Testing for a Change in Persistence under Non-Stationary Volatility

Robert Taylor

School of Economics
University of Nottingham

(joint work with G. Cavaliere)

1. Introduction

This paper is about testing for a change in persistence in the presence of non-stationary volatility in the driving shocks.

We look at the effects of a general class of non-stationary variance processes, including trending variances and (smooth transition) variance breaks, on the ratio-based persistence change tests of Kim (2002,JoE), Busetti and Taylor (2004,JoE) and Leybourne and Taylor (2004,EL).

In finite samples it is likely to be hard to differentiate between a change in persistence and, especially, a break in volatility.

Non-stationary volatility introduces a deformation in the time domain which can effect severe over-sizing, although statistics do not diverge unless there is a change in persistence.

We propose wild bootstrap-based versions of the persistence change tests. The form of the volatility process is not assumed known to the practitioner.

Motivation

A number of recent applied studies have argued that volatility in many macro and financial series has evolved over time.

Kim and Nelson (1999) and McConnell and Perez Quiros (2000) argue that the volatility of US GDP has declined over the past 20 years.

McConnell *et al.* (1999) report decline in volatility in all major components of GDP. Chauvet and Potter (2001) find decreased volatility in aggregate consumption, income and employment.

Watson (1999) finds evidence of a decrease in the variability of short-term US interest rates since 1985. Sensier and van Dijk (2004) find that most of the variables in the Stock and Watson (1999) data-set show a decline in variability since the 1980s.

Structure of the Talk

1. Introduction
2. The Heteroskedastic Persistence Change Model
3. Impact on Standard Persistence Change Tests
4. Bootstrap Persistence Change Tests
5. Simulation Results
6. Empirical Illustration
7. Conclusions

2. The Heteroskedastic Persistence Change Model

Generalising Kim (2000,p.99), we consider the null hypothesis H_0, that a scalar series y_t is the sum of a deterministic component, d_t, and a short memory ($I(0)$) component which displays time-varying unconditional volatility:

$$\begin{aligned} y_t &= d_t + z_{t,0}\ , \quad t = 1, ..., T & (1) \\ d_t &= \mathbf{x}_t'\beta & (2) \\ z_{t,0} &= \sigma_t \varepsilon_t & (3) \end{aligned}$$

We consider two alternative hypotheses: (i) H_{01}, a change in persistence from $I(0)$ to $I(1)$ behaviour at time $t = \lfloor \tau^* T \rfloor$; (ii) H_{10}, a change in persistence from $I(1)$ to $I(0)$ behaviour at time $t = \lfloor \tau^* T \rfloor$.

Both cases may be expressed conveniently within a generalization of the persistence change data generating process (DGP) of Kim (2000,p.100)

$$y_t = d_t + z_{t,1}, \quad t = 1, ..., \lfloor \tau^* T \rfloor, \ \tau^* \in (0, 1) \tag{4}$$

$$y_t = d_t + z_{t,2}, \quad t = \lfloor \tau^* T \rfloor + 1, ..., T. \tag{5}$$

The $I(0)$-$I(1)$ persistence change alternative is obtained for

$$\begin{aligned} \mathsf{H}_{01} \quad : \quad & z_{t,2} = z_{t-1,2} + \sigma_t \varepsilon_t \\ & z_{t,1} = \sigma_t u_t \\ & z_{\lfloor \tau^* T \rfloor,2} = z_{\lfloor \tau^* T \rfloor,1} \end{aligned} \tag{6}$$

while the $I(1)$-$I(0)$ alternative is given by

$$\begin{aligned} \mathsf{H}_{10} \quad : \quad & z_{t,1} = z_{t-1,1} + \sigma_t \varepsilon_t \\ & z_{t,2} = \sigma_t u_t + z_{\lfloor \tau^* T \rfloor,1}. \end{aligned} \tag{7}$$

Both (6) and (7) embody end-effect corrections, as are also used in Banerjee, Lumsdaine and Stock (1992,p.278) and BT, which ensure that a given realization of the process will not display a spurious sharp jump in level at the break point.

We assume that the following conditions hold on σ_t, ε_t and d_t in (1)-(3). The error u_t is assumed to satisfy the same condition as for ε_t with long run variance λ_u^2.

Assumption $\mathcal{V}$. The term $\{\sigma_t\}$ satisfies $\sigma_{\lfloor sT \rfloor} = \omega(s)$, where $\omega(\cdot) \in \mathcal{D}$ (cadlag) is a non-stochastic function with a finite number of points of discontinuity, and satisfies a (uniform) first-order Lipschitz condition except at the points of discontinuity.

Assumption $\mathcal{E}$. $\{\varepsilon_t\}$ is a zero-mean, unit variance, strictly stationary mixing process with $E|\varepsilon_t|^p < \infty$ for some $p > 2$ and with mixing coefficients $\{\alpha_m\}$ satisfying $\sum_{m=0}^{\infty} \alpha_m^{2(1/r-1/p)} < \infty$ for some $r \in (2, 4]$, $r \leq p$. The long run variance $\lambda_\varepsilon^2 := \sum_{k=-\infty}^{\infty} E(\varepsilon_t \varepsilon_{t+k})$ is strictly positive and finite.

Assumption $\mathcal{X}$. $\mathbf{x}_t$ is a $(k+1) \times 1$ deterministic vector with $\mathbf{x}_{1t} = 1$, all t, and satisfying the condition that there exists a scaling matrix δ_T and a bounded piecewise continuous function $F(\cdot)$ on $[0, 1]$ such that $\delta_T \mathbf{x}_{\lfloor \cdot T \rfloor} \rightarrow \mathbf{x}(\cdot)$ uniformly on $[0, 1]$, and where, for all $\tau \in \Lambda$, $\Lambda = [\tau_l, \tau_u]$ the compact subset of $[0, 1]$ used in section 3 below, $\int_0^\tau \mathbf{x}(s) \mathbf{x}(s)' ds$ and $\int_\tau^1 \mathbf{x}(s) \mathbf{x}(s)' ds$ are both positive definite.

Assumptions $\mathcal{E}$ and $\mathcal{X}$ are both standard.

Standard theory also assumes *global stationarity*. This entails that $\omega(s) = \sigma$ for all s.

We generalize this set-up through $\mathcal{V}$ which only requires the innovation variance to be bounded and to display a limited number of jumps.

Examples ($s \in [0,1]$ in all cases):

1. *Single Volatility Shift*: $\omega(s)^2 := \sigma_0^2 + (\sigma_1^2 - \sigma_0^2)\mathbb{I}(s \geq \tau)$, $\tau \in (0,1)$.

2. *Double Volatility Shift*: $\omega(s)^2 := \sigma_0^2 + \left(\sigma_1^2 - \sigma_0^2\right)\mathbb{I}(\tau_1 \leq s < \tau_2) + \left(\sigma_2^2 - \sigma_0^2\right)\mathbb{I}(\tau_2 \leq s \leq 1)$, $\tau_1, \tau_2 \in (0,1)$, $\tau_1 < \tau_2$.

3. *Trending Volatility*: $\omega(s)^2 := \sigma_0^2 + \left(\sigma_1^2 - \sigma_0^2\right) s$.

