Smart Beta, Scrabble and Simian Indices

Presented by
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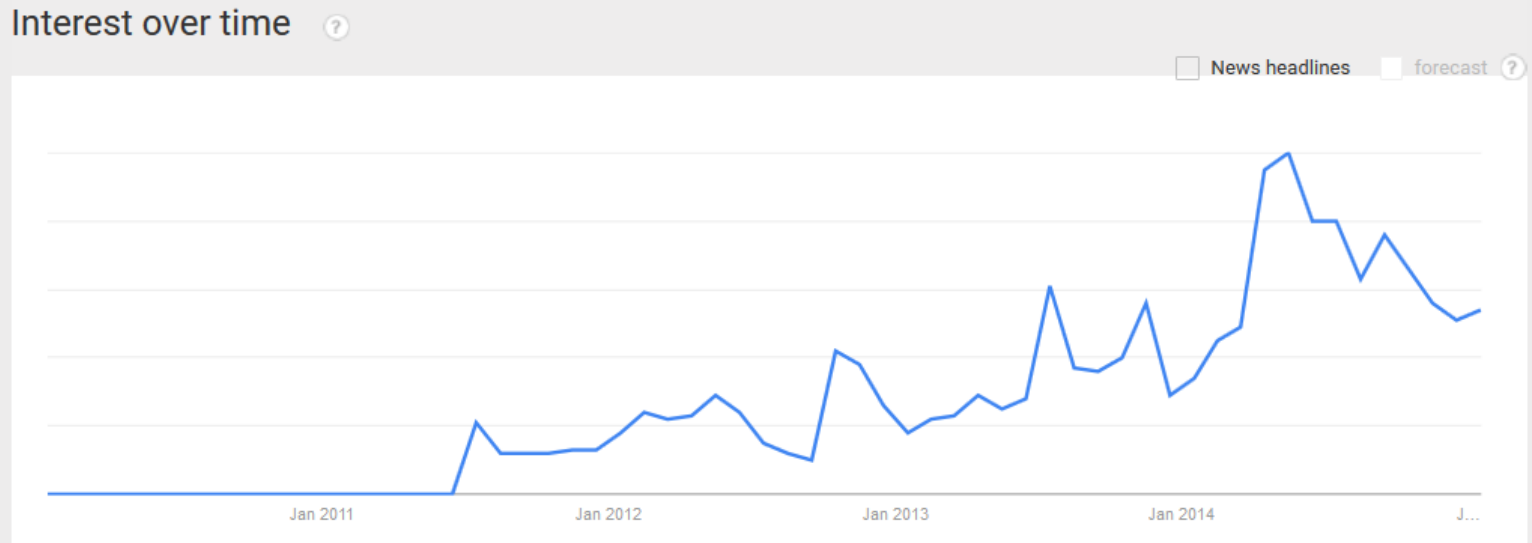
INTRODUCTION

INVESTOPEDIA

DEFINITION OF 'SMART BETA'

Smart beta defines a set of investment strategies that emphasize the use of alternative index construction rules to traditional market capitalization based indices. Smart beta emphasizes capturing investment factors or market inefficiencies in a rules-based and transparent way.

Google Trends



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INTRODUCTION

Smart beta gaining ground in battle for investment supremacy

David Oakley, Investment Correspondent

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The business model of active fund managers is under threat from a relatively new form of investing as customers increasingly demand lower fees and better value for their money.

Smart beta, a hybrid form of active and passive management, is rapidly gaining market share, growing to $544bn in assets under management globally, a fivefold increase since the financial crisis in 2008, says data company Morningstar.

September 23, 2015, 12:27 P.M. ET

Finra Warns On 'Smart Beta' ETF Risks

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INTRODUCTION

- In 2013 we released two research papers commissioned by AON Hewitt[1][2]
- We examined 8 alternative index methodologies which are all available as commercial products and have become collectively known as "Smart Beta"
- We used a common set of stocks* over a 43 year period in order to make an "apples to apples" comparison
- Today I will share some updated insights from that research based on a series of white papers commissioned by Invesco PowerShares

**In the original papers we use the largest 1,000 US stocks from CRSP, today I will use the results from the largest 500 stocks, where our market-cap benchmark is 99.71% correlated with the S&P 500 and extend the results through 2014.*

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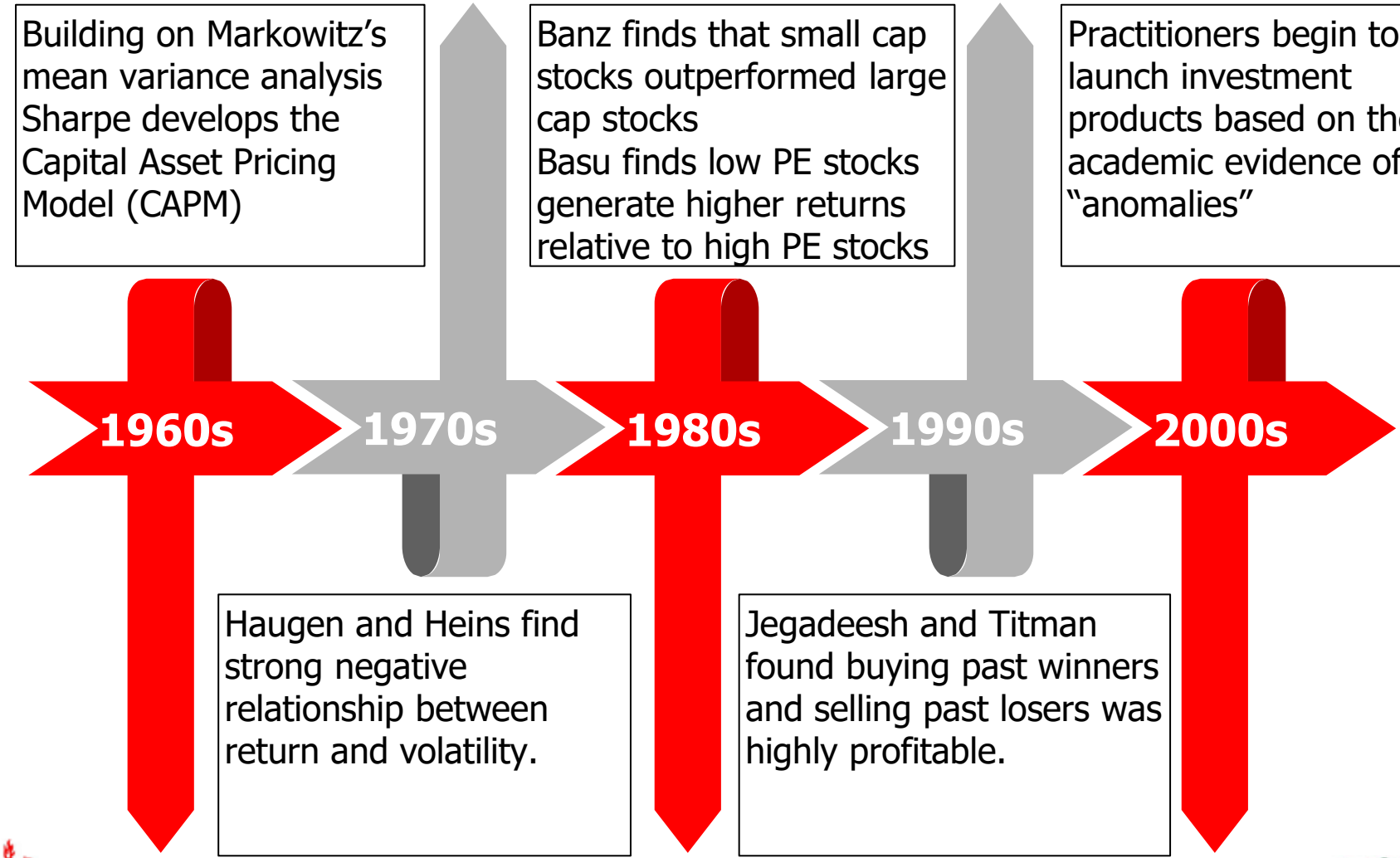
THE ACADEMIC ORIGINS OF SMART BETA



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WHY ALTERNATIVE WEIGHTING SCHEMES?

