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Abstract: Social security systems for old age have been explicitly studied in a public choice framework for over 30 years. They illustrate extremely well the problems of allocating economic resources through a system of voting. Pension systems also currently provide some of the most significant threats to the long-term budget positions of developed countries, a point that was made in the Nobel Laureate lecture of Professor James Buchanan. In this paper, we look at the costs and benefits that will be faced by different groups of voters as a result of state pension reform in the UK. It is shown that state pension systems will be very difficult to reform in ways that reduce government provision. However, an exception to this general rule is that reform by raising retirement ages may well be politically feasible. These results are in accordance, not just with theoretical work, but with other empirical work and practical observations.

Keywords: *Public choice economics; pensions policy; state pension reform; social security*

JEL classifications: H55; J14; H42; D72.

Introduction

The original work by James Buchanan and Gordon Tullock[1] in the field of public choice economics and rent seeking has spurned a literature on public choice and state pensions. This is not surprising given the importance of state pensions in redistributing income and providing income in old age in most democracies. However, it is also the case that state pensions are particularly amenable to investigation using certain forms of public choice model, for reasons discussed below. The issue is important given that the implicit social security debts represent some of the biggest financial obligations of developed-country governments.

Public choice economists, of course, do not argue that self interest is the only motivating factor when people take decisions at the ballot box. Rather they argue that it is prudent to assume that self interest could be one amongst many motivating factors. As Buchanan points out, public choice

[1] For example, Buchanan and Tullock (1962) and Tullock (1967).

theory suggests that self interest is a “positively valued good” in influencing decisions and it does not say to what extent it dominates: “This assumption [of self interest] does not place economic interest in a dominating position and it surely does not imply imputing evil or malicious motives to political actors…” (Buchanan, 1986).

Even if only a proportion of voters vote according to their perceived self interest, as the electorate ages there will be increased pressure to expand payments under public pension schemes. One way of assessing the influence of changes in age structure on pensions policy is through median voter models. Though the criticisms and qualifications made by other authors, such as Verbon (1983), should not be dismissed, median-voter public choice models of voter behaviour do have predictive power and we will use such models here.

Precisely how changes in the composition of the electorate will affect public policy will vary from country to country and depend on voting systems and the structure of state pension systems. However, as Galasso (2006) shows, some lessons are remarkably general.

In the context of a public choice analysis of state pension schemes, it is, in fact, worth noting a particularly prescient statement made by Edward Marshall, former president of the Actuarial Society of America, at the First Business Meeting of the Centenary Assembly of the Institute of Actuaries on 22nd June 1948:

> There was at the existing time great political pressure from Governments to adopt or maintain ambitious programmes of so-called social security, with perhaps too little understanding of their ultimate effect on the social and economic structure. A sound social insurance and superannuation programme could sustain and strengthen a nation; on the other hand, a sufficiently unsound one could ultimately destroy it. Furthermore, once such a programme was put into effect it became *politically impossible to discard it or to reduce the benefit scales* which it was beyond the ability of the nation’s economy to support. (Marshall, 1950, my italics)

It is precisely this last observation that is analysed here. It is also notable that economist, Alfred Marshall, told the 1893 Royal Commission on the aged poor, ‘they [universal state pension systems] do not contain in themselves the seeds of their own disappearance. I am afraid that, if started, they would tend to become perpetual’. If one accepts the analysis below, this statement is prophetic.

Public choice models and pay-as-you-go-pension systems

A theoretical model of social security in a democracy was developed by Browning (1975). He demonstrated that, in a pure democracy, there are

strong incentives to increase the size of social security pension systems beyond their optimal level. When pay-as-you-go[2] (PAYGO) pension systems are expanded, the only cohort that pays the full cost of the expansion is that just beginning its working life. That group will pay a working lifetime of higher contributions (social security taxes) to obtain a higher pension from retirement age. Older workers, however, will receive some form of subsidy from later entrants to the workforce. This is because older workers will pay the higher contributions needed to finance higher benefits for only a portion of their working life yet they will receive a higher pension throughout their retirement. This problem is inherent in the nature of PAYGO systems[3]. At the extreme, of course, people who have already retired will be able to vote for higher pensions without bearing any of the costs themselves. Buchanan (1986) specifically mentioned this problem of implicit debt in social security systems in his Nobel Laureate lecture.

There are recognised inconsistencies in the Browning model (see, for example, Verbon, 1987 and the references therein such as Tullock, 1983). For example, the model implicitly assumes that the electorate votes for a policy change which it believes to be permanent whereas, in fact, future electorates will be able to change the policy. However, Browning's conclusions about the ability of an electorate to expand existing social security systems beyond their optimal size is a good starting point for discussion and analysis. It is especially robust if the median age of voters is expected to continue to rise: there will then be little pressure to reverse a policy change that has involved raising the level of benefits.

There is, of course, no single accepted model of behaviour in public choice theory. Various models are discussed in the theoretical literature (see Mueller, 2003). Median voter models are more straightforward when choices are clear and linear. As it happens, policies related to state pension provision fit into this model of public choice fairly well because the net benefits of an increase in state pensions are, more or less, a linear function of age[4]. Also, those who benefit from an increase in the state pension usually have relatively "coherent" preferences – i.e. they are interested in a relatively small number of issues at an election (such as pensions, social care provision and healthcare). This might suggest that median voter models are, if anything, conservative in their predictions.

Predictions of public choice models

Using public choice models of voter behaviour, we can make a range of a priori predictions about voter behaviour in relation to PAYGO pension systems, specifically:

[2] "Pay as you go" state pension schemes are those pension schemes where current pensions are paid using tax receipts from the current generation of taxpayers.
[3] As will be discussed below, it is possible to have some sort of quasi-contractual arrangement whereby the accruals principle is applied so that if voters support an increase in pensions, this will only apply to entitlement earned in future years.
[4] This is slightly complicated by the fact that state pension schemes often contain an element of redistribution. This will be discussed below.