The variance function, $\omega(\cdot)$, is assumed non-stochastic although this is not strictly necessary.

Many exogenous stochastic volatility processes are also allowed. Here our results should be read as *conditional* on a given realization of $\omega(\cdot)$. Valid examples include Markov-switching variances and models of (near-) integrated stochastic volatility.

Example 4. Near-Integrated Stochastic Volatility:

$$\omega(s)^2 := \sigma_0^2 \exp(\nu J_c(s)) \ , \ s \in [0,1]$$

for $\nu > 0$, with $J_c(\cdot)$ either an Ornstein-Uhlenbeck process ($c > 0$) or a Brownian motion ($c = 0$).

In what follows we define the *variance profile* of the process as

$$\eta(s) := \left(\int_0^1 \omega(r)^2 \, dr \right)^{-1} \int_0^s \omega(r)^2 \, dr.$$

Notice that $\eta(s) = s$ under stationary volatility while it deviates from s in the presence of non-stationary volatility.

The denominator, $\omega^2 := \int_0^1 \omega(r)^2 \, dr$, is, by Assumption $\mathcal{V}$, equal to the limit of $T^{-1} \sum_{t=1}^T \sigma_t^2$, i.e. the (asymptotic) average innovation variance.

3. Impact on Persistence Change Tests

Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004) [BT], test the I(0) null against the alternative of an I(0)-I(1) change in persistence at some point τ, rejecting for large values of the statistic

$$\mathcal{K}(\tau) := \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} (\breve{S}_t(\tau))^2}{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} (\hat{S}_t(\tau))^2}$$

where

$$\breve{S}_t(\tau) := \sum_{i=\lfloor \tau T \rfloor + 1}^{t} \breve{\varepsilon}_{i,\tau}, \quad \hat{S}_t(\tau) := \sum_{i=1}^{t} \hat{\varepsilon}_{i,\tau}$$

with $\hat{\varepsilon}_{t,\tau}$ and $\breve{\varepsilon}_{t,\tau}$ the OLS residuals from regressing y_t on $\mathbf{x}_t$, for $t = 1, \ldots, \lfloor \tau T \rfloor$, and $t = \lfloor \tau T \rfloor + 1, ..., T$, respectively.

Where the true breakpoint is unknown consider:

$$
\begin{aligned}
K_1 &\equiv \int_{\tau \in \Lambda} \mathcal{K}(\tau) d\tau \\
K_2 &\equiv \ln \left\{ \int_{\tau \in \Lambda} \exp\left(0.5\mathcal{K}(\tau)\right) d\tau \right\} \\
K_3 &\equiv \max_{\tau \in \Lambda} \mathcal{K}(\tau)
\end{aligned}
$$

where $\Lambda = [\tau_l, \tau_u]$ is a closed subset of $(0, 1)$, as standard solutions to the problem where a nuisance parameter is present only under the alternative. It is assumed that where a change in persistence occurs, that $\tau \in \Lambda$.

These tests are $O_p(T^2)$ under the I(0)-I(1) model.

BT propose tests which are $O_p(T^2)$ against an I(1)-I(0) change: formed as above, replacing $\mathcal{K}(\cdot)$ with $(\mathcal{K}(\cdot))^{-1}$ throughout. Denote these K_1', K_2' and K_3'. K_1, K_2 and K_3 are each of $O_p(1)$ against the I(1)-I(0) model, while K_1', K_2' and K_3' are of $O_p(1)$ against the I(1)-I(0) model.

Consequently, where the direction of change is unknown, BT propose

$$K_{j+3} = \max\{K_j, K_j'\},\ j = 1, 2, 3,$$

in each case rejecting for large values of the statistics. Each is of $O_p(T^2)$ under either the I(1)-I(0) or the I(0)-I(1) model.

Leybourne and Taylor (2004) [LT] propose modifications of the above statistics whereby the numerator and denominator of $\mathcal{K}(\tau)$ are scaled by (long-run) variances estimators appropriate for the given sub-sample. Call these tests $\mathcal{K}^*(\tau)$, K_j^*, $j = 1, ..., 6$, and $K_j'^*$, $j = 1, ..., 3$.

LT show that this greatly improves the finite sample size properties of the tests when ε_t is serially correlated.

The null distributions of the resulting statistics are unaltered. However, rates of consistency are slowed from T^2 to T/ℓ, where ℓ is the bandwidth used in the LRV estimator.

LT recommend the use of $\ell = 1$ in practice.

In what follows we will make use the limiting process

$$B_\omega(s) := \frac{\int_0^s \omega(r) dB(r)}{\left(\int_0^1 \omega(r)^2 dr\right)^{1/2}}$$

which, up to a scaling factor, is the diffusion solving the stochastic differential equation, $dB_\omega(s) = \omega(r) \, dB(r)$, $B(\cdot)$ a standard Brownian motion.

Since B_ω is Gaussian, has independent increments and unconditional variance $E(B_\omega(s)^2) = \eta(s)$, B_ω is called a time-deformed Brownian motion.

Under global stationarity B_ω reduces to B.

The limiting null distribution of the ratio statistic for known τ under non-stationary volatility satisfying Assumption $\mathcal{V}$ is:

$$\mathcal{K}(\tau) \xrightarrow{w} L_{\omega}(\tau) := \frac{(1-\tau)^{-2}\int_{\tau}^{1}\breve{B}_{\omega}(s,\tau)^{2}\,ds}{\tau^{-2}\int_{0}^{\tau}\hat{B}_{\omega}(s,\tau)^{2}\,ds}$$

where $\breve{B}_{\omega}$ and $\hat{B}_{\omega}$ are the residuals from the non-orthogonal Hilbert projections of $B_{\omega}(s)$ on the space spanned by $\mathbf{x}(s)$, $s \in [\tau, 1]$ and $s \in [0, \tau]$, respectively.

The statistic of Leybourne and Taylor (2004) satisfies

$$\mathcal{K}^{*}(\tau) \xrightarrow{w} \kappa_{\omega}(\tau)\, L_{\omega}(\tau) =: L_{\omega}^{*}(\tau)$$

where $\kappa_{\omega}(\tau) := \frac{1-\tau}{\tau}[\eta(\tau)/(1-\eta(\tau))]$ is the ratio of the asymptotic average volatilities in the first and second sub-samples.

Under non-stationary volatility the ratio statistics therefore do not have their usual asymptotic null distributions. Rather, their distributions depend on the sample path of the volatility process, $\omega(\cdot)$. Only where $\omega(\cdot) = \sigma$ do pivotal distributions obtain.

For the Leybourne-Taylor statistic the additional term $\kappa_\omega(\tau)$ also depends on the time-path of the volatility process, and equals unity if and only if the asymptotic average volatilities are equal in the first and second sub-samples.

In the special case of a single break in volatility occurring at time $\lfloor \tau_\varepsilon T \rfloor$, it can be shown that the limiting null distribution of the Leybourne-Taylor statistic for a change in persistence at time $\lfloor \tau_\varepsilon T \rfloor$ is pivotal.