- A major advantage of a cap-weighted index is low turnover, (the weights of the stocks in the index change as the market value of the stocks change) hence tracking is easy and expenses will be low.
  - However, a counter argument is that market-cap weighting can put more weight on companies that might be most overvalued by the stock market and less weight on companies that are possibly undervalued
  - Cap-weighted indices can also be quite concentrated with a small proportion of stocks making up the bulk of the index weights
  - Alternative indices might seek to exploit other risk premium or known anomalies such as size, value, momentum or low-beta

THE SET OF ALTERNATIVES

- The eight alternative approached considered:
  - Equally Weighted
  - Diversity Weighted
  - Inverse Volatility
  - Equal Risk Contribution
  - Minimum Variance
  - Maximum Diversification
  - Risk Efficient
  - Fundamentally Weighted

**Please see the appendix for the research justifying each alternative as well as the methodology*

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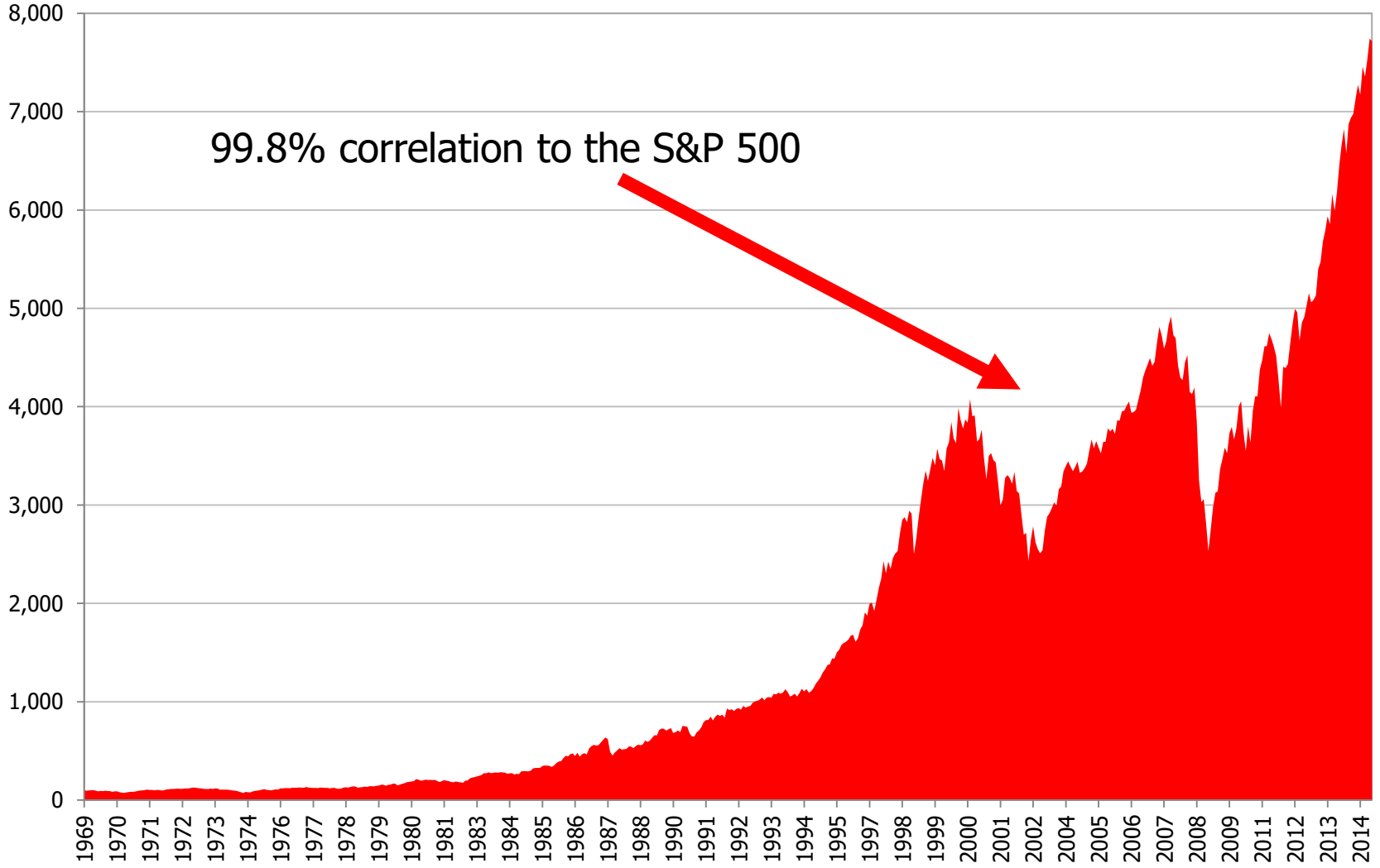
DATA AND METHODOLOGY

- Equity data from CRSP
  - Selected 500 largest market cap universe with requirement of 5 years of continuous historical returns
  - Sample period December 1968 to December 2014
  - Rebalanced annually in December

- We followed as closely as possible the index providers methodology but stress we were looking at the *spirit* as opposed to the *law* of construction using the academic papers as our guide

THE BENCHMARK MARKET CAP INDEX



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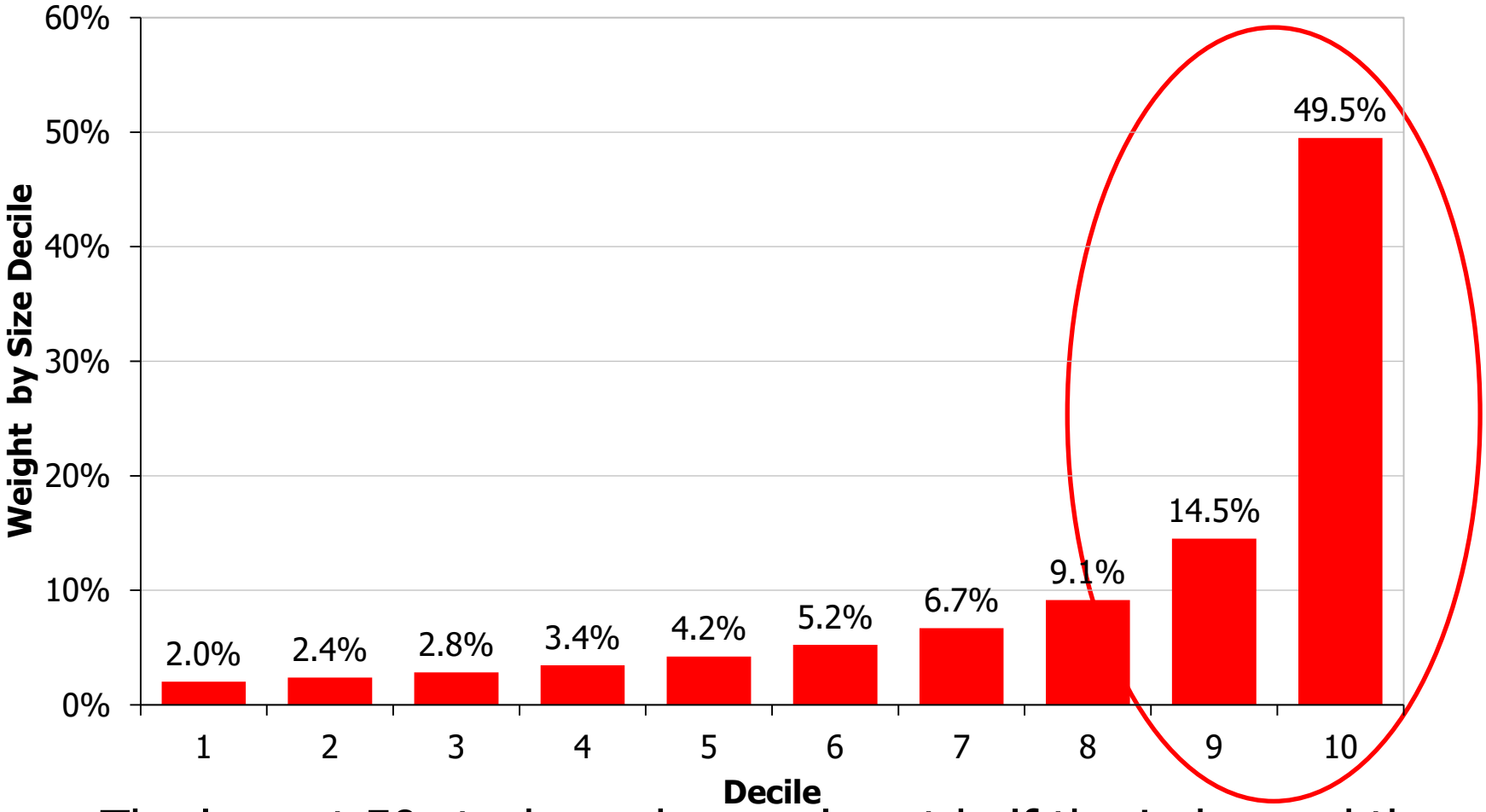
THE MARKET CAP INDEX IS INDEED QUITE CONCENTRATED



- The largest 50 stocks make up almost half the index and the largest 100 make up almost two thirds

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FULL SAMPLE RESULTS 1969-2014
RETURN AND RISK

	Mean Return	Standard Deviation	Sharpe Ratio
Market Cap-Weighted	10.62%	15.00%	0.38
Equally-Weighted	11.93%	16.15%	0.43
Diversity-Weighted	10.98%	15.27%	0.39
Inverse Volatility-Weighted	11.79%	14.13%	0.48
Equal Risk Contribution	11.88%	14.93%	0.46
Minimum Variance Portfolio	10.83%	12.04%	0.49
Maximum Diversification	11.62%	14.16%	0.47
Risk Efficient	12.03%	15.62%	0.45
Fundamentally-Weighted	11.89%	14.81%	0.47