- The greater the proportion of older voters, the higher the level of social security pensions.
- The greater the level of political freedom, the less likely it is that a country will have social security reform that involves reducing the size of PAYGO pension systems.
- The more efficiently taxes can be levied, the higher will be the level of social security pensions. This is a subtle but important point. If a tax system is inefficient then an economy reaches its maximum taxable capacity at lower tax rates. In this case, middle-age people will be less willing to vote for an increase in pensions because the increase in taxes necessary to generate a given increase in revenue to finance the increase in pensions will be higher.
- Systems based on earnings-related pensions are likely to grow bigger than those which are not because earnings-related pensions distort labour markets less than systems that provide flat-rate benefits. In an earnings-related system, a higher level of earnings gives rise to an entitlement to higher pensions. This, in turn, partially offsets the disincentive effect of higher levels of social security contributions required on higher earnings. As a result, taxes levied to finance earnings-related pensions distort the labour market less than taxes levied to finance flat-rate pensions.
- It will be harder for voters to expand systems based on an accruals[5] principle, but, at the same time, it will be harder to reduce accrued pensions that have already been promised in such a system. The reason for this is that those who have already accrued pension have a very strong interest in fighting any reduction in their entitlement and such groups will be willing to invest a lot of time and money in campaigning to protect their benefit.
- Reform based on increasing the age at which state pensions become payable may be achievable when other reforms are not.

There is empirical evidence to confirm these hypotheses. For example, Breyer and Craig (1997) test public choice models for 20 OECD countries over four decades. They found a very strong relationship between median voter age and pension programme size. An increase in median voter age of one year added 0.5% to the share of national income taken by the pensions programme. Cremer and Pestiau (2000) confirm that a contributory, accruals-based system can set up “entrenched interests” that are harder to overcome than other interests in a public choice model as individuals who have accrued entitlements hold a political weight stronger than their numbers suggest. Wang and Davis (2003) show that the proportion of older people in a country and the level of political freedom help explain resistance to the contraction of PAYGO pension systems – though the analysis is complicated by the strong correlation between political freedom and the proportion of the population aged 65 or over (their variable to represent ageing).

[5] In a system based on accruals, individuals “earn” entitlement to pension for each year they are in the labour force. It is a quasi-contractual system. This contrasts with a system where the level of pension is simply determined, for all pensioners, by the government of the day.

The experience of successful reform programmes also helps to confirm some of the above predictions. Pension reform has recently been undertaken in Sweden. This is perhaps surprising given the demographic background there. In the 20th century[6] the age of the median voter has increased by eight years to 47.5 and young people between the ages of 20 and 25 fell by half as a proportion of the electorate. Despite this background, a new pension system was introduced during the 1990s that reduced both costs and pension benefits. Unsurprisingly, the package was attractive to younger voters. Voters over 57 years of age were not affected and had no incentive to oppose the proposals. The voting behaviour of middle-aged groups would depend on their specific income characteristics but, overall, the reforms were designed in such a way that, according to Kruse (2005), votes in favour of the reform would have been expected to outnumber votes against the reform if voters voted according to their financial best interests. In a public choice analysis of reforms, Kruse (2005, page 14) says, that it was such a "smart use of the transition rules" that made reform possible.

Other pension system reforms have only been achieved by removing the costs of reform from the older generation of the population entirely – effectively "buying off" interest groups. For example, where Chilean-type[7] reforms based on compulsory personal accounts have been implemented, older people and middle-aged people have often been excluded from the new arrangements altogether (see Stroinski, 1998 for an analysis of Polish reforms). This has made reform acceptable to electorate. Alternatively, the value of the accrued rights of people affected by reform has been crystallised in the form of "recognition bonds" – non-tradable or tradable bonds that reflect accrued rights within an existing system.

In many other countries, the only type of pension reform that has been possible has involved raising the retirement age[8]. Galasso (2006) has a major study of this type of reform and it is covered in detail below.

UK population modelling

Galasso (2006) has used very sophisticated general-equilibrium economic models to analyse pensions in a public choice framework in a number of EU countries. The investigations below use less sophisticated partial equilibrium models but with more explicit assumptions which make the results clearer, and the application of the model more flexible. It should be mentioned that, where possible, the results have been cross checked against the results of Galasso (2006) and there is a high degree of correspondence. Galasso's general equilibrium models pick up second order effects which, for our purposes, are not important.

[6] The state pension system was established at the beginning of the 20th century.
[7] Of course, Chile itself was not a democracy at the time of reform but, even there, some groups were exempted from pension reforms as a result of parliamentary pressure from representatives with close ties to those groups.
[8] This has happened twice in the UK, with little opposition.

The model was used to examine public choice pressures in the UK Basic State Pension (BSP) system. This system offers little protection for accrued rights and therefore the scope and size of the system can be expanded and contracted by the elected government: in particular, the level of pension can be changed and the method of indexation can be changed by ministers acting through Parliament. This arrangement means that the system falls between the tax transfer systems and insurance-based systems that are discussed by Verbon (1987) and Buchanan (1983). The implicit liability that Buchanan describes as being very difficult to default upon is less clear in the UK BSP and thus it would seem amenable to median-voter analysis. The other UK state pension benefit, known as the State Second Pension, provides stronger guarantees that past accrual will not be altered in either direction and we will therefore ignore this aspect of the state pension system.

The population model

A population model of the UK economy was developed that shows the actual and anticipated numbers in different age groups from 2004 to 2078. The model was based on the Government Actuary's Department (GAD) UK population projections, using 2004 as a base year[9]. In turn, those projections are based on the UK government's Registrar General's estimate of the resident population of the UK. The population includes all residents, irrespective of nationality. For each age, the starting population plus net migrants less deaths gives the number in the population, one year older, at the end of the year. To this number births are added. Age is defined as completed years at the last birthday. The number of births in the year is estimated by taking the average number of women at each single year of age during the year and multiplying by the fertility rate applicable to them during that year. The total number of births in a year is divided between the sexes in the ratio of 105 males to 100 females, in line with recent experience. Mortality is assumed to improve at 1.0% per annum at each age in line with GAD projections. Migration is assumed to continue at its current rate, which is derived by GAD using International Passenger Survey (IPS) as a principal source though the population was projected with and without migrants.

Developments in the age structure of the UK voting population

The population model was used to project the proportion of the voting population aged over 55 and the median age of voters in the UK electorate using four different assumptions:

1. The likelihood of a member of the electorate voting was the same for people of all ages and there was no migration.
2. The likelihood of a member of the electorate voting was the same for people of all ages but the population was subject to migration.

[9] Full details can be found on www.gad.gov.uk

3. People of different ages had differing propensities to vote, based on the propensities to vote at the 2005 general election in the UK but the population was not subject to migration.
4. People of different ages had different propensities to vote but the population was subject to migration.