For the unknown τ case, the limiting null distributions of the K_j, $j = 1, ..., 6$, K'_j, $j = 1, ..., 3$, K^*_j, $j = 1, ..., 6$ and K'^*_j, $j = 1, ..., 3$, statistics follow using the CMT. For example,

$$\mathcal{K}_1 \xrightarrow{w} \sup_{\tau \in \Lambda} L_\omega(\tau).$$

Again, these all depend on the sample path of the volatility process, reducing to the representations for the homoskedastic case only where $\omega(\cdot) = \sigma$.

The rates of consistency of all of the tests under both I(0)-I(1) shifts and I(1)-I(0) shifts are exactly as before, although the limiting distributions of the (scaled) statistics depend on the underlying volatility process. Consequently, one would expect the finite sample power properties of the tests to differ under non-stationary volatility from the homoskedastic case.

4. Bootstrap Persistence Change Tests

In the paper we discuss bootstrap procedures for both a known and unknown possible persistence change point. For the purposes of the talk I will just outline the latter case.

We make use of the wild bootstrap (or fixed regressor - Hansen 2000) procedure which replicates the pattern of heteroskedasticity present in the original data in the re-sampled data. We show that this allows us to conduct asymptotically pivotal inference under the null in the presence of non-stationary stochastic volatility of unknown form satisfying Assumption $\mathcal{V}$.

In what follows we confine our discussion for expositional purposes, but without loss of generality, to the $\mathcal{K}_1$ test.

The steps in our bootstrap procedure are as follows:

1. Compute the full sample residuals, say $\tilde{\varepsilon}_t$, obtained by regressing y_t on $\mathbf{x}_t$ for $t = 1, ..., T$. The bootstrap sample is then generated as

$$y_t^b := \tilde{\varepsilon}_t w_t \ , \ t = 1, ..., T,$$

with $\{w_t\}_{t=1}^T$ an independent $N(0, 1)$ sequence.

2. For $\tau \in [\tau_l, \tau_u]$, let $\breve{\varepsilon}_{t,\tau}^b$ be defined as the residuals obtained from the OLS projection of y_t^b on $\mathbf{x}_t$ for $t = \lfloor T\tau \rfloor + 1, ..., T$; similarly, let $\hat{\varepsilon}_{t,\tau}^b$ be defined as the residuals obtained from the OLS projection of y_t^b on $\mathbf{x}_t$ for $t = 1, ..., \lfloor T\tau \rfloor$.

3. Construct the bootstrap analogue of $\mathcal{K}_1$; *viz*,

$$\mathcal{K}_1^b := \max_{s \in \{\lfloor \tau_l T \rfloor, \dots, \lfloor \tau_u T \rfloor\}} \mathcal{K}^b(s/T)$$

where

$$\mathcal{K}^b(\tau) := \frac{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \left(\sum_{i=\lfloor \tau T \rfloor + 1}^{t} \breve{\varepsilon}_{i,\tau}^b \right)^2}{\lfloor \tau T \rfloor^{-2} \sum_{t=1}^{[\tau T]} \left(\sum_{i=1}^{t} \hat{\varepsilon}_{i,\tau}^b \right)^2}.$$

4. The associated bootstrap p-value is given by $p_{1,T}^b := 1 - G_{1,T}^b(\mathcal{K}_1)$, where $G_{1,T}^b(\cdot)$ denotes the cdf of $\mathcal{K}_1^b$.

In practice the cdf $G^b_{1,T}(\cdot)$ will be unknown. However, it can be approximated in the usual way.

5. Generate N conditionally independent bootstrap statistics, $\mathcal{K}^b_{i,1}(\tau)$, $i = 1, ..., N$, computed as above but from

$$y^b_{i,t} := \tilde{\varepsilon}_t w_{i,t} \ , \ t = 1, ..., T,$$

with $\{\{w_{i,t}\}_{t=1}^T\}_{i=1}^N$ a doubly independent $N(0,1)$ sequence. The simulated bootstrap p-value is then given by

$$\tilde{p}^b_{1,T} := N^{-1} \sum_{i=1}^N \mathbb{I}\left(\mathcal{K}^b_{i,1}(\tau) \geq \mathcal{K}_1\right).$$

By standard arguments, see e.g. Hansen (1996), $\tilde{p}^b_{1,T}$ is consistent for $p^b_{1,T}$ as $N \to \infty$.

In the paper we establish that each bootstrap statistic attains the same limiting null distribution as its original counterpart statistic, and that the bootstrap p-values are asymptotically pivotal and uniformly distributed. Consequently the bootstrap tests are correctly sized for samples of sufficiently large dimension.

Moreover, we show that all of the bootstrap statistics are of $O_p(1)$ under persistence change processes (either switches from I(0)-I(1) or vice versa). This implies that each bootstrap test is also consistent at the same rate as its original counterpart.

Under persistence change, the limiting distributions of the (scaled) bootstrap statistics are not the same as the original scaled statistics, however, so finite sample power may be expected to differ.

5. Simulation Results

Standard tests run using the published asymptotic 0.05 level CVs.

Results for tests run on de-meaned data. Very similar results were obtained for de-trended data.

10,000 MC reps and 400 BS reps.

As is usual, we set $\Lambda = [0.2, 0.8]$.

For the standardized ratio tests we set $\ell = 1$, as suggested by Leybourne and Taylor (2004), and, accordingly, we also set a bandwidth of $\ell^b = 1$ in their bootstrap counterparts.

We report finite sample size and power properties of the persistence change tests for the following non-stationary volatility process:

Model 1 (single break): set $\sigma_0^2 = 1$ and $\sigma_1 \in \{1, 1/3, 3\}$ and $\tau_\varepsilon = 0.5$. Allows positive ($\sigma_1 < 1$) and negative ($\sigma_1 > 1$) breaks, and also the no break case ($\sigma_1 = 1$).

Model 2 (trending volatility): set $\sigma_0^2 = 1$ and vary $\sigma_1 \in \{1/3, 3\}$, allowing positively and negatively trending variances.

Model 3 (exponential near-integrated SV): set $\sigma_0^2 = 1$, with $\nu = 5$ and vary $c \in \{0, 10\}$.

Size Properties

Data were generated according to the DGP

$$
\begin{aligned}
y_t &= \sigma_t \varepsilon_t \\
\varepsilon_t &= \phi \varepsilon_{t-1} + v_t - \theta v_{t-1}, \;\; v_t \sim IIDN(0,1)
\end{aligned}
$$

for the cases of σ_t outlined above, for $(\phi,\ \theta) \in \{(0,0),\ (0.5,0),\ (0,0.5)$.

Model 1:

Extreme over-sizing seen in many cases in the basic tests. Bad under-sizing also seen. Size distortions vary slightly with ϕ and θ: increase for $\phi > 0$ and decrease for $\theta > 0$, relative to $\phi = \theta = 0$.

Studentized tests appear much better behaved avoiding the large over-size problems that are seen with the basic tests. Also somewhat less dependence on ϕ and θ.

Bootstrap tests also avoid the size distortions seen in the basic tests. Appear to also deliver a further improvement on the studentized tests.

Model 2:

In general, linear trending volatility has a lower impact on the size of the standard tests than abrupt changes. The basic conclusions drawn for the relative performance of the various tests is pretty much as for Model 1.

Model 3:

Severe over-sizing is again seen in the basic tests which is greatest, other things equal, for the case of $c = 0$.