- All 8 of the alternative indices have a higher return
- 5 out of 8 have lower volatility
- All 8 have a higher Sharpe Ratio

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RESULTS BY DECADE

	1970s		1980s		1990s		2000s	
	Mean Return	Sharpe Ratio	Mean Return	Sharpe Ratio	Mean Return	Sharpe Ratio	Mean Return	Sharpe Ratio
Market Cap-Weighted	6.76%	0.04	16.84%	0.51	17.57%	0.97	1.03%	-0.11
Equally-Weighted	8.91%	0.15	17.41%	0.53	16.13%	0.83	5.52%	0.17
Diversity-Weighted	7.33%	0.07	17.06%	0.52	17.08%	0.93	2.34%	-0.03
Inverse Volatility-Weighted	9.04%	0.18	18.71%	0.68	13.51%	0.72	6.06%	0.24
Equal Risk Contribution	9.03%	0.17	18.18%	0.61	14.61%	0.77	5.84%	0.21
Minimum Variance Portfolio	7.73%	0.12	19.78%	0.90	10.31%	0.49	6.46%	0.31
Maximum Diversification	7.92%	0.10	19.61%	0.77	13.28%	0.68	5.48%	0.22
Risk Efficient	9.45%	0.18	18.17%	0.60	14.56%	0.74	6.06%	0.21
Fundamentally-Weighted	9.40%	0.21	18.21%	0.63	16.19%	0.92	4.15%	0.09

- Similar pattern in the 1970s, 1980s & 2000s
- In the bull market of the 1990s Market Cap beat everything

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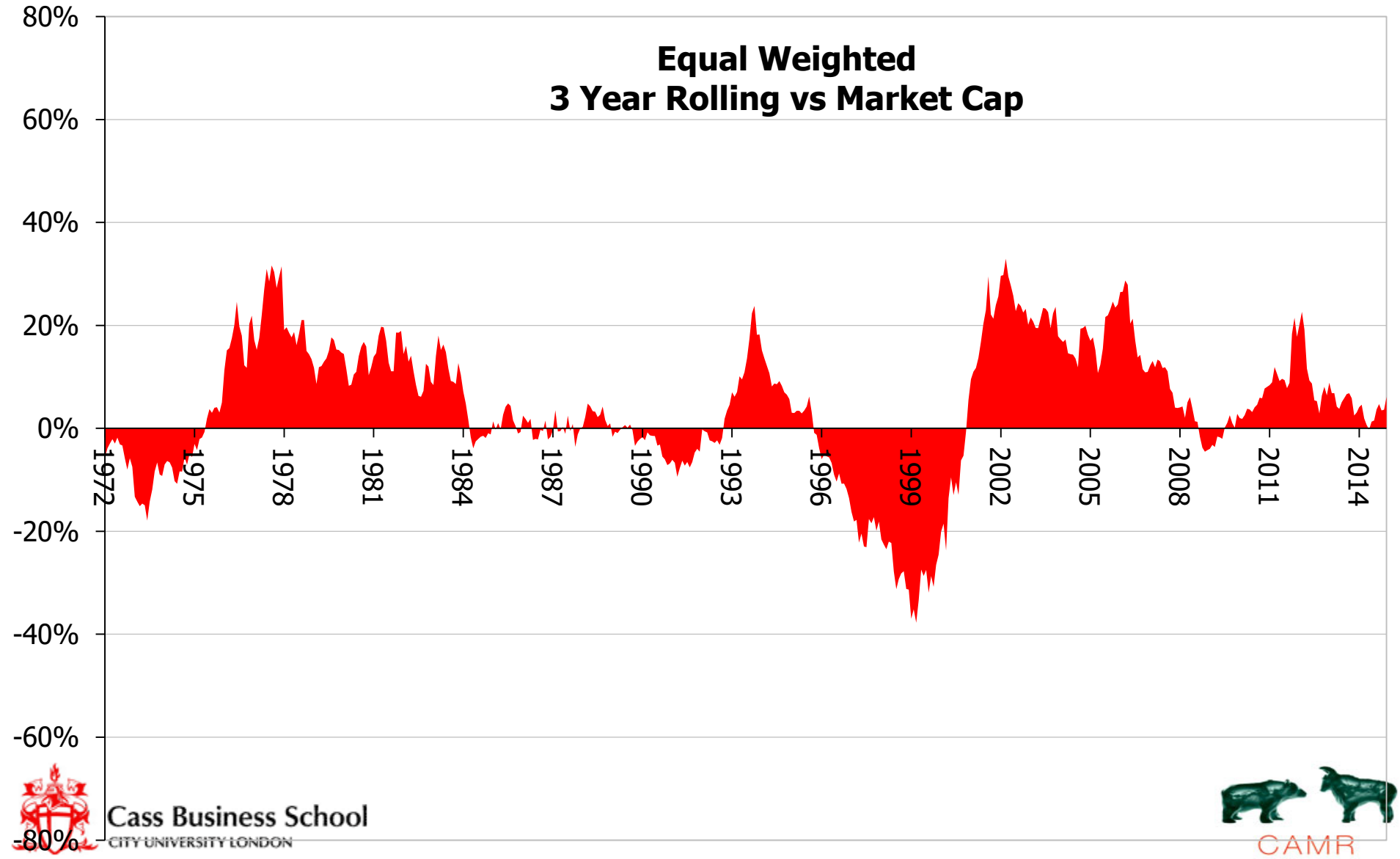
MOST OF THE ALTERNATIVES SUFFER FROM PERIODS OF SOMETIMES SEVERE UNDERPERFORMANCE



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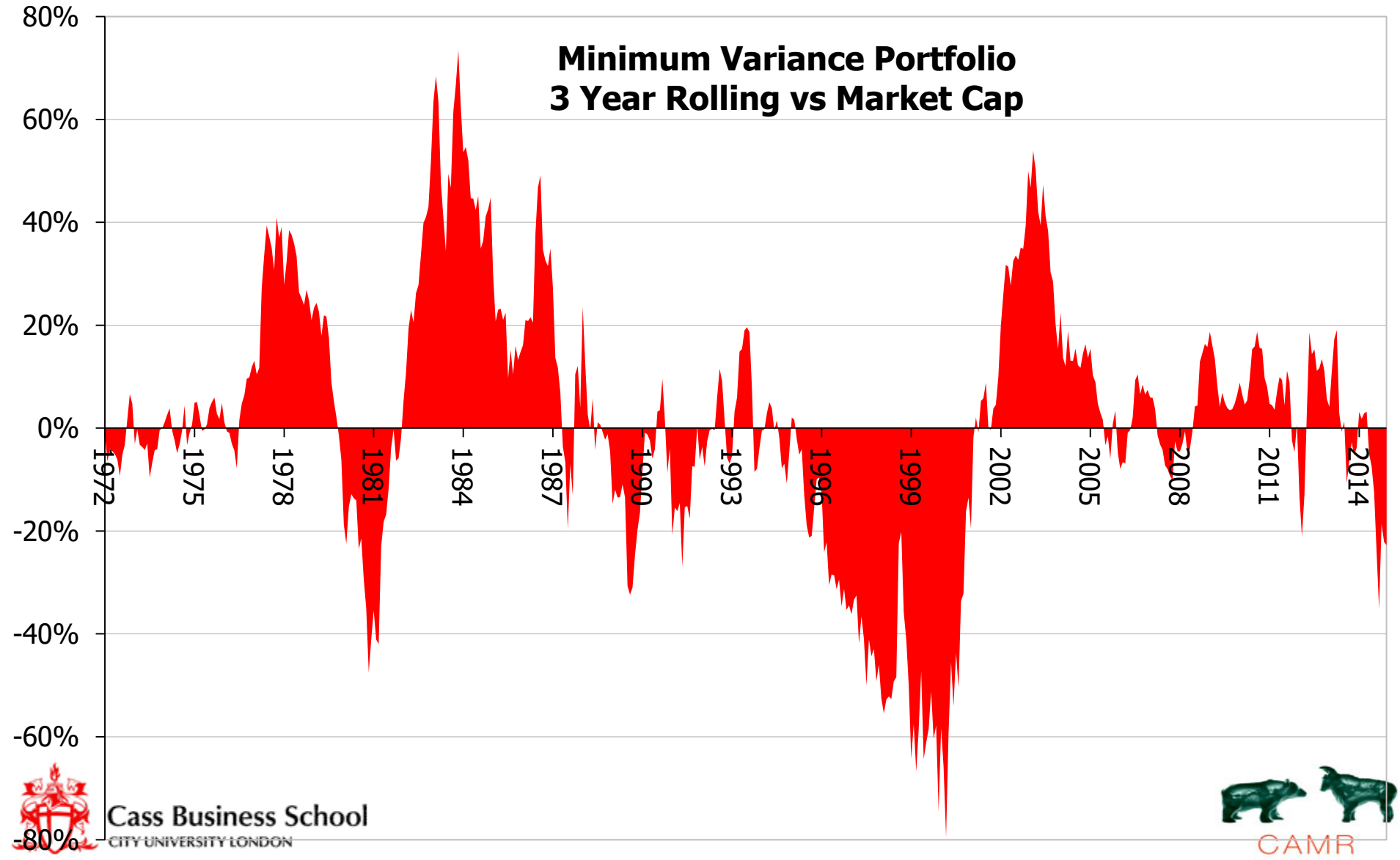