It is a subjective matter how one should treat the two variables here – migration and the propensity to vote. In general, migrants will lower the median age of the population (at least at first) and continued migration is likely to limit the expected increase in the median age of the electorate in the coming generation. However, though it is too early to observe how the current generation of migrants will behave, migrants tend not to vote until settled in a country for a long time[10]; they also often return to their country of origin after a few years.

Regarding the propensity to vote, in most countries there is a tendency for the propensity to vote to rise with age. There are various possible reasons for this. It is possible that older people have a lower marginal cost of time in which case one would expect current patterns to prevail into the future. It is also possible that today's older generation value their vote to a greater extent than the young because of their knowledge of the threat of fascist and communist dictatorships in other countries during the last century, or because they are more aware of recent generations not having a universal franchise. If this were the case, voting propensities across the age groups would tend to equalise over time (at a lower level).

In most of the financial calculations below, we have assumed that there is no migration and ignored age-related propensities to vote. Overall, as will be seen, the impact of ignoring voting propensities leads to conclusions being more optimistic than they otherwise would be. The impact of ignoring migration gives rise to slightly less optimistic results – though this effect does not completely offset the effect of ignoring voting propensities. On balance, our results are probably slightly optimistic and understate voter pressure to expand state pension systems.

The demographic composition of the population was examined under the four different assumptions noted above. The projected proportion of voters who will be age 55 or older between 2004 and 2055 is shown in Figure One.

It can be seen that, even under the most optimistic assumptions[11], the proportion of people over the age of 55 (that is within sight of current state pension age) will rise from about 35% of the population to about 47% of the population over the next 50 years. If we allow for older people being more likely to vote, the proportion of the active voting population over age 55 is already nearly 45% and, on the assumption also that migrants do not vote, will rise to nearly 60% over the next 50 years. Given that, in the UK, a party needs only about 40% of the voting electorate (about 28% of the total electorate) to form a working government and given also what has

[10] Indeed, they are often not permitted to vote.
[11] That is that all people are equally likely to vote and assuming migrantion.

been noted about the coherence of the interests of older voters, these figures are stark. Work published in Booth (2008a) shows comparable figures for other European countries and New Zealand. In all cases other than New Zealand the ageing of the electorate is even more pronounced than in the UK.

The age of the median voter

The population model was used, together with actuarial mathematics techniques, to estimate the age of the median voter. This was then used in the further computations below. The progression of the age of the median voter is shown in figure two, below. It is notable that, using an independent population model, Galasso (2006) came to similar conclusions for the UK as those presented here.

Under the most unfavourable set of assumptions (no migration but incorporating age-related propensities to vote) the median age of the voting electorate will rise to about 60 over the coming 50 years. If we make allowances for migration then the median age will rise to 57. In most of our calculations we have assumed no migration but a uniform propensity to vote. This produces a median age for voters of 55 in 2055. Reversing this sharp upward trend in the age of the median voter would require a considerable rise in the birth rate very soon. Ignoring migration, the age profile of the electorate until 2034 is more or less settled today.

It is worth noting that, once the median voter is close to state pension age there is a strong incentive for voters to expand the state pension system to its maximum possible size, which would occur when tax rates are at the top of the Laffer curve. This is potentially in prospect in a number of EU countries where, on average, the electorate is older and government spending already high (see Booth, 2008a, Booth 2008b and Smith, 2006).

Actuarial and financial analysis of Policy Change One[12]: increasing the state pension

After projecting the age structure of the electorate, we examined the rate of return to voters from an increase in pensions. The increase in pensions was assumed to be financed by an increase in National Insurance contributions[13] paid by the working population. workers would have to pay this increase until their retirement age. The increased rate of National Insurance contributions can be regarded as the “price” that voters pay for the “benefit” of an increase in pension. The increase in National Insurance

[12] Many of the results relating to “Policy Change One” have previously been presented and discussed in Booth (2008b). However, they provide useful context for the analysis of “Policy Change Two” so they are repeated here.

[13] The payroll tax used to finance various aspects of government social insurances – including pensions – in the UK.

contributions required to finance the increase in state pension whilst keeping income and expenditure in the National Insurance "fund" in balance was calculated using the working and retired population projections from the population model. The necessary increase will change over time as the age structure of the population changes. The rate of interest which makes the present value of the expected increase in pension equal to the present value of the expected increase in National Insurance contributions, after allowing for mortality, was then calculated for a voter of median age and for a new entrant to the workforce (assumed to be aged 18). All variables are calculated in real terms.

There are simplifying assumptions in this approach in order to isolate the age-dependent effects of the proposed policy change. It is assumed that a flat increase in pension is financed by a flat increase in National Insurance contributions. This is not in accordance with the administrative mechanisms of the National Insurance system by which contributions are levied as a percentage of earnings but a flat-rate pension is paid. However, we want all distributional issues to be set to one side or, equivalently, we want to ensure that the cross-sectional distribution of income remains unchanged as a result of the increase in the level of the pension. This could be achieved by flat-rate National Insurance contribution increases paying for flat-rate pension increases as we have assumed here. Alternatively, it could be achieved by increasing pensions by a flat-rate amount whilst raising the *percentage rate* of National Insurance contributions levied on earnings and whilst taking other action to ensure that the cross-sectional distribution of net income is unchanged. Such action could include, for example, simultaneously reducing the basic rate of income tax and not raising the basic tax allowance in line with earnings: a policy that has, in fact, been followed for most of the last forty years.[14] Thus, although the precise approach to financing a pension increase has been chosen for computational simplicity, the approach is financially equivalent to an alternative that is certainly realistic in political terms.

It should be noted that, if there are distributional effects of a rise in the state pension that (for example) benefit the less well off, then the proposal will become better value to some younger voters on low earnings. In turn, it will be worse value to some older voters on higher earnings. We are simply setting such considerations to one side. Our assumption is not unrealistic in the sense that we are denying the existence of the distributional implications of the state pension system. Rather, we are setting aside distributional issues and focusing on one specific policy change – the raising of the state pension whilst the distribution of income is otherwise unchanged. The alternative approach of segmenting the population by income would increase the realism of the analysis but reduce the extent to which the effect of the particular variable of ageing was understood. Clearly, this is an area for further research.

[14] It should also be noted that there have been various changes to tax and benefits systems in the UK recently where the government has introduced consequential changes to ensure that the distributional effects of the policy were neutralised. The most recent case was when the 10% tax band was withdrawn in the 2008 Finance Act.