Studentized tests again better behaved but still significantly over-sized for $c = 0$.

Bootstrap tests again give a further improvement, especially for $T = 200$.

Power Properties

Here we consider the switching AR(1) DGP:

$$\begin{aligned} y_t &= \rho_t y_{t-1} + z_{t,0} \\ z_{t,0} &= \sigma_t \varepsilon_t, \;\; \varepsilon_t \sim NIID(0,1) \end{aligned}$$

and where

$$\rho_t = \begin{cases} 0.8, & t = -100, ..., \lfloor \tau^* T \rfloor \\ 1.0, & t = \lfloor \tau^* T \rfloor + 1, ..., T \end{cases}$$

Same cases for σ_t considered as before. We vary the persistence change-point among $\tau^* \in \{0.25, 0.50, 0.75\}$.

For homoskedastic errors there is a modest drop in power for the basic bootstrap vis-à-vis basic ratio tests, so that our bootstrap procedure does not seem to cause significant power losses when unnecessary.

Studentized tests have much lower power than the basic and basic bootstrap tests under homoskedasticity. This ranking seems to hold true, in general, for the non-stationary volatility models considered.

The effect of non-stationary volatility on power is mixed and depends on whether we look at raw or size-adjusted power for the basic tests (recall that these can be heavily over-sized and also badly under-sized).

Different volatility models have different impacts on the power rankings. E.g., under Model 3 the size-adjusted power of the basic tests is much lower than for their bootstrap equivalents. Under Models 1 and 2 the opposite tends to be the case.

6. Empirical Illustration

Monthly U.S. producer price inflation series from Stock and Watson (1999). We identify the data by the same reference codes as in Stock and Watson (1999, pp.35-41): PWFSA, PWFCSA, PWIMSA, PWFXSA, PWCMSA, PW160A and PW150A. Sensier and van Dijk (2004), reject the hypothesis of fixed variability against a single structural break in the variance (at an unknown point) at the 5 % level for all of these series.

All of the statistics were computed on de-meaned data. For each outcome two bootstrap p-values are reported: p_{hom}, obtained from a standard bootstrap, and p_{het} using our wild bootstrap.

For the standardized ratio tests we set $\ell = 1$ and $\ell^b = 1$.

Data are graphed in Figure 1 together with an estimate of the variance profile for each series.

For each series the estimated variance profile shows substantial deviations from the 45° line which pertains to a constant variance process.

The profiles for the first five series all suggest multiple breaks in variance with a positive followed by negative break in variance within the first third of the sample. The fifth series also shows evidence of a further positive followed by negative variance break in the last third of the data.

The estimated variance profiles for the final two series both follow a relatively smooth (more so for PW160A) arc above the 45° line, consistent with negatively trending volatility, or possibly a smooth-transition variance break centred on the early 1980s.

For all but PWFXSA, visual evidence in the graphs is consistent with volatility being lower on average from the early 1980s onwards than prior to that in line with arguments on the efficiency of the Fed's monetary policy in the Volcker-Greenspan era and the stabilizating of the US economy.

PWFSA: of the standard and bootstrap persistence change tests all but $\mathcal{K}_2$ reject the null of constant $I(0)$ at the 5% level. Generally, slightly larger p-values (less significant) for the bootstrap tests: even more so for standardized tests.

PWFCSA: standard tests reject the constant $I(0)$ null, as do all but the bootstrap $\mathcal{K}_j'^b$, $j = 1, 2, 3$, tests which reject only at the 10% level. The standardized tests are again less significant overall, as are the bootstrap tests.

PWIMSA: there are strong differences between the standard tests and bootstrap tests. The standard ratio tests clearly indicate the rejection of the constant $I(0)$ null. In contrast, the studentized and bootstrap tests deliver very little evidence against the null.

PWFXSA: all of the standard and bootstrap tests reject at the 5% level. Similar results obtain for the standardized ratio tests. Again though, smaller p-values for the standard tests.

For the remaining series (PWCMSA, PW150A and PW160A) none of the standard or bootstrap tests are able to reject the constant $I(0)$ null at the 5% level.

Overall, the bootstrap and standardized tests yielded fewer significant outcomes than did the standard tests, as we might expect from the patterns of estimated variance profiles.

7. Conclusions

We have analyzed the behaviour of tests for a change in persistence in cases where the innovation process displays non-stationary volatility.

The tests were shown to have non-pivotal limiting null distributions in such cases, with a tendency in some of the tests to large over-size.

Consequently it is hard for practitioners to discriminate between true persistence change processes and constant persistence processes which display non-stationary volatility on the basis of these tests.

Wild bootstrap tests proposed as a solution to the inference problem. Shown to work well in finite samples being approximately correctly sized in the presence of a range of time-varying volatility processes, yet not significantly losing power relative to the standard tests against persistence changes.

Table 1: Empirical Size of Standard Persistence Change Tests: De-meaned Data. Tests Based on Asymptotic 5% Critical Values.

				Model 1			Model 2		Model 3	
T	ϕ	θ		$\delta = 1$	$\delta = 1/3$	$\delta = 3$	$\delta = 1/3$	$\delta = 3$	$c = 0$	$c = 0$
100	0.0	0.0	$\mathcal{K}_1$	3.5	61.7	0.2	35.2	0.1	39.1	16.0
			$\mathcal{K}_1'$	3.3	0.3	60.2	0.1	31.6	39.7	15.0
			$\mathcal{K}_4$	3.5	52.4	48.5	26.3	21.5	69.5	20.4
			$\mathcal{K}_1^*$	2.9	3.8	8.0	3.7	3.2	7.9	5.5
			$\mathcal{K}_1'^*$	2.7	6.0	3.4	3.0	2.2	9.3	4.9
			$\mathcal{K}_4^*$	2.2	5.2	4.8	2.9	2.8	10.6	4.6
	0.5	0.0	$\mathcal{K}_1$	9.2	67.3	2.1	44.0	1.0	40.9	22.6
			$\mathcal{K}_1'$	8.1	1.5	65.0	1.3	40.3	41.5	18.6
			$\mathcal{K}_4$	11.8	60.7	57.0	34.9	32.3	71.9	26.7
			$\mathcal{K}_1^*$	2.9	4.2	7.1	3.5	3.8	7.7	4.4
			$\mathcal{K}_1'^*$	2.6	6.1	2.7	3.4	2.5	8.3	3.9
			$\mathcal{K}_4^*$	3.3	4.9	4.5	3.1	3.0	8.3	4.4
	0.0	0.5	$\mathcal{K}_1$	0.8	45.8	0.0	20.2	0.0	35.0	9.6
			$\mathcal{K}_1'$	0.5	0.0	43.5	0.0	16.1	35.8	7.7
			$\mathcal{K}_4$	0.4	32.8	31.1	11.9	9.6	63.1	10.8
			$\mathcal{K}_1^*$	1.7	2.7	4.2	2.0	1.9	6.9	4.2
			$\mathcal{K}_1'^*$	0.9	4.1	1.6	1.9	0.8	7.6	3.8
			$\mathcal{K}_4^*$	0.7	3.1	2.2	1.1	0.8	7.4	4.2

Table 1: Empirical Size of Standard Persistence Change Tests: De-meaned Data. Tests Based on Asymptotic 5% Critical Values.