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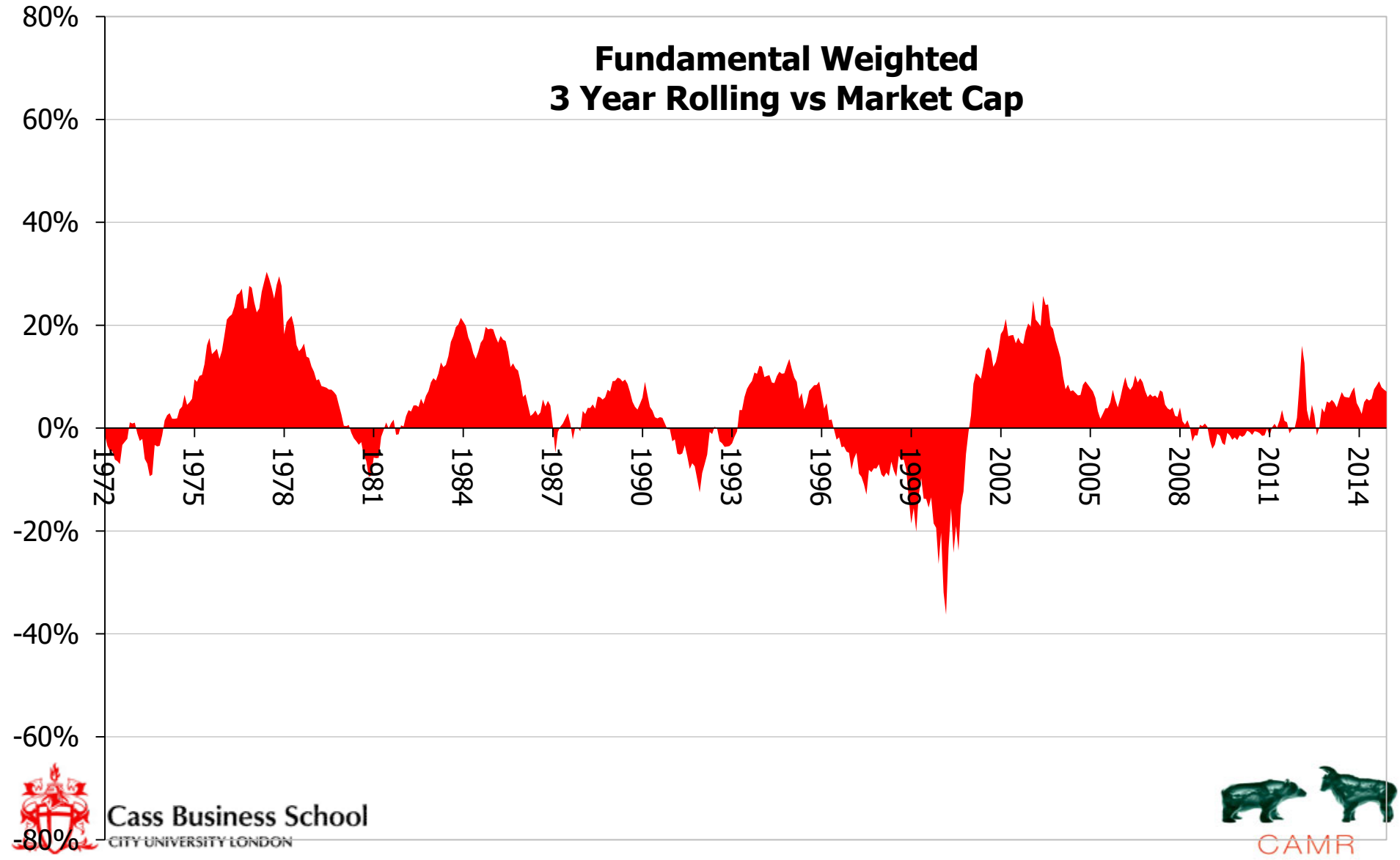

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ARE THE SHARPE RATIOS SIGNIFICANTLY DIFFERENT FROM THE MARKET CAP INDEX?

- From the previous slides it might be argued that in fact all we are seeing is noise and the Sharpe ratios are in fact not different.
- Testing whether Sharpe ratios are statistically different from each other is non-trivial. We use a robust procedure proposed by Ledoit and Wolf (2008) [12]

	Sharpe Ratio	p-value
Market Cap-Weighted	0.38	-
Equally-Weighted	0.43	17.3%
Diversity-Weighted	0.39	17.6%
Inverse Volatility-Weighted	0.48	5.1%
Equal Risk Contribution	0.46	5.2%
Minimum Variance Portfolio	0.49	37.4%
Maximum Diversification	0.47	12.9%
Risk Efficient	0.45	11.0%
Fundamentally-Weighted	0.47	5.5%

- Only 3 of the 8 are statistically different……..

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DO SMART BETA INDICES HAVE ALPHA?

	CAPM Beta	CAPM Alpha
Market Cap-Weighted	1.00	0.00%
Equally-Weighted	1.06	0.84%
Diversity-Weighted	1.02	0.20%
Inverse Volatility-Weighted	0.89	2.31%**
Equal Risk Contribution	0.96	1.68%*
Minimum Variance Portfolio	0.51	4.73%*
Maximum Diversification	0.82	2.34%*
Risk Efficient	1.01	1.44%*
Fundamentally-Weighted	0.96	1.75%**

- A single factor CAPM model would suggest that 6 of the 8 alternatives have positive and significant alpha

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DO SMART BETA INDICES HAVE ALPHA?

	Alpha	Rm-Rf	SMB	HML	R^2
Equally-Weighted	0.01%	0.98**	-0.17**	0.02**	100%
	[1.00]	[318.06]	[-39.49]	[4.77]	
Diversity-Weighted	0.02%	1.02**	0.04**	0.17**	97%
	[0.47]	[124.44]	[3.45]	[13.85]	
Inverse Volatility-Weighted	0.01%	0.99**	-0.12**	0.07**	99%
	[0.76]	[276.46]	[-22.82]	[12.29]	
Equal Risk Contribution	0.03%	0.91**	-0.08**	0.30**	93%
	[0.66]	[84.14]	[-4.92]	[17.97]	
Minimum Variance Portfolio	0.03%	0.96**	-0.03*	0.24**	95%
	[0.70]	[104.00]	[-2.00]	[17.02]	
Maximum Diversification	0.06%	0.64**	-0.18**	0.39**	62%
	[0.67]	[29.42]	[-5.77]	[11.89]	
Risk Efficient	0.08%	0.86**	-0.01	0.16**	87%
	[1.27]	[57.29]	[-0.30]	[6.89]	
Fundamentally-Weighted	0.00%	0.99**	0.04*	0.27**	95%
	[0.08]	[101.53]	[2.55]	[18.38]	

- However adding the Fama French factors for size and value removes all sign of any alpha