Rate of Return Calculations – voters of median age

It was assumed that the increase in pension was £2,000 per annum in 2008 prices. The level of the pension increase is irrelevant to determining the real rate of return to the median voter because the increase in National Insurance contributions is proportional to the assumed increase in the pension. The real rate of return to the median voter (*j)*, from successfully supporting policies to raise pensions by £2,000 per year in a given year, is calculated from:

$$2{,}000\sum_{t=65-x}^{t=110-x}\frac{l_{x+t,date+t}}{l_{x,date}}(1+j)^{-t}-\sum_{t=0}^{t=(64-x)}NI_{date+t}\times\frac{l_{x+t,date+t}}{l_{x,date}}(1+j)^{-t}=0$$

x,date = the age of the median voter in year *date*
$l_{x+t,date+t}/l_{x,date}$ is the probability of a voter still being alive *t* years after they reach the age of the median voter in year *date.* Mortality was as assumed in the population model.
NI_{date+t} = the rate of National Insurance contributions necessary to finance the proposed pension increase.

The rate of return from a given increase in pension, financed by the necessary increase in National Insurance contributions, is the solution to the above equation, *j.* If *date* is equal to 2009, for example, the second term in the equation would use the age of the median voter in 2009 (*x* = 47) and discount all the expected increased National Insurance contributions necessary to finance a £2,000 increase in the state pension from age 47 to age 64 (*date+t* = 2009, 2010, 2011 etc, *x+t* = 47, 48, 49 etc). The first term in the equation would sum the present value of all the expected extra pension payments of £2,000 from the date at which the individual would reach 65.

The extent of the increase in National Insurance contributions necessary to finance the increase in pensions depends on the age structure of the population which largely determines the balance of contributors and recipients of benefits. The trend of the increase is shown in Table One below.

Economic rationale

The rationale for this analysis is straightforward. Voters in the political market can choose to maximise their financial best interests when they vote. The median voter can obtain a rate of return from deferring income in two ways. He can save in the financial markets and obtain a guaranteed rate of return above inflation from government index-linked bonds (or higher expected returns if investment risk is taken). Alternatively, the voter can act in the political market and obtain a rate of return on increased National Insurance contributions in the form of a higher state pension. Neither of these options is risk free, though the risks attached to each are

different.[15] The above equation allows us to compute the rate of return to a voter from a decision to increase National Insurance contributions. If this is more profitable to the median voter than saving through financial markets, one can expect the median voter to have a preference for raising his pension by voting for higher pensions in the political marketplace.

The rate of return to the median voter from increased state pensions increases as the median voter gets older because a given pension increase will be financed by fewer years of increased National Insurance contributions for that voter. The rate of return will also be related positively to life expectancy and be negatively related to the increase in National Insurance contributions necessary to finance a given increase in the state pension. The increase in National Insurance contributions necessary to finance a given increase in the state pension whilst keeping the National Insurance fund in balance on a PAYGO basis will, of course, increase as the age of the median voter increases (because there will be fewer contributors per retiree). This causes a slight offset to the tendency of the rate of return to voters to rise as the age of the median voter rises.

The results

The variation in the real rate of return to the median voter who successfully votes to increase pensions is shown in Figure Three[16]. At the current time, the rate of return is 5.8%. The rate of return will rise quickly to 7.6% by 2025 and then rise more slowly to 8.8% by 2050. This suggests very strong, and increasing, incentives for the median voter to vote to increase the size of state pensions. One would expect politicians to respond to those incentives, at least at the margin, by offering financial packages that benefit the median voter and raise the level of government help to pensioners. These packages might not necessarily involve raising state pensions directly - other benefits may be provided to the old instead. On the other hand, the results suggest that reducing benefits to pensioners will be very difficult.

These results can be compared with the risk-free real rate of return that the median voter can obtain from saving in financial markets – currently about 1% per annum. However, this comparison is not straightforward (see Galasso, 2006). The return to the median voter from successfully voting for a higher pension is not a risk-free return. Future generations of voters could take the increased pension away. Indeed, the voter of median age could pay the higher contributions and then find that, because of fiscal constraints, future electorates decide that they do not receive a higher pension at all – and thus they could receive a return on their additional contributions of -100%. Indeed Weaver (1983) suggests that increasing the risk that social security pension benefits will not be paid is one way to

[15] See below.
[16] It should be remembered that in calculating the rates of return we have taken a conservative approach by not adjusting the age profile of the electorate for the variation in the propensity to vote at different ages.

reduce support for the system. However, the influence over the electoral system of ageing median voters will reduce the risk that benefits will be taken away. If individuals save privately, even through risk-free investments such as index-linked government bonds, they are exposed to the risk that annuity rates may have become more expensive in the time between investment and retirement.

Rate of return calculation – new voters

New voters will have to pay increased contributions all their lives in return for an increased pension at retirement. Therefore new voters face both the costs and the benefits from a decision to raise the level of pensions.

The changes in the rate of return over time from new voters voting for an increase in pension are shown in Figure Four below. It can be seen that there is a gradual decline in the rate of return from zero to just below -1%. These rates of return are in line with those that can be reasonably expected from a Bismarkian PAYGO system during an era of ageing population (see Samuelson, 1958, for the original discussion of rates of return in PAYGO systems).

It is clear that new voters would gain from a contraction of the state pension system and a move towards other methods of pension provision. Even if young people were to invest solely in more-or-less risk-free, index-linked investment instruments they would obtain a higher rate of return than the risky rates of return that can be promised through the political system. Again, this merely illustrates the result of Browning (1975): social security systems can be expanded beyond their optimal level for those who pay the full costs of that expansion because the median voter does not pay all the costs. The youngest voters, who do pay all the costs of expansion because they contribute for a full lifetime as well as receiving a pension throughout their retirement, are not sufficiently influential in the electoral process to stop the social security system expanding as they form only a small proportion of the electorate.

Actuarial and financial analysis of Policy Change Two: changing the state pension age

In many countries, it seems to have been possible to reform state pensions by raising the state pension age. It might be asked whether this casual empirical observation can be reconciled with the predictions of a public choice model which, in most respects, suggests that the pension system will continue to grow as the population ages. In fact, it can be reconciled, as has been shown by Galasso (2006).

We can begin by considering the trivial situation where the median voter is older than current state pension age. In this case, any immediate rise in the state pension age will have no detrimental impact on the median voter.