				Model 1			Model 2		Model 3	
T	ϕ	θ		$\delta=1$	$\delta=1/3$	$\delta=3$	$\delta=1/3$	$\delta=3$	$c=0$	$c=0$
200	0.0	0.0	$\mathcal{K}_1$	4.9	60.8	0.5	33.8	0.5	37.4	15.6
			$\mathcal{K}_1'$	3.3	0.2	59.3	0.1	33.6	37.8	14.1
			$\mathcal{K}_4$	4.9	49.9	48.6	24.3	24.3	65.3	19.8
			$\mathcal{K}_1^*$	3.9	5.2	8.7	3.6	4.8	12.1	6.0
			$\mathcal{K}_1'^*$	2.9	7.1	5.2	4.1	3.0	11.9	6.6
			$\mathcal{K}_4^*$	3.6	6.7	8.3	4.2	4.5	14.4	6.8
	0.5	0.0	$\mathcal{K}_1$	7.8	63.9	1.2	38.5	1.1	39.4	17.6
			$\mathcal{K}_1'$	5.2	0.8	60.8	0.5	37.8	39.8	16.7
			$\mathcal{K}_4$	8.4	53.5	51.9	28.4	28.6	67.6	23.9
			$\mathcal{K}_1^*$	4.1	4.6	8.1	4.1	5.0	9.3	5.3
			$\mathcal{K}_1'^*$	2.8	6.5	3.9	3.0	2.7	9.3	4.7
			$\mathcal{K}_4^*$	3.4	6.2	6.4	3.8	4.3	10.5	5.8
	0.0	0.5	$\mathcal{K}_1$	2.8	52.1	0.0	24.1	0.0	34.0	12.2
			$\mathcal{K}_1'$	1.1	0.0	50.4	0.0	22.7	36.2	8.5
			$\mathcal{K}_4$	1.6	38.5	39.5	14.9	14.0	62.2	13.4
			$\mathcal{K}_1^*$	2.7	3.4	7.1	2.3	2.8	8.7	5.2
			$\mathcal{K}_1'^*$	2.1	5.1	4.1	3.1	2.3	11.8	4.3
			$\mathcal{K}_4^*$	1.8	3.9	4.7	2.0	3.0	12.4	4.9

Table 2: Empirical Size of Bootstrap Persistence Change Tests: De-meaned Data.

				Model 1			Model 2		Model 3	
T	ϕ	θ		$\delta=1$	$\delta=1/3$	$\delta=3$	$\delta=1/3$	$\delta=3$	$c=0$	$c=0$
100	0.0	0.0	$\mathcal{K}_1$	2.1	3.2	2.7	2.9	1.5	6.4	2.2
			$\mathcal{K}_1'$	1.9	2.4	3.2	1.9	2.4	6.0	2.7
			$\mathcal{K}_4$	1.5	3.2	3.2	2.9	2.4	10.1	2.1
			$\mathcal{K}_1^*$	5.8	6.9	8.2	6.6	6.1	8.7	7.0
			$\mathcal{K}_1'^*$	5.4	7.1	7.0	6.1	5.6	9.8	6.9
			$\mathcal{K}_4^*$	5.7	7.0	8.3	6.7	5.5	9.3	8.1
	0.5	0.0	$\mathcal{K}_1$	3.5	6.9	4.4	5.6	2.8	11.7	3.5
			$\mathcal{K}_1'$	3.5	4.3	5.7	3.5	4.9	11.1	3.7
			$\mathcal{K}_4$	4.0	6.9	5.8	5.5	4.9	19.2	3.6
			$\mathcal{K}_1^*$	6.0	6.8	6.9	6.2	5.5	6.5	5.5
			$\mathcal{K}_1'^*$	5.4	6.2	5.3	5.6	4.8	8.0	5.5
			$\mathcal{K}_4^*$	5.6	6.3	6.3	6.0	5.5	7.1	5.8
	0.0	0.5	$\mathcal{K}_1$	0.5	0.7	0.2	0.4	0.0	2.9	1.0
			$\mathcal{K}_1'$	0.2	0.2	0.2	0.0	0.2	2.8	1.0
			$\mathcal{K}_4$	0.0	0.7	0.2	0.4	0.1	4.7	0.8
			$\mathcal{K}_1^*$	3.9	4.4	3.6	4.1	3.6	5.4	4.7
			$\mathcal{K}_1'^*$	2.4	4.9	2.8	2.8	2.6	6.3	4.7
			$\mathcal{K}_4^*$	2.7	4.6	3.3	2.8	2.3	6.2	4.7

Table 2: Empirical Size of Bootstrap Persistence Change Tests: De-meaned Data.

				Model 1			Model 2		Model 3	
T	ϕ	θ		$\delta = 1$	$\delta = 1/3$	$\delta = 3$	$\delta = 1/3$	$\delta = 3$	$c = 0$	$c = 0$
200	0.0	0.0	$\mathcal{K}_1$	3.4	3.2	3.9	3.5	3.1	6.4	3.3
			$\mathcal{K}_1'$	2.6	3.4	3.6	2.9	2.6	6.5	3.4
			$\mathcal{K}_4$	3.6	3.2	3.6	3.5	2.6	9.0	3.3
			$\mathcal{K}_1^*$	5.7	6.0	6.3	5.4	5.1	7.1	5.4
			$\mathcal{K}_1'^*$	4.5	6.0	6.3	5.0	5.4	7.1	6.0
			$\mathcal{K}_4^*$	5.5	5.7	7.9	5.4	5.3	7.6	6.6
	0.5	0.0	$\mathcal{K}_1$	4.3	4.5	5.3	5.1	4.7	10.2	4.0
			$\mathcal{K}_1'$	3.1	3.8	4.9	3.2	3.4	7.7	4.1
			$\mathcal{K}_4$	4.8	4.5	5.0	5.2	3.5	13.9	4.4
			$\mathcal{K}_1^*$	5.4	4.4	5.4	5.1	5.1	5.3	4.9
			$\mathcal{K}_1'^*$	4.0	5.1	5.0	3.8	4.3	5.4	4.4
			$\mathcal{K}_4^*$	4.5	4.9	5.7	4.5	4.7	5.4	4.9
	0.0	0.5	$\mathcal{K}_1$	1.8	1.0	0.7	1.5	1.4	2.3	1.6
			$\mathcal{K}_1'$	1.0	0.6	1.4	0.8	1.2	1.9	1.2
			$\mathcal{K}_4$	0.5	1.0	1.4	1.5	1.2	2.9	1.1
			$\mathcal{K}_1^*$	3.4	3.3	4.9	3.1	3.5	4.8	4.5
			$\mathcal{K}_1'^*$	3.3	3.4	4.6	3.9	3.4	5.9	3.8
			$\mathcal{K}_4^*$	3.6	3.4	3.8	3.1	3.5	5.5	4.3

Table 3: Empirical Power of Standard Persistence Change Tests: De-meaned Data. Tests Based on Asymptotic 5% Critical Values.