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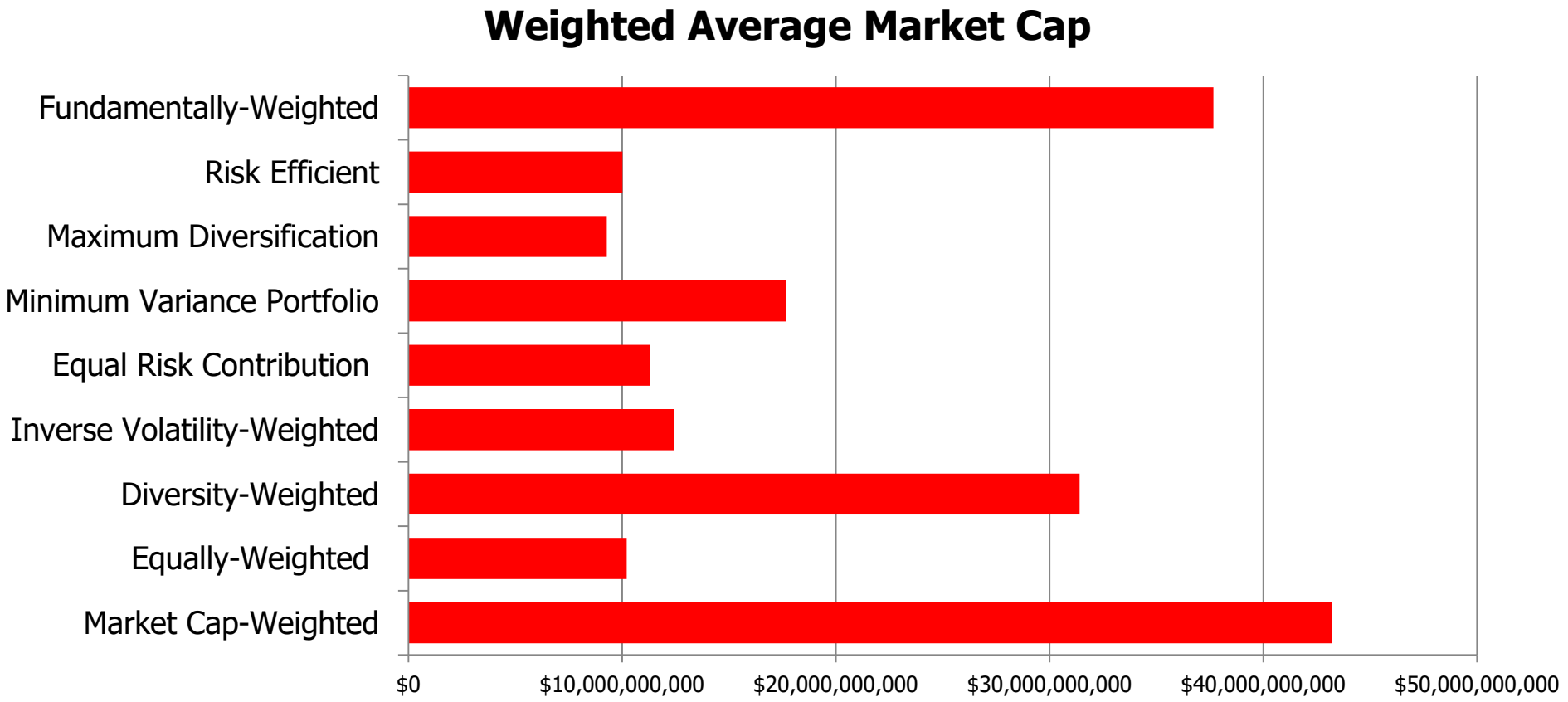
SEEMS THE ALTERNATIVES HAVE A SMALL CAP BIAS



- All of the alternatives hold much smaller stocks
- Except perhaps the fundamentally weighted index

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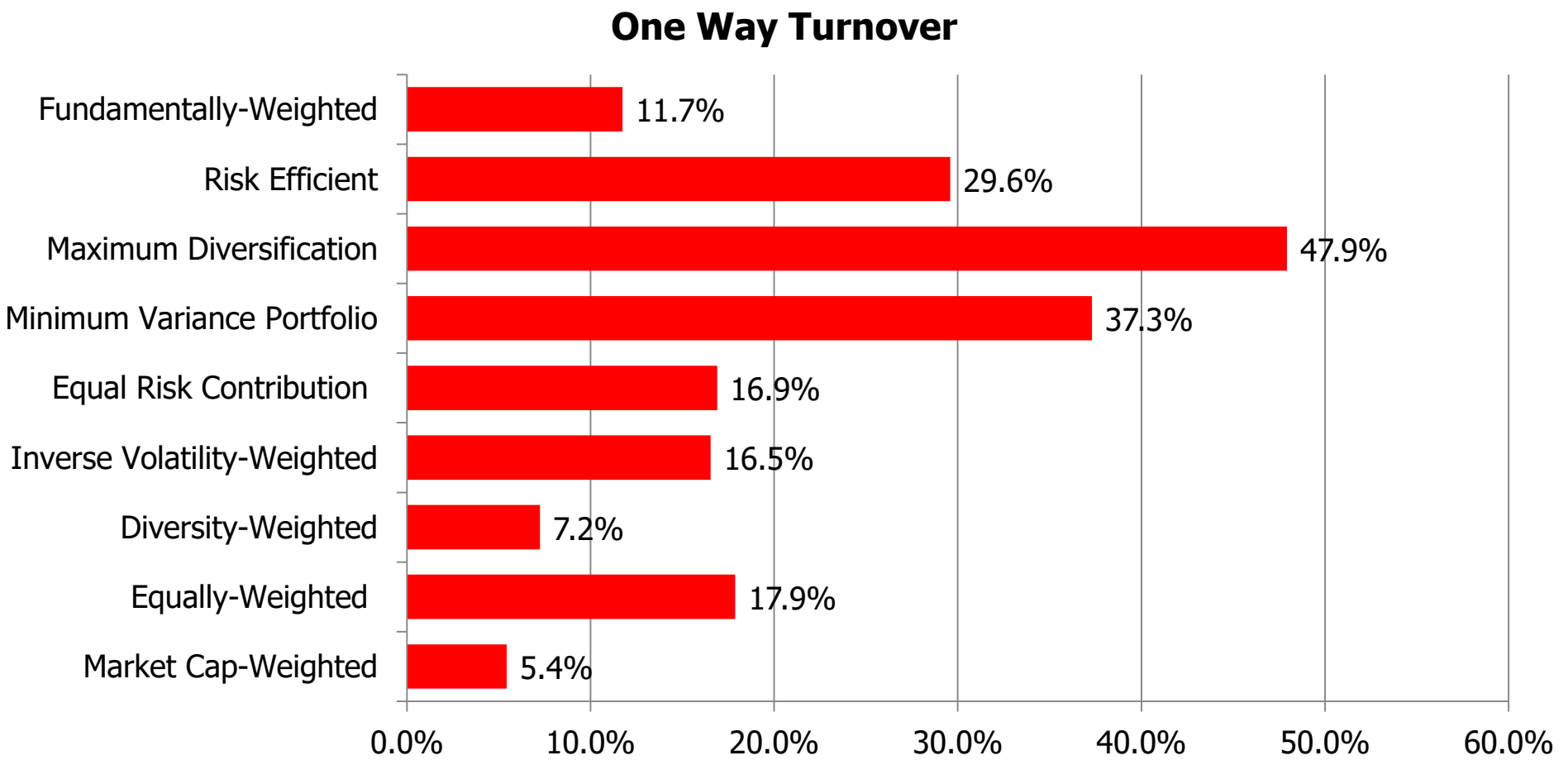
WHAT ABOUT TRANSACTIONS COSTS?

- Good performance is one thing, but none of our results incorporate transactions costs

- What we did was:
  - calculate the turnover involved in rebalancing the index every year and
  - then calculate how high transactions costs would need to be to eliminate the performance difference

- In our view, in most cases bid-ask spreads would need to have been unbelievably high to have eliminated all of the difference

TURNOVER



- All of the alternatives have higher turnover than market cap

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TRANSACTIONS COSTS

- So how high would transactions costs have to have been to eliminate the performance difference?

	1-Way Turnover	Transaction Cost to Equalize Return	Transaction Cost to Equalize Sharpe Ratio
Market Cap-Weighted	5.4%	-	-
Equally-Weighted	17.9%	5.4%	3.8%
Diversity-Weighted	7.2%	10.3%	7.9%
Inverse Volatility-Weighted	16.5%	5.4%	6.7%
Equal Risk Contribution	16.9%	5.6%	5.8%
Minimum Variance Portfolio	37.3%	0.3%	2.1%
Maximum Diversification	47.9%	1.2%	1.6%
Risk Efficient	29.6%	3.0%	2.6%
Fundamentally-Weighted	11.7%	10.3%	10.7%

- In almost all cases, transaction costs would have had to be unfeasibly high to eliminate the alternative index's outperformance of the cap-weighted index

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THE STORY SO FAR…….

- We have 8 alternative indices all of which have had a higher Sharpe ratio than market-cap over a 46 year sample period.
  - 3 have statistically different Sharpe ratios than market-cap
  - Nearly all have CAPM alpha
  - None have FF alpha
  - All are being offered as commercial products

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WE INTERRUPT THIS PRESENTATION FOR

A COMMERCIAL BREAK

THE SCRABBLE™ INDEX

- Knowing that the market cap index is poorly diversified
  - 10% of the stocks make up 50% of the weight
- We can construct a better diversified index by using an "innovative" weighting scheme
  - Using the ticker symbol for each stock we calculate the Scrabble™ score for each stock
    - (1 point)-A, E, I, O, U, L, N, S, T, R.
    - (2 points)-D, G.
    - (3 points)-B, C, M, P.
    - (4 points)-F, H, V, W, Y.
    - (5 points)-K.
    - (8 points)- J, X.
    - (10 points)-Q, Z
  - We then sum the scores and divide each stocks score by the total to calculate the weight e.g. XOM has twice the weight of AAPL