Similarly, a deferred increase in the state pension age will have no detrimental impact on the median voter as long as the deferral period is longer than the difference between the age of the median voter and the current state pension age. In the UK, for example, the state pension age will be increased from 2024. Anybody who will be 65 before 2024 (i.e. who was 48 when the measure was passed in 2007) will not lose from this policy change. Interestingly, the median voter, after allowing for voting propensities, was 49 at the time the increase in pension age was passed into law, thus ensuring that over 50% of voters would not lose from the change.[17]

More generally, we have to identify the losers and gainers from a rise in the state pension age and this will depend on how the savings are used. In general, those countries that have raised the state pension age have done so in order to prevent social security taxes from rising to the level that would otherwise have prevailed and that is the form of policy change that we analyse below.

If the financial savings from a rise in the state pension age are used to reduce National Insurance contributions the main losers will be people who retire just after the change takes effect. The gainers are new entrants into the system whose National Insurance contributions will be reduced but who do not suffer from the loss of a year's pension for many years[18]. Those who retire before the deferral period ends are not directly affected by the change as neither their pension nor National Insurance contributions will change[19].

As has been noted, the greatest impact is felt by those who retire shortly after the rise in pension age takes affect. The impact then falls as age falls. Younger people will enjoy a larger number of years of the reduced level of National Insurance contributions than middle-aged people; they will also be further away from the loss of a year's pension and the payment of an extra year's National Insurance contributions that result from the rise in the state pension age. To put it another way, an older person who was affected by the increase in the state pension age would have to invest a greater sum of money than a younger person to be able to privately fund the year of retirement that is lost because of the rise in the state pension age. Of course, all members of the population may gain from improved labour market efficiency arising from lower labour taxes, but this effect can only be captured by general equilibrium models (see Galasso, 2006) and this is second order.

[17] When proposals were passed to increase the state pension age for women in 1995, a longer deferral period was chosen, consistent with a lower age of median voter.
[18] The extent of gains and losses and how they are distributed here depends on whether the population is increasing or declining. There may also be second-round effects from a decrease in labour market taxes.
[19] If the savings had been used to finance an increase in the pension, they would gain from the change, of course.

Financial calculations for Policy Change Two

We now examine the public choice implications of a hypothetical recent decision by the UK government to increase the state pension age by one year in 2008, deferred until 2020.[20] This is not unlike the recent policy change by the UK government regarding changes to the state pension age. It is assumed that the rise in the state pension age is used to finance a reduction in National Insurance contributions which is revenue neutral, thus maintaining financial balance under the pay-as-you-go principle. The decrease in National Insurance contributions that would be financed by the rise in the state pension age varies each year and is determined by the age structure of the population and the balance between pensioners and workers at any particular time. All variables were assumed to be constant in real terms.

Again, we abstracted from distributional considerations within cohorts and it was assumed that a *level* decrease in National Insurance contributions would be financed by an increase in the state pension age. It could have been assumed that the increase in state pension age financed a decrease in the percentage rate of National Insurance contributions. If that had been the case, then some younger, poorer people would have been worse off and some middle-age, richer people better off than our results found. It should be stressed, once again, that it is not being suggested that distributional considerations are not part and parcel of pensions policy changes. It was desired here to isolate the impact of a change in the pensions system which had no effect on income distribution within generations.

The proposed change in the retirement age would not affect anybody who was due to retire before 2020. Individuals aged 54 in 2008 would retire as normal in 2019; on the other hand, individuals aged 53 in 2008 would not be able to retire at age 65 in 2020 but would have to wait until they were aged 66 in 2021. Therefore, nobody aged 54 or over would be affected by the policy change. Somebody aged 53 would lose from the change and would have no offsetting reduction in National Insurance contributions. Anybody below age 53 would pay a reduced level of National Insurance contributions (the reduction being made possible by the rise in the state pension age) in the years before retirement; then, when they are aged 65, such people pay an additional year of National Insurance contributions at the lower prevailing level and also lose a year of pension, as compared with the situation that would have prevailed before the change in the state pension age.

Figure Five shows the reduction in National Insurance contributions in constant prices that is made possible by an increase of one year in the state pension age, if the balance of revenue and expenditure in the National Insurance fund is to remain unchanged. The reduction in National Insurance contributions is directly proportional to the level of the state

[20] This has been chosen because, as will be clear, the decision is a marginal one in public choice terms and therefore illustrates the issues well.

pension and, for illustration, is shown per £1,000 per annum of state pension in constant prices. It can be seen that the change in National Insurance contributions fluctuates somewhat on a generally upward trend. The reduction is greatest when the number of 65-year-olds is high relative to the size of the working population. The number of 65-year-olds is affected by the general process of population ageing as well as by occasional baby booms and other fluctuations.

Given a state pension of £5,000 per annum (approximately the current level in the UK), a new entrant to the workforce would benefit from a reduction of about £150 a year in National Insurance contributions throughout most of his working life and then, at age 65, lose a year of pension worth £5,000 as well as paying additional National Insurance contributions in that year.

The equation of value for a person aged *x*, who is young enough to be affected by the change in the state pension age, allowing for mortality, is:

x = the age of a given voter in 2008
l_{x+t}/l_x is the proportion of voters initially aged *x* in 2008 that is still alive *t* years later.

ΔNI_{2008+t} = is the change in the rate of National Insurance contributions that arises in year 2008+t in order to keep the financial balance in the National Insurance fund if the retirement age is increased by one year.

The real rate of return, *j,* paid by a voter of age *x*, arising from a rise in the retirement age of one year is found by solving the equation:

$$5{,}000\frac{l_{65}}{l_x}(1+j)^{-(65-x)} - \sum_{t=12}^{t=(65-x)} \Delta NI_{2008+t} \times \frac{l_{x+t}}{l_x}(1+j)^{-t} = 0$$

This is an equation of value where *j* (the rate of return to the voter from the policy change) is allowed to vary so that the present value of the expected change in National Insurance contributions (including the additional year of contributions required before the later state pension age) is equal to the present value of the lost year of pension at age 65. It has been assumed that all cash flows occur at the end of the year. Clearly, the equation only applies for those between age 18 and 52. As has been noted, nobody who is aged 54 or above is affected by the policy change and those aged 53 bear only costs with no benefit.