			Model 1			Model 2		Model 3	
T	τ^*		$\delta = 1$	$\delta = 1/3$	$\delta = 3$	$\delta = 1/3$	$\delta = 3$	$c = 0$	$c = 0$
100	0.25	$\mathcal{K}_1$	78.8	96.8	50.0	93.7	57.8	71.0	78.8
		$\mathcal{K}_1'$	48.1	30.1	82.2	31.9	74.4	52.8	51.1
		$\mathcal{K}_4$	83.7	96.3	83.9	93.9	82.4	90.9	85.0
		$\mathcal{K}_1^*$	27.8	30.4	25.3	29.9	26.9	29.9	29.5
		$\mathcal{K}_1'^*$	9.8	9.7	10.4	9.7	11.4	12.8	13.1
		$\mathcal{K}_4^*$	26.8	27.0	25.1	28.3	26.2	28.1	27.2
	0.50	$\mathcal{K}_1$	76.7	97.3	34.0	92.5	53.0	68.1	76.8
		$\mathcal{K}_1'$	30.4	25.8	52.1	19.1	50.0	43.5	34.5
		$\mathcal{K}_4$	75.5	96.7	62.6	91.5	67.0	86.5	79.3
		$\mathcal{K}_1^*$	29.5	33.2	23.4	33.3	26.8	32.3	30.8
		$\mathcal{K}_1'^*$	5.6	7.9	4.1	5.8	5.7	7.6	6.6
		$\mathcal{K}_4^*$	24.1	28.2	19.2	27.6	22.4	27.2	25.8
	0.75	$\mathcal{K}_1$	59.3	91.3	18.8	85.9	30.7	60.4	60.6
		$\mathcal{K}_1'$	8.4	3.8	37.5	2.7	25.6	30.5	14.5
		$\mathcal{K}_4$	57.6	89.4	43.1	81.7	42.4	78.9	63.9
		$\mathcal{K}_1^*$	19.9	24.0	14.3	24.4	15.4	22.6	19.2
		$\mathcal{K}_1'^*$	1.9	2.8	1.2	2.1	1.8	1.9	1.1
		$\mathcal{K}_4^*$	13.8	18.0	9.0	17.3	11.0	17.4	12.8

Table 3: Empirical Power of Standard Persistence Change Tests: De-meaned Data. Tests Based on Asymptotic 5% Critical Values.

			Model 1			Model 2		Model 3	
T	τ^*		$\delta = 1$	$\delta = 1/3$	$\delta = 3$	$\delta = 1/3$	$\delta = 3$	$c = 0$	$c = 0$
200	0.25	$\mathcal{K}_1$	89.1	97.9	71.0	96.6	76.8	77.4	88.8
		$\mathcal{K}_1'$	49.0	31.1	80.0	35.3	71.1	54.2	53.7
		$\mathcal{K}_4$	91.2	98.0	87.2	96.8	88.0	92.8	91.6
		$\mathcal{K}_1^*$	44.8	45.6	41.8	46.7	43.9	44.6	46.7
		$\mathcal{K}_1'^*$	14.8	12.5	14.9	13.6	15.8	15.3	14.6
		$\mathcal{K}_4^*$	40.9	41.9	39.0	43.6	41.3	42.3	42.2
	0.50	$\mathcal{K}_1$	89.0	100.0	52.1	96.8	70.8	73.3	88.2
		$\mathcal{K}_1'$	29.8	26.3	42.4	20.0	45.7	43.5	32.5
		$\mathcal{K}_4$	88.4	99.5	65.4	96.3	78.1	88.5	88.7
		$\mathcal{K}_1^*$	47.9	52.4	42.5	53.6	46.7	46.0	51.0
		$\mathcal{K}_1'^*$	8.0	9.6	4.2	7.5	7.8	7.2	8.2
		$\mathcal{K}_4^*$	40.8	44.5	34.7	45.5	39.0	40.3	45.3
	0.75	$\mathcal{K}_1$	73.6	95.9	26.1	92.4	41.9	61.3	72.0
		$\mathcal{K}_1'$	2.2	0.6	17.3	0.3	11.0	23.8	5.3
		$\mathcal{K}_4$	70.6	95.2	34.3	90.5	42.8	77.8	69.2
		$\mathcal{K}_1^*$	34.9	43.9	27.6	41.7	29.5	36.8	36.4
		$\mathcal{K}_1'^*$	0.3	0.9	0.3	0.3	0.5	0.9	0.5
		$\mathcal{K}_4^*$	26.1	33.1	19.9	32.8	20.7	29.6	26.8

Table 4: Size-Adjusted Power of Standard Persistence Change Tests: De-meaned Data.

			Model 1			Model 2		Model 3	
T	τ^*		$\delta = 1$	$\delta = 1/3$	$\delta = 3$	$\delta = 1/3$	$\delta = 3$	$c = 0$	$c = 0$
100	0.25	$\mathcal{K}_1$	81.4	83.9	76.2	84.6	82.7	22.0	69.1
		$\mathcal{K}_1'$	49.6	51.7	50.3	51.7	53.9	15.3	41.5
		$\mathcal{K}_4$	84.4	84.4	55.1	86.8	69.2	24.3	73.8
		$\mathcal{K}_1^*$	39.7	39.3	24.3	44.5	35.3	19.5	30.0
		$\mathcal{K}_1'^*$	15.6	8.9	15.2	13.0	18.3	12.3	13.8
		$\mathcal{K}_4^*$	37.5	28.2	23.0	39.0	31.3	20.4	29.8
	0.50	$\mathcal{K}_1$	79.8	85.8	64.1	84.7	78.7	20.9	66.4
		$\mathcal{K}_1'$	32.0	46.7	19.8	38.0	32.5	9.9	26.8
		$\mathcal{K}_4$	78.9	86.0	25.5	85.4	53.8	21.0	66.6
		$\mathcal{K}_1^*$	41.8	43.6	24.7	48.7	36.5	22.0	32.3
		$\mathcal{K}_1'^*$	8.1	6.1	5.6	6.8	8.6	6.3	8.6
		$\mathcal{K}_4^*$	34.6	28.2	17.7	38.5	28.6	19.1	28.3
	0.75	$\mathcal{K}_1$	63.3	69.2	47.7	71.1	58.0	15.0	48.9
		$\mathcal{K}_1'$	8.4	10.9	7.6	9.0	11.3	6.6	8.3
		$\mathcal{K}_4$	59.6	69.3	9.3	71.4	24.1	14.7	45.0
		$\mathcal{K}_1^*$	30.2	33.2	17.6	36.2	23.2	15.5	22.2
		$\mathcal{K}_1'^*$	1.9	0.9	1.3	1.3	1.9	2.3	2.0
		$\mathcal{K}_4^*$	21.3	18.5	11.5	24.5	16.0	11.9	17.9

Table 4: Size-Adjusted Power of Standard Persistence Change Tests: De-meaned Data.