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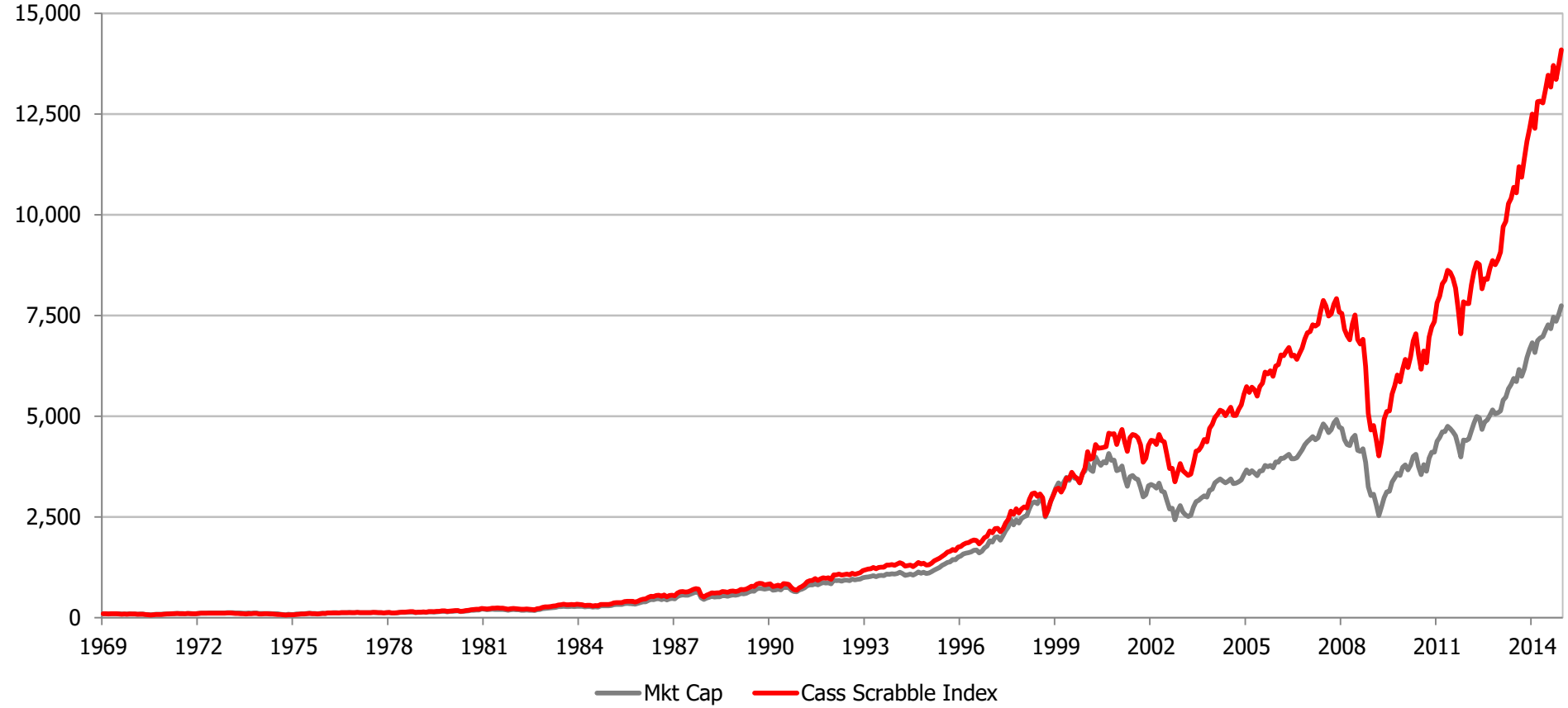
THE SCRABBLE™ INDEX PERFORMED WELL (MUCH BETTER THAN MARKET-CAP)



	Mean Return	Standard Deviation	Sharpe Ratio	Maximum Drawdown	Tracking Error	Information Ratio	Active Share	1-Way Turnover
Cass Scrabble Index	12.1%	16.3%	0.44	-49.3%	4.0%	0.38	46.9%	18.3%
Market Cap-Weighted	10.6%	15.0%	0.38	-48.5%	-	-	-	5.4%

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IS THE MARKET CAP INDEX REALLY THE CORRECT BENCHMARK?

- Obviously the Cass Scrabble™ Index is not a real investment proposition
  - Though if you're interested drop me an email

- There are an infinite number of combinations of weights for 500 stocks that sum to 1
  - The real question is where the market cap index and the 8 smart beta indices sit in the distribution of possible weighting schemes
  - Just beating market cap might not be that difficult

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BRING ON THE MONKEYS!

"If one puts an infinite number of monkeys in front of (strongly built) typewriters and lets them clap away (without destroying the machinery), there is a certainty that one of them will come out with an exact version of the 'Iliad.' Once that hero among monkeys is found, would any reader invest [their] life's savings on a bet that the monkey would write the 'Odyssey' next?"[13]

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WHERE TO FIND AN INFINITE NUMBER OF MONKEYS? (NO MONKEYS WERE HARMED IN THIS EXPERIMENT)

- Even though Cass is located in the square mile we did not have access to an infinite number of monkeys (or typewriters for that matter)

- Instead we devised a robust procedure* to generate 500 random weights that sum to one with a minimum increment of 0.2% and then relied on some serious computer power.

- We constructed 10 million randomly weighted indices and calculated various performance metrics

**See Appendix 2 for details of the algorithm*

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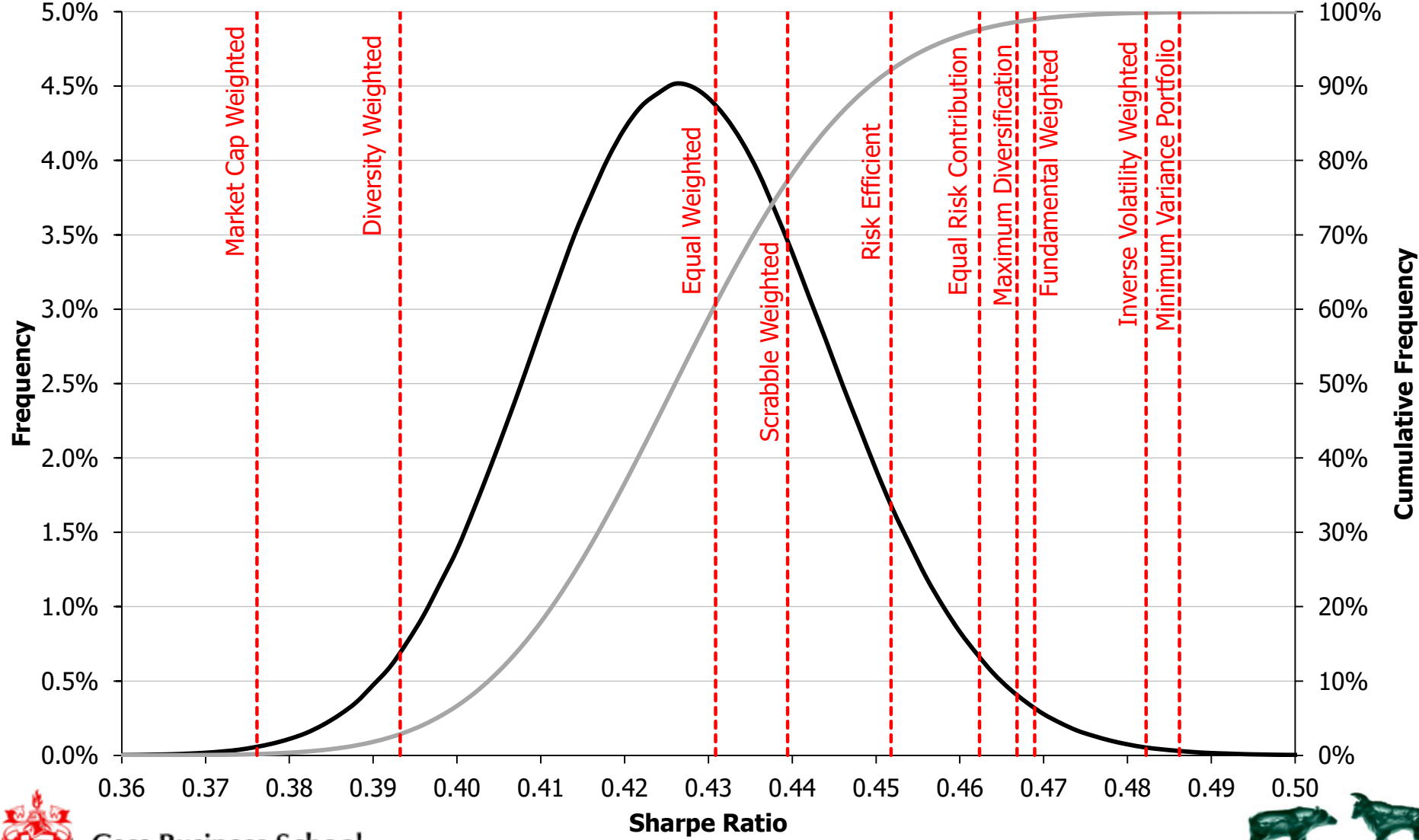
10 MILLION SIMIAN INDICES vs SMART BETA & SCRABBLE INDICES