Unlike the situation where we considered an increase in state pensions financed by an increase in National Insurance contributions, a lower internal rate of return is more favourable for the voter in this case. When we consider an increase in state pension age, the policy change involves an individual receiving an initial financial benefit that entails a cost at a later stage (a lost year of pension). The higher the cost, the worse is the outcome for the individual. This situation can be thought of as involving an individual borrowing from the government (through reduced National

Insurance contributions) and then repaying the government at a later stage (through reduced pension payments to the individual). An alternative way of thinking about this is as follows: the voter can neutralise the effect of the proposed policy change by investing any reduction in National Insurance contributions and using the proceeds of the investment to purchase the state pension that has been taken away at age 65. A higher internal rate of return is an indication that the individual would need a higher return in investment markets to compensate for the lost year of pension.

There is, of course, a risk attached to the proposed policy change. A young voter might vote for the policy change and find that, at the time of retirement, the retirement age has been increased further – though it is also possible that the retirement age could be reduced back to its previous level. In theory, a break-even rate of return should be determined that reflects the risk of the policy change to a marginal young elector who may just be favourably inclined towards it. In practice it is not possible to observe what that rate of return would be from market data and it will vary from individual to individual. We will assume that the real rate of return that electors would be happy to pay is 2.5% per annum effective (approximately 1.5% above the real rate of return from risk-free government instruments), but also look at results at a rate of return of 2% per annum effective. In effect, therefore, we are assuming that electors believe that they will be able to replace the lost year of pension, with an equivalent degree of risk, by investing their reduced National Insurance contributions in investment instruments bearing a real rate of return of 2.5% per annum or 2% per annum.

The rate of return to voters of different ages is shown in Figure Six below. The rates of return paid by the older voters who are affected by the change are very high: for example a 52 year old would experience a reduction in National Insurance contributions in 2020 of just £122.33 (assuming a state pension of £5,000 per annum) and suffer a cost of £6,633.95 in 2021 as a result of paying an extra year of National Insurance contributions and losing a year of pension. For this reason, to prevent the scale of the graph from being too small, the rates of return arising from the policy change are shown only up to age 45, the rates of return for ages from 46 to 52 range from 41.7% to 640%.

The steep rise in the rate of return with age from this kind of policy is very clear from Figure Six. A government wishing to phase in an increase in the state pension age can expect a large measure of indifference from the old (in this case, defined as those age 54 and above) and from the young; but some age groups will be very severely affected. It is also of interest to note that the government cannot change this *general* pattern by the way it phases in a change: a rise in the retirement age will always adversely affect those just below the cut-off age. The government can lengthen the phasing in period so that fewer people in total will be affected, but there will always be some people who are adversely affected.

Voters aged 18-23 inclusive have a rate of return of 2.5% and, according to our assumptions, may be willing to support the policy change. Those at the margins or who are not directly affected might be expected to benefit slightly from the policy change because of the second-order effects of increased labour-market efficiency arising from lower labour-market taxes.

The proportion of voters who benefit from the change is shown in Table Two. The proportion is calculated assuming required rates of return of 2% and 2.5%, with and without making an allowance for age-related differential turnout of voters.

It can be seen that the decision to raise the retirement age by one year deferred for 12 years in 2008 would have been an extremely marginal one from a public choice point of view in the median voter model. Assuming that a government is attempting to build a coalition of *active* voters favourable to reform then, if the required rate return from the expected financial cash flows from the policy change were 2.5% (an assumption that does not seem unreasonable), then there would just be a majority benefiting from reform.

An analysis of actual policy changes

Policy Change Two is on the margins of electoral palatability. The most significant related recent policy change in the UK has been the recent decision to raise gradually the state retirement age by three years from 65 to 68. The first part of the increase will take place over two years from 2024. Here, we just note briefly the differences between the public choice impact of that change and the specific change analysed in detail here.

The major difference is that the increase in the state pension age is being deferred four years more than has been assumed in the above calculations. The direct effect of this is to add four years of age to the group that is unaffected by the first round effects of the change – nobody aged 50 or older will be affected. This is roughly an additional 7% of the electorate.[21] There will be some other, second order, effects which are likely to be small. For example, younger voters will have to wait longer before they benefit from a reduction in National Insurance contributions than we assumed in Policy Change Two because of the phasing-in. However, some voters will also have to wait longer before the loss of a year's pension – thus reducing the present value of that loss.

Perhaps most crucially, it should be noted that the age of the median voter in 2008 was 51, after allowing for voting propensities. This means that the increase in the state pension age has been determined so that just over 50% of the electorate will not be affected at all by the change (and could benefit slightly from the general economic benefits arising from lower

[21] This varies, of course, according to whether an allowance is made for age-related propensities to vote, but it does not vary substantially. This percentage will be increased slightly further by the phasing of the increase in retirement age over two years.

labour market taxes) and thus it should gain electoral support, even ignoring the possibility that some young voters will benefit. So it is clear that the one significant cost-reducing pension reform in the UK in the last 20 years has been determined in such a way that the majority of voters either benefit from the change or will not lose from it.

As has been noted, the specific way in which changes to state pension benefits are being brought in will also have some impact on income distribution. We have abstracted from these effects for reasons already discussed. In this particular case, if there is a reduction in the percentage rate of National Insurance contributions financed by an increase in pension age, then there will be some younger, poorer people who will gain less than the average young person from the rise in the retirement age. At the same time, there will be some older, better-off people who will lose less than the average older person. In any case, with regard to the UK state pension age, over 50% of active voters were exempt from any impact of the change as a result of its deferral. Any younger people who benefited from the increase in state pension age would simply add more voters to the 50% who might already be expected to not oppose the change.

Conclusion

The calculations in this paper reveal that there will be strong democratic pressures in the UK to expand government pension systems as the population ages. The pressures in most other European countries are likely to be greater because of the greater maturity of the age structure of the population. However, it has also been shown that it is possible to scale back pension systems by raising retirement ages because the public choice impact of such a policy is often more favourable than the impact of, for example, reducing the size of pensions in real terms.

The calculations are remarkably consistent with practice in most European countries. Thus the formal theory and related calculations seem to explain public policy in practice. This is a result that has been found by other authors in different contexts. Retirement ages in state pension schemes have often been increased – with deferral periods being chosen so that the results are acceptable in public choice terms. At the same time, it has been difficult to scale back benefits. It is also interesting that this result is consistent with the strategy for short-run reform proposed by Buchanan (1983).