			Model 1			Model 2		Model 3	
T	τ^*		$\delta=1$	$\delta=1/3$	$\delta=3$	$\delta=1/3$	$\delta=3$	$c=0$	$c=0$
200	0.25	$\mathcal{K}_1$	90.2	89.6	87.0	92.0	90.0	27.2	79.9
		$\mathcal{K}_1'$	50.3	50.2	52.2	52.8	53.5	14.1	41.6
		$\mathcal{K}_4$	92.9	89.9	61.5	92.9	76.2	24.2	82.4
		$\mathcal{K}_1^*$	48.0	46.7	32.1	50.4	40.0	26.0	39.9
		$\mathcal{K}_1'^*$	15.9	7.8	13.9	12.8	16.2	8.1	10.6
		$\mathcal{K}_4^*$	44.4	36.8	30.2	44.0	42.8	20.8	35.0
	0.50	$\mathcal{K}_1$	89.4	92.6	73.0	90.1	88.3	25.9	77.8
		$\mathcal{K}_1'$	29.7	45.0	14.3	35.0	27.9	7.4	22.9
		$\mathcal{K}_4$	89.3	92.7	25.2	90.5	60.4	20.2	76.6
		$\mathcal{K}_1^*$	53.5	52.4	34.8	57.1	46.8	31.5	42.8
		$\mathcal{K}_1'^*$	9.0	6.0	3.7	7.9	7.6	4.6	5.9
		$\mathcal{K}_4^*$	48.2	41.7	28.4	48.2	42.5	23.7	35.9
	0.75	$\mathcal{K}_1$	73.1	76.8	53.5	76.6	66.0	18.4	60.3
		$\mathcal{K}_1'$	2.7	4.1	1.9	2.9	3.4	3.2	3.1
		$\mathcal{K}_4$	71.4	76.9	6.0	76.6	23.8	13.7	56.8
		$\mathcal{K}_1^*$	41.5	42.5	21.0	48.2	30.1	21.4	29.8
		$\mathcal{K}_1'^*$	0.5	0.4	0.4	0.5	0.6	0.7	0.7
		$\mathcal{K}_4^*$	32.7	27.2	14.6	35.1	24.5	15.0	21.3

Table 5: Empirical Power of Bootstrap Persistence Change Tests: De-meaned Data.

			Model 1			Model 2		Model 3	
T	τ^*		$\delta=1$	$\delta=1/3$	$\delta=3$	$\delta=1/3$	$\delta=3$	$c=0$	$c=0$
100	0.25	$\mathcal{K}_1$	67.5	82.1	56.6	79.9	59.1	61.5	66.0
		$\mathcal{K}_1'$	43.7	37.9	60.4	35.3	58.3	46.7	47.0
		$\mathcal{K}_4$	67.5	82.7	63.9	79.4	66.2	74.0	67.7
		$\mathcal{K}_1^*$	33.9	36.8	27.5	37.4	33.1	30.8	34.4
		$\mathcal{K}_1'^*$	14.5	12.9	16.8	13.9	15.9	15.4	15.3
		$\mathcal{K}_4^*$	30.1	31.2	27.0	32.1	30.6	29.9	30.9
	0.50	$\mathcal{K}_1$	57.4	81.5	33.2	72.2	45.8	54.2	58.8
		$\mathcal{K}_1'$	34.9	37.4	34.9	28.2	45.0	40.0	37.5
		$\mathcal{K}_4$	59.0	81.6	40.3	72.0	54.2	68.1	61.4
		$\mathcal{K}_1^*$	37.9	40.5	28.3	41.6	35.5	34.1	37.3
		$\mathcal{K}_1'^*$	8.7	9.2	6.6	8.7	8.4	8.6	8.4
		$\mathcal{K}_4^*$	28.3	31.2	23.0	31.1	27.7	27.8	28.6
	0.75	$\mathcal{K}_1$	31.0	58.2	18.4	49.4	22.7	37.2	33.0
		$\mathcal{K}_1'$	7.5	8.2	10.8	5.7	12.7	20.4	10.2
		$\mathcal{K}_4$	31.4	58.3	16.7	49.7	23.8	47.5	34.5
		$\mathcal{K}_1^*$	24.9	29.0	17.8	27.8	22.3	21.5	23.8
		$\mathcal{K}_1'^*$	2.0	1.7	1.5	1.7	1.7	2.8	1.8
		$\mathcal{K}_4^*$	17.1	19.4	14.6	18.8	16.8	16.0	17.2

Table 5: Empirical Power of Bootstrap Persistence Change Tests: De-meaned Data.

			Model 1			Model 2		Model 3	
T	τ^*		$\delta=1$	$\delta=1/3$	$\delta=3$	$\delta=1/3$	$\delta=3$	$c=0$	$c=0$
200	0.25	$\mathcal{K}_1$	79.2	87.8	73.2	85.0	74.2	70.3	78.6
		$\mathcal{K}_1'$	42.2	37.4	60.5	34.5	56.3	48.6	47.3
		$\mathcal{K}_4$	77.2	87.7	69.7	84.9	71.5	77.7	77.2
		$\mathcal{K}_1^*$	40.8	45.8	36.0	43.9	38.8	38.1	42.1
		$\mathcal{K}_1'^*$	14.6	11.4	17.0	13.2	15.9	14.8	13.8
		$\mathcal{K}_4^*$	36.6	37.7	35.1	37.7	37.5	36.0	38.2
	0.50	$\mathcal{K}_1$	71.4	88.7	46.6	80.6	61.6	61.4	69.6
		$\mathcal{K}_1'$	33.1	35.9	29.3	27.2	43.3	34.3	35.4
		$\mathcal{K}_4$	73.3	88.7	45.4	80.7	64.3	66.8	70.3
		$\mathcal{K}_1^*$	48.2	51.4	40.0	52.1	46.6	43.8	47.0
		$\mathcal{K}_1'^*$	7.9	9.3	4.6	7.7	7.5	6.9	7.0
		$\mathcal{K}_4^*$	40.2	42.0	33.6	42.9	38.8	37.2	39.6
	0.75	$\mathcal{K}_1$	42.9	67.8	20.7	60.6	28.4	39.1	43.4
		$\mathcal{K}_1'$	3.4	4.2	4.6	2.7	6.5	11.4	6.2
		$\mathcal{K}_4$	42.7	67.8	16.3	60.6	26.4	43.9	43.2
		$\mathcal{K}_1^*$	35.8	39.9	24.0	41.0	31.1	30.6	33.5
		$\mathcal{K}_1'^*$	0.5	0.5	0.4	0.5	1.0	1.3	0.8
		$\mathcal{K}_4^*$	25.4	28.9	17.6	29.3	22.7	23.4	23.8

Table 6: Persistence Change Tests for US Producer Price Inflation Series.