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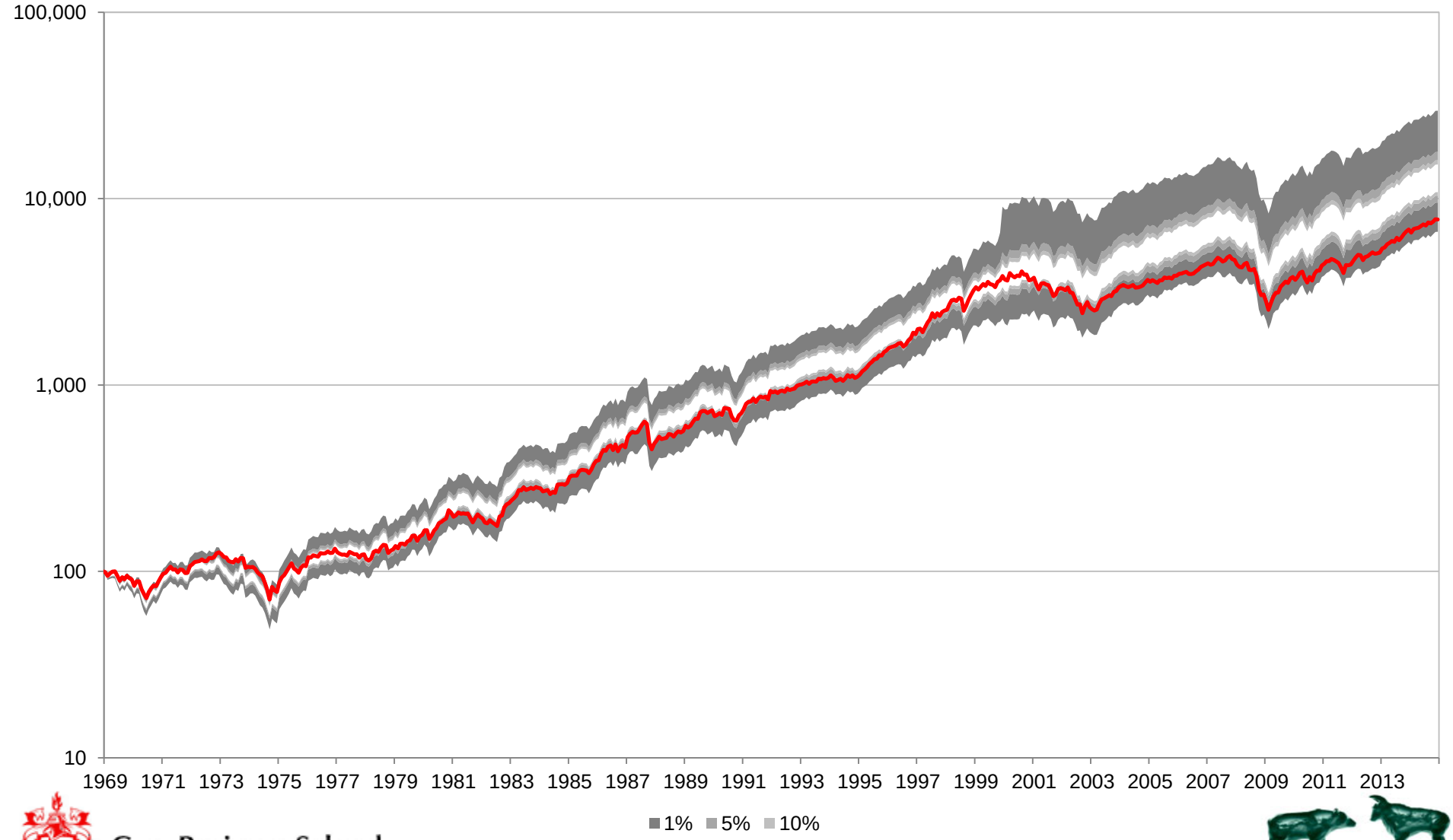
10 MILLION SIMIAN INDICES VERSUS MARKET CAP OVER TIME



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OUR CONCLUSIONS FROM THIS RESEARCH

- The back-tested historical risk adjusted returns of "smart beta" indices look good when compared to a market cap weighted index
- The outperformance can be explained by exposure to value and size factors
- There have been periods of sometimes severe underperformance for all of the "smart beta" approaches
- 99.82% of random (or simian) indices would also have beaten market cap over the same period BUT "smart beta" generally beat over 90% of the monkeys.
- A Scrabble weighted index might be a tough sell.

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THE PRESS CONCLUSIONS FROM THIS RESEARCH

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Cass Business School: 'Monkeys' outperform passive funds

3 April 2013 | By Jonathan Yarker

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Fund management

The rise of smart beta

Terrible name, interesting trend

Jul 6th 2013 | From the print edition

INVESTORS face a quandary. Cash offers a return of virtua
countries; government-bond yields may have risen in recen
unattractive. Equities have suffered two big bear markets si
again. It is hardly surprising that pension funds, insurers an
for new sources of return.

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Standard index tracker funds are designed in the wrong way - 50 years worth of data suggests

9 | 0 | 51 | 60 | Email

The 'scrabble index' was made up of the 500 largest US stocks, weighted according to their scrabble scores.

Appendix 1:
Constructing The Alternatives

CONSTRUCTING THE SET OF ALTERNATIVES

- Equally-weighted
  - Each stock is given a weight of 1/n. This very simple and perhaps somewhat naive approach to determining weights was examined by DeMiguel, Garlappi and Uppal (2009) [3] and found to outperform many more sophisticated methods due to the avoidance of parameter estimation errors. Numerous investable equal weight indices are available e.g. S&P, MSCI, Russell etc.

- Diversity-weighted
  - This approach was first proposed by Fernholz et al (1998) [4] Effectively it involves raising the Market-cap weight (w) of each constituent by the value p, that is w^p, where p is bounded between 1 and 0. The weight of each index constituent is then calculated by dividing its w^p weight by the sum of all w^ps of all of the constituents in the index. When p is set to 1 then the constituent weights are equal to Market-cap weights and when p is set to 0 the weights are equivalent to equal weights. We use p=0.76 which is the value used in the original paper. INTECH Investment Management LLC apply the diversity approach.

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CONSTRUCTING THE SET OF ALTERNATIVES

- Inverse volatility
  - In the mid-1970s Haugen and Heins [5] published a paper that demonstrated that low volatility stocks tended to outperform high volatility stocks, since then there has been much research on the "low-volatility anomaly". We calculate the historical return variance of each stock using five years of monthly data. We then calculate the inverse of this value, so that the stock with the lowest volatility will have the highest inverted volatility. We then simply summed these inverted variances. The weight of stock i is then calculated by dividing the inverse of its return variance by the total inverted return variance. This process therefore assigns the biggest weight to the stock with the lowest volatility, and the lowest weight to the stock with the highest return volatility. Various investable indices based on this concept are available e.g. S&P Low Vol, MSCI Risk Weighted

CONSTRUCTING THE SET OF ALTERNATIVES

- Equal risk contribution
  - Maillard et al (2008) [6] propose weighting each stock such that that the contribution of each stock to the risk of the overall portfolio is equal. We use a covariance matrix based on 5 years history (shrunk using Ledoit and Wolf [7]) and the algorithm proposed in this paper to calculate equal risk contribution weights. Investable indices based on this concept are produced by Lyxor e.g. FTSE Lyxor SmartIX ERC

- Minimum variance
  - The minimum variance approach uses historical data in an attempt to identify the weights of the global minimum variance portfolio. Authors such as Clarke, de Silva, and Thorley (2006) [8] have identified strong performance of minimum variance portfolios. We use the same shrunk covariance matrix as before and cap individual weights at a maximum of 5%. Various investable version of minimum variance indices are available e.g. FTSE Global Minimum Variance Index Series, MSCI Minimum Variance etc.

CONSTRUCTING THE SET OF ALTERNATIVES

- Maximum diversification
  - Choueifaty and Coignard (2008)[9] introduce a measure of portfolio diversification, called the "Diversification Ratio", which is defined as the ratio of a portfolio's weighted average volatility to its overall volatility. Poorly diversified portfolios that have either concentrated weights, highly correlated holdings or even both will exhibit relatively low diversification ratios. Choueifaty and Coignard propose an optimisation process to identify the 'most diversified portfolio' which is defined as the portfolio with the highest diversification ratio. Intuitively it is apparent that if expected returns are proportional to their volatility, the maximum diversification portfolio will be the same as the maximum Sharpe ratio portfolio (this can be proven mathematically). Again we use the same shrunk covariance matrix and cap individual weights at a maximum of 5%. This index is investable via the FTSE TOBAM Maximum Diversification Indices.

CONSTRUCTING THE SET OF ALTERNATIVES

- Risk efficient
  - Amenc, Goltz, Martellini, and Retkowsky (2010) [10] propose a very similar methodology to maximum diversification except that they assume that the expected return on each constituent is assumed to be linearly related to the downside-deviation of its return. They also group stocks into deciles of semi-deviation and assign each stock the median of its decile. The second stage then involves finding the portfolio with the maximum expected return (proxied by the median downside deviation of each stock's decile) with the lowest portfolio return standard deviation. To prevent the optimiser from creating a portfolio with concentrated single stock exposures, they impose restrictions on the constituent weights:
    - lower limit = $1/(\lambda \times N) \times 100\%$
    - upper limit = $\lambda/N \times 100\%$

    where N represents the total number of stocks under consideration and λ is a free parameter. We set λ equal to 2 and use the same shrunk covariance matrix. This index is investable via the FTSE EDHEC Risk Efficient Indices.