Further research would be useful to investigate the way in which changes to state pension schemes affect the distribution of income *within* generations. This would require detailed information of income distribution on a cross-sectional basis. Research can also be undertaken using general equilibrium models that can capture second-order effects. However, the results of the above work, where possible, have been checked against the results from a general equilibrium model which has

been used for very detailed calculations in a number of countries (see Galasso, 2006).

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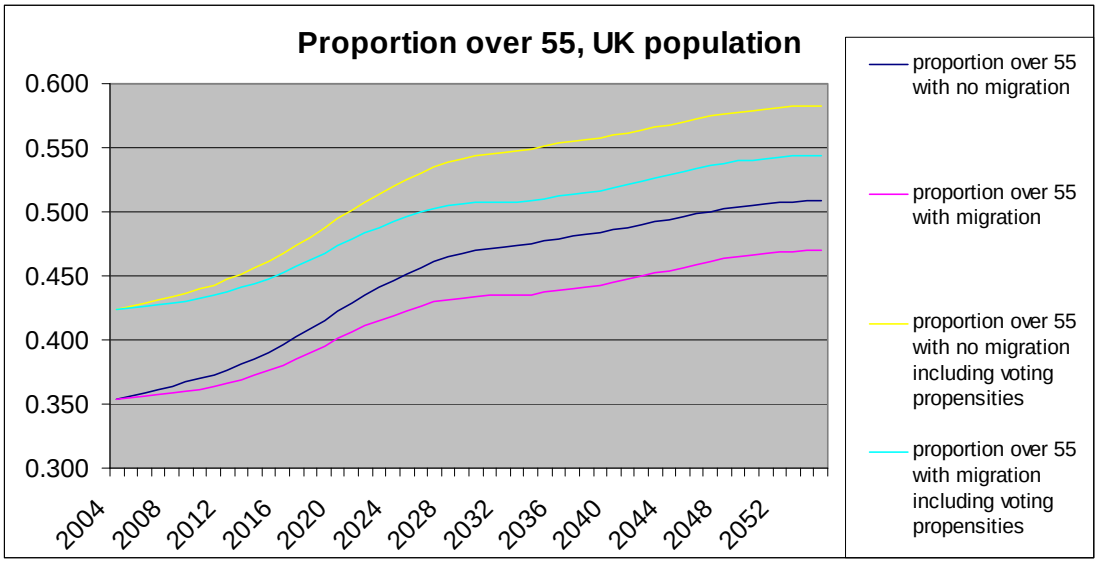