	$\mathcal{K}_1$	$\mathcal{K}_1'$	$\mathcal{K}_4$	$\mathcal{K}_2$	$\mathcal{K}_2'$	$\mathcal{K}_5$	$\mathcal{K}_3$	$\mathcal{K}_3'$	$\mathcal{K}_6$
PWFSA	140.102	45.281	140.102	2.396	9.657	9.657	64.671	17.891	64.671
$(T = 360)$	0.000	0.003	0.000	0.198	0.010	0.018	0.000	0.003	0.000
	0.000	0.038	0.003	0.093	0.028	0.028	0.000	0.038	0.003
PWFCSA	83.172	30.491	83.172	1.577	7.059	7.059	36.206	10.832	36.206
$(T = 360)$	0.000	0.015	0.000	0.398	0.020	0.035	0.000	0.015	0.000
	0.003	0.080	0.005	0.281	0.058	0.058	0.003	0.075	0.005
PWIMSA	30.435	34.049	34.049	1.050	10.219	10.219	10.240	12.996	12.996
$(T = 360)$	0.013	0.010	0.018	0.652	0.008	0.015	0.015	0.010	0.020
	0.113	0.148	0.208	0.494	0.068	0.068	0.115	0.140	0.201
PWFXSA	421.056	47.362	421.056	13.354	9.339	13.354	205.148	18.867	205.148
$(T = 359)$	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000
	0.000	0.020	0.000	0.000	0.010	0.003	0.000	0.020	0.000
PWCMSA	4.355	15.859	15.859	0.678	2.326	2.326	0.378	2.946	2.946
$(T = 360)$	0.564	0.085	0.150	0.802	0.271	0.481	0.802	0.153	0.273
	0.837	0.226	0.469	0.842	0.351	0.619	0.915	0.286	0.546
PW160A	1.096	18.813	18.813	0.387	4.375	4.375	0.200	5.646	5.646
$(T = 275)$	0.965	0.068	0.115	0.932	0.088	0.170	0.947	0.073	0.123
	0.647	0.358	0.361	0.581	0.474	0.474	0.586	0.366	0.368
PW150A	3.629	14.996	14.996	0.758	3.499	3.499	0.474	4.212	4.212
$(T = 275)$	0.667	0.103	0.183	0.774	0.135	0.248	0.739	0.098	0.175
	0.256	0.401	0.406	0.343	0.514	0.531	0.328	0.406	0.411

Table 7: Standardized Persistence Change Tests for US Producer Price Inflation Series.

	$\mathcal{K}_1^*$	$\mathcal{K}_1^{*\prime}$	$\mathcal{K}_4^*$	$\mathcal{K}_2^*$	$\mathcal{K}_2^{*\prime}$	$\mathcal{K}_5^*$	$\mathcal{K}_3^*$	$\mathcal{K}_3^{*\prime}$	$\mathcal{K}_6^*$
PWFSA	41.511	20.334	41.511	1.613	4.472	4.472	15.379	6.234	15.379
$(T = 360)$	0.000	0.053	0.003	0.396	0.080	0.133	0.000	0.053	0.005
	0.000	0.070	0.010	0.441	0.040	0.090	0.000	0.058	0.013
PWFCSA	25.132	16.236	25.132	1.283	3.931	3.931	7.203	4.458	7.203
$(T = 360)$	0.023	0.078	0.053	0.526	0.105	0.180	0.038	0.078	0.078
	0.013	0.103	0.030	0.541	0.063	0.118	0.023	0.088	0.048
PWIMSA	6.104	11.291	11.291	1.036	3.556	3.556	0.789	3.135	3.135
$(T = 360)$	0.373	0.155	0.278	0.642	0.120	0.223	0.531	0.133	0.231
	0.707	0.276	0.541	0.737	0.083	0.228	0.739	0.206	0.406
PWFXSA	109.316	31.734	109.316	4.725	6.228	6.228	49.481	11.551	49.481
$(T = 359)$	0.000	0.005	0.000	0.048	0.033	0.053	0.000	0.005	0.000
	0.000	0.010	0.000	0.053	0.013	0.038	0.000	0.010	0.000
PWCMSA	4.592	12.336	12.336	0.973	1.901	1.901	0.551	1.878	1.878
$(T = 360)$	0.519	0.128	0.243	0.659	0.338	0.639	0.684	0.253	0.446
	0.694	0.190	0.401	0.694	0.293	0.674	0.764	0.243	0.529
PW160A	3.074	5.940	5.940	1.197	1.385	1.385	0.659	0.999	0.999
$(T = 275)$	0.714	0.444	0.724	0.559	0.521	0.917	0.612	0.486	0.807
	0.674	0.333	0.664	0.566	0.461	0.915	0.609	0.406	0.807
PW150A	7.282	6.002	7.282	1.841	1.296	1.841	1.414	1.029	1.414
$(T = 275)$	0.313	0.444	0.599	0.326	0.559	0.697	0.306	0.476	0.619
	0.263	0.326	0.474	0.331	0.519	0.649	0.286	0.406	0.539

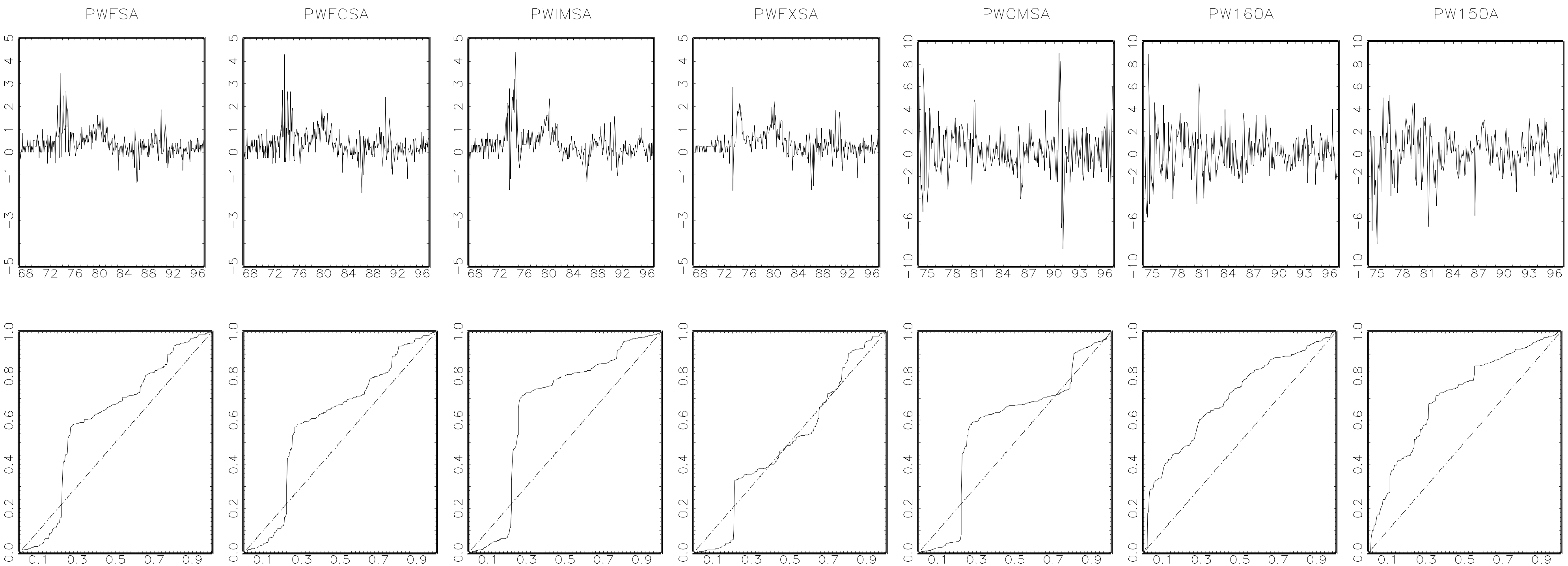

Figure 1: Inflation Rates from the seven Seasonally Adjusted Producer Price Indices from the Stock-Watson (1999) Database (top row), and Estimated Variance Profiles (bottom row).