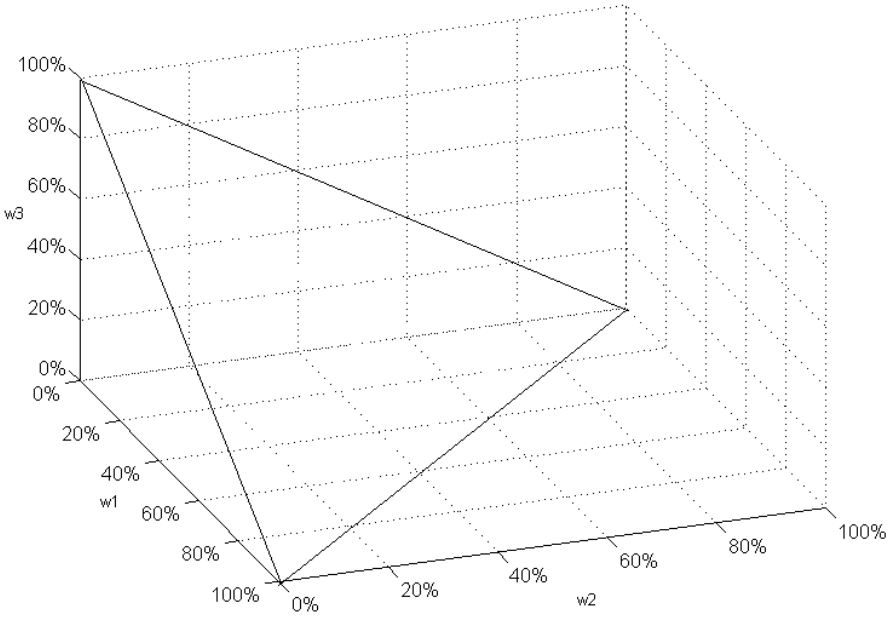
CONSTRUCTING THE SET OF ALTERNATIVES

- Fundamental Indexing
  - Arnott et al. (2005) [11] argue that alternative measures of the size or scale of a company may be just as appropriate a basis for determining constituent weights as the more commonly used metric of market capitalisation. We calculate four different indices that weight stocks according the 5 year historical average of total dividends, cash-flow, book value of equity and sales. We then take the average weights of these four indices to form a fundamental composite index. Research Affiliates, LLC produce investable fundamental indices.

Appendix 2:
Constructing The Random Indices

WHERE TO FIND AN INFINITE NUMBER OF MONKEYS? (NO MONKEYS WERE HARMED IN THIS EXPERIMENT)

- There are an infinite number of combinations of weights for 500 stocks that sum to 1
  - 1st step is to make this a finite universe by specifying a minimum increment Δw *of* 0.2%
  - Objective is to sample randomly and uniformly from the set of feasible weights
  - For example with 3 stocks, the set of feasible weights form a hyper-plane



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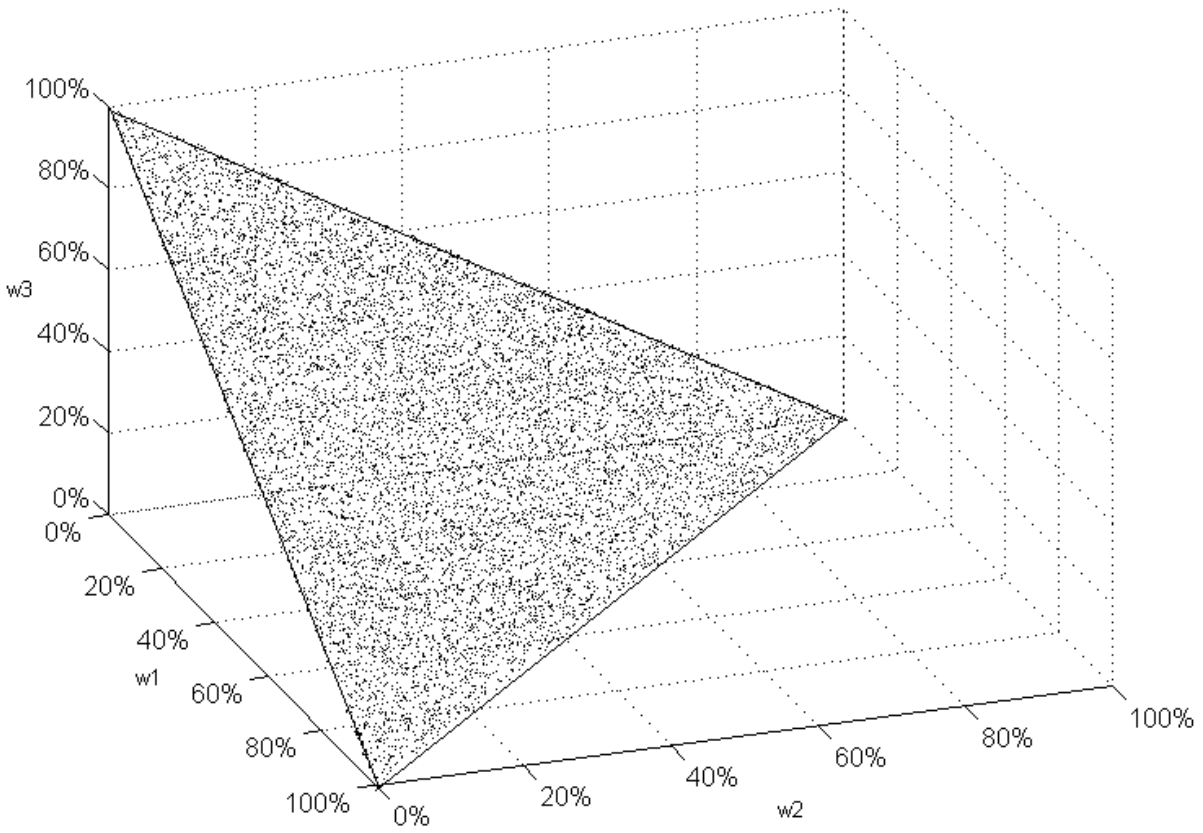
WHERE TO FIND AN INFINITE NUMBER OF MONKEYS? (NO MONKEYS WERE HARMED IN THIS EXPERIMENT)

- Use an algorithm adapted from Smith and Tromble (2004)[14]

- Given *n* stocks, 4 steps:
  1. Sample *n-1* numbers uniformly at random from the set {1, 2, ... $(1/\Delta w)+n-1)$} *without replacement.*
  2. Sort the numbers in ascending order and append a zero to the beginning of the sequence and $(1/\Delta w + n)$ to the end of the sequence.
  3. Take the difference between successive numbers in the sample and subtract 1 from each.
  4. Multiply these numbers by Δw.

WHERE TO FIND AN INFINITE NUMBER OF MONKEYS? (NO MONKEYS WERE HARMED IN THIS EXPERIMENT)

- Scatter plot of the result of 10,000 repetitions of the above algorithm for n=3 and Δw=0.1%



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PROOF OF ROBUSTNESS

- Though the *mean* of our weights will be the same as equal weight there is no bias towards any weighting scheme:
  - Consider the example of a portfolio containing 100 stocks (n=100) where the minimum increment is set at 1% (Δw =0.01)
    - The first step involves selecting 99 random numbers from the set {1, 2, ... 199}. If we suppose that the numbers chosen are {2, 4, 6, … 198}
    - then step 2 will result in the following set of 101 numbers {0, 2, 4, 6,…..,198, 200}.
    - Step 3 produces 100 identical numbers {1, 1,….1}
    - Hence step 4 will generate an equally weighted portfolio with each stock given a weight equal to 1% or 1/n.

PROOF OF ROBUSTNESS

- If instead the 99 random numbers chosen had been {1, 2, 3, ... 99} then the set of weights produced would be zero for the first 99 stocks and 100% in the 100th stock.

- Since choosing {2, 4, 6, … 198} and choosing {1, 2, 3, …, 99} are equally likely hopefully this demonstrates that the randomly generated portfolio weights are unbiased.

CONSTRUCTING THE RANDOM INDICES

- Using the algorithm we generate 500, weights that sum to one, with a minimum increment of 0.2%.
  - Apply these weights to the universe of 500 stocks sampled at December 1968
  - Calculate the performance of the resulting index over the next twelve months.
  - Apply another set of randomly generated weights to the 500 stocks sampled in December 1969, and again calculate the performance of this randomly constructed index over the next 12 months.
  - Repeated for each year in our sample until we produce an index spanning January 1969 to December 2014.

- Repeat the whole process ten million times…….

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