 0.05 |

| **Appreciation** | | | | | | |
|---|---|---|---|---|---|---|
| | Europe | | Asia | | Latin America | |
| | 5 percent Tail | 2.5 percent Tail | 5 percent Tail | 2.5 percent Tail | 5 percent Tail | 2.5 percent Tail |
| $(\Delta s, \Delta m_{-1}, +)$ | 0.05 | 0.02 | 0.08 | 0.06 | 0.05 | 0.03 |
| $(\Delta s, \Delta y_{-1}, -)$ | 0.04 | 0.00 | 0.05 | 0.00 | 0.05 | 0.03 |
| $(\Delta s, \Delta p_{-1}, +)$ | 0.02 | 0.01 | 0.07 | 0.03 | 0.01 | 0.00 |
| $(\Delta s, \Delta i_{-1}, +)$ | 0.08 | 0.05 | 0.05 | 0.02 | 0.05 | 0.05 |
| $(\Delta s, \Delta i_{-1}, -)$ | 0.09 | 0.06 | 0.06 | 0.03 | 0.03 | 0.03 |

Figure 1: Plots of the conditional probability