Figure One

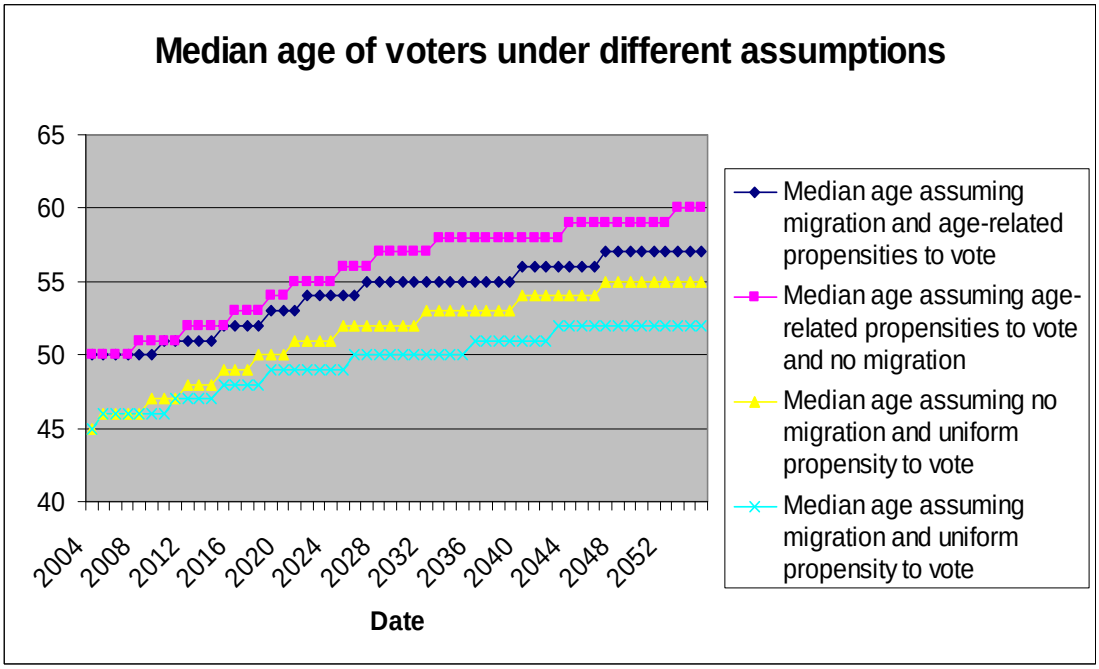

Figure Two

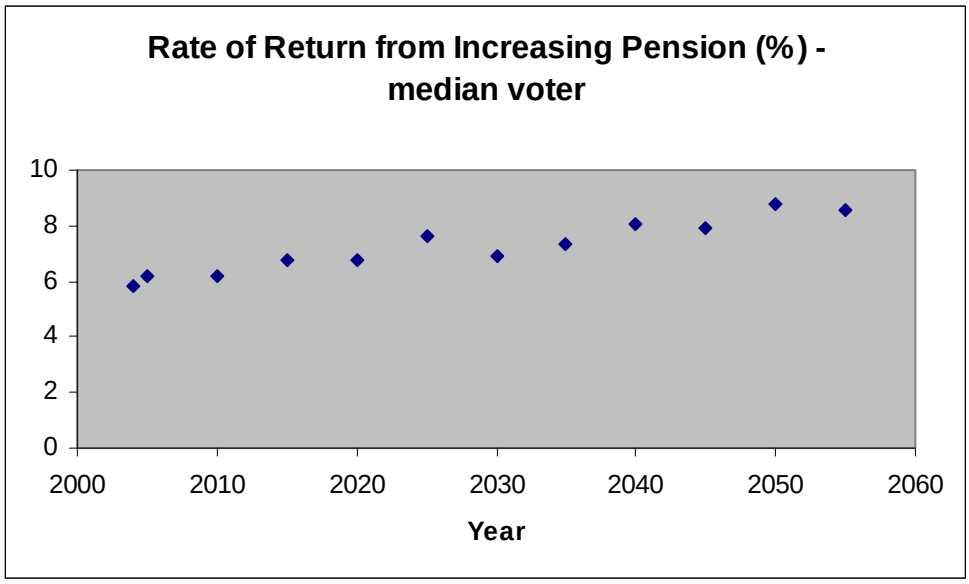

Figure Three

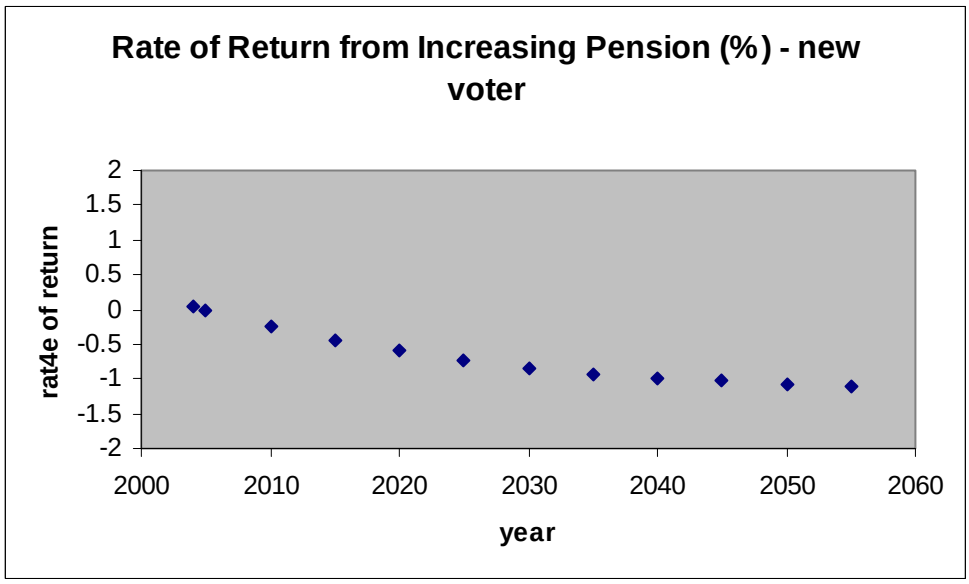

Figure Four

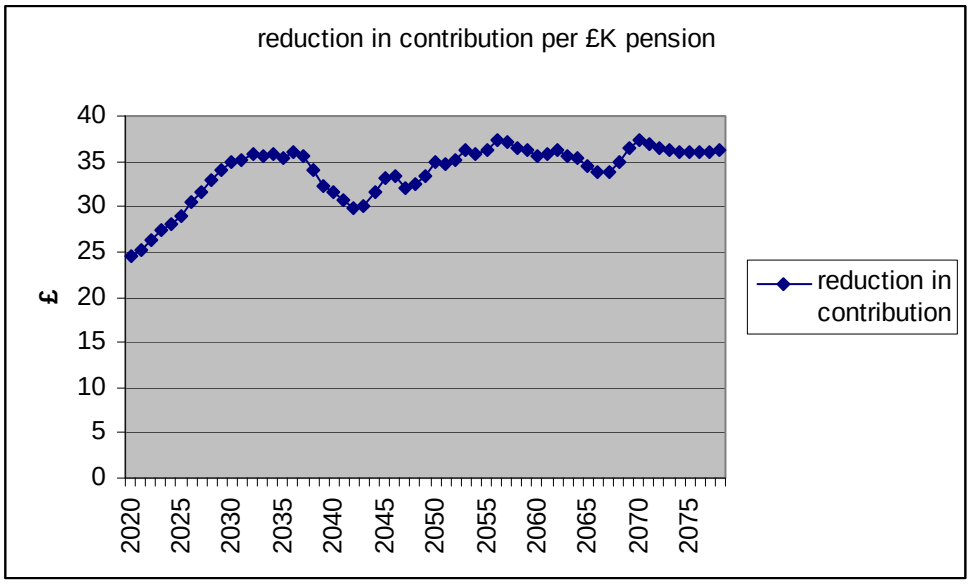

Figure Five

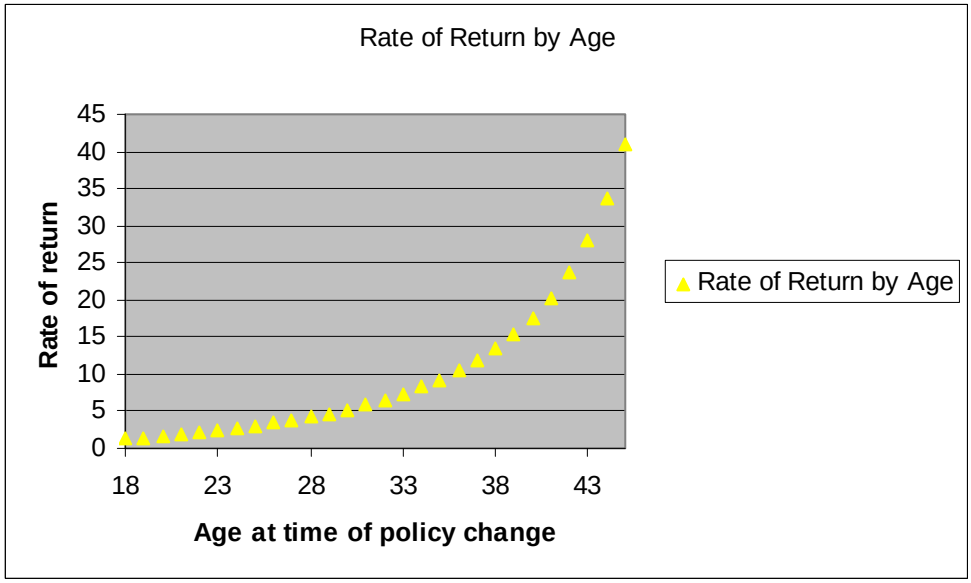

Figure Six

Date	Projected increase in national insurance contributions £ p.a.
2008	532
2018	665
2028	832
2038	1035
2048	1080
2058	1157
2068	1189
2078	1240

Table One – Increase in National Insurance contributions necessary to finance a £2,000 increase in the basic state pension

	Proportion of active voters who benefit %	Proportion of those registered to vote who benefit %
Required return 2%	49%	44.6
Required return 2.5%	51%	47.9

Table Two - Proportion of voters benefiting from Policy Change Two

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