GRaM for Ox Professional (Windows) - User Manual and Documentation

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July 30, 2004

Abstract

GRaM (Growth Rates and cointegration Means) is a program for estimating long-run properties in cointegrated VAR models. As other program packages for cointegration analysis, the cointegrating vectors and their corresponding adjustment parameters can be estimated. In addition, the program decomposes the parameters for the deterministic terms into interpretable counterparts such as growth rate parameters and cointegration mean parameters. These parameters express long-run properties of the model. For example, the growth rate parameters tell us how much to expect (unconditionally) the variables in the system to grow from one period to the next, and will therefore be especially important to identify if the system is to be used for forecasting.

GRaM also allows for different types of hypotheses testing. Both linear tests on the cointegrating vectors and on the 'growth rates' are allowed for, i.e. we can test for both the long-run relationship between the time series and the long-run growth in the different time series. (Testing restrictions on the adjustment parameters for the cointegrating vectors are not allowed in this version.)

This document describes the program, and shows how to use it. A tutorial replicates some of the results in "Identifying Structural Breaks in Cointegrated VAR Models"; see Hungnes (2004), where the program is used to test for different types of structural breaks in the money demand in a reunified Germany.

Please note: This is a very preliminary documentation of GRaM for Ox Professional. The program is still under construction.

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1.1 System requirements

Requirements:
- Windows
- Ox Professional 3.30
- Simulated Annealing (Ox package)

1.2 Installation

You need Ox Professional, (c) J. A. Doornik, in order to run the program. (Please note that you cannot use the console version of Ox.) If you do not have Ox Professional you can buy if from the following address:

- http://www.timberlake.co.uk/software/Oxmetrics/Ox/ox.html

If you have purchased Ox Professional 3.0, 3.1 or 3.2, and have a valid licence code, then you can download the upgrade to version 3.3 from this address:

- http://www.nuff.ox.ac.uk/users/Doornik/oxformpro.html

It is important that you upgrade Ox Professional to the version 3.3 (or newer). This is because the program applies a function (MaxSQP) that is not implemented in earlier versions of Ox.

You must also run Windows to run GRaM for Ox. (I apologise for this...) The reason is that the program use the OxPack interface in Ox Professional, and this runs (unfortunately) only under Windows.

Finally, you need to include the MaxSA package in the directory /Ox/Packages/. (The MaxSA package provides an implementation in Ox of the Simulated Annealing, see Goffe et al. (1994) for a documentation of the algorithm.) You can download this package from Charles Bros' home page:

- http://www.tinbergen.nl/~cbos/software/maxsa.html

The next step is to download GRaM. You can download it from the following page: [Not yet available for downloading!]

- http://folk.ssb.no/hhu

Unzip the files and save them in the directory /Ox/Packages/. To run the program start OxPack, which is a part of Ox Professional. In OxPack, choose 'Add/Remove Package..' under the 'Package' menu. Choose 'Browse' to find gram.oxo. (If you followed the instruction above, you will find it in the directory /Ox/Packages/.) Finally, press the button 'Add'. Now you can choose 'GRaM' in the 'package' menu. And that is it!

1.3 User license and citation

GRaM (Growth Rates and cointegtartion Means) 0.7 is copyright (c) Håvard Hungnes 2003-2004. It is free to use under the following conditions:

1. Its use in published research is acknowledged. Please cite this document in your list of references.
2. (Disclaimer) The code is distributed "as is", with no warranties as to fitness for any purpose. Use it at your own risk.

Ox Professional 3.3 (or newer) is required to the package, see 'Installation'. Ox should be cited whenever it is used. For example you could say in the text: "the results are generated using Ox version 3.30 (see Doornik, 2001)" and then give the references: "Doornik (2001), Object-Oriented Matrix Programming Using Ox, 4th ed. London: Timberlake Consultants Press and Oxford: www.nuff.ox.ac.uk/Users/Doornik."

2 Theoretical introduction

2.1 Introduction

In analysing a dynamic econometric model we are often interested in identifying and testing its long run properties. The cointegrating vectors are examples of long run relationships between different variables. However, also the underlying growth rates (i.e. steady state growth rates) can be identified in cointegrated vector autoregressive (VAR) models.

The growth rates tell us how much to expect (unconditionally) the variables in the system to grow from one period to the next. If the system is used for forecasting, the vector of growth rates will be very important in providing good forecasts. In fact, as the forecasting horizon approaches infinity, the forecast will rely on this vector only.

GRaM decomposes all deterministic terms in a cointegrated VAR model into interpretable counterparts. The corresponding coefficients describe the long run (steady state) growth rates for the variables, and possibly shifts in level and growth rates (the latter depending on the type of deterministic variables that are included in the system). Combined with the coefficients for the cointegrating vectors, they also describe level and trends (and possibly shifts in these) in the cointegrating vectors.

Examples of applications:

- Hungnes (2002); cointegrated VAR model where some variables were allowed to grow and some were not.
- Hungnes (2004); coitnegrated VAR with structural breaks.

Another example of application for the procedure is to test for "zero-mean" convergence. "Zero-mean" convergence implies that the difference between two variables (say; output per capita in two countries) is stationary with zero mean. To test if the mean is zero, the deterministic terms must be decomposed.

Throughout the paper we define the orthogonal complement of the full column rank matrix A as $A_\perp$ such that $A_\perp' A = 0$ and $(A, A_\perp)$ has full rank. (The orthogonal complement of a nonsingular matrix is 0, and the orthogonal complement of a zero matrix is an identity matrix of a suitable dimension.) Furthermore, for a matrix A with dimension $n \times m$ ($m \leq n$), we define $\overline{A} = A\left(A'A\right)^{-1}$.

2.2 Granger's representation theorem

In (1) Y_t is an n-dimensional vector of variables that are integrated of order one at most. α and β are matrices of dimension $n \times r$ (where r is the number of cointegrating vectors) and $\beta' Y_t$ is an $r \times 1$ vector where all row elements are $I(0)$. Furthermore, Γ_i ($i = 1, 2, ..., p-1$) are $n \times n$ matrices of coefficients, where p is the number of lags. Δ is the difference operator. D_t^* is a vector of deterministic variables. The errors ε_t are for simplicity assumed to be Gaussian white noise ($\varepsilon_t \sim N(0, \Omega)$).

$$\Delta Y_t = \alpha \left(\beta' Y_{t-1}\right) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \delta D_t^* + \varepsilon_t, \;\; t = 1, 2, ..., T. \tag{1}$$

It is common to distinguish between deterministic variables that are restricted to lie in the cointegration space and those which are not. Table 1 on the following page summarises the normal specifications. Let $\delta D_t^* = \delta_0 D_{0,t}^* + \delta_1 D_{1,t}^*$, where $D_{0,t}^*$ includes the deterministic variables restricted to lie in the cointegrating space (i.e such that the corresponding coefficient matrix can be decomposed as $\delta_0 = -\alpha\rho$). Disregarding different types of dummies (such as impulse dummies, shift dummies and seasonal dummies), the most common two specifications for these deterministic variables are $\left(D_{0,t}^*, D_{1,t}^*\right) = (1, \emptyset)$ (restricted constant, excluding a linear drift in Y_t, see H_c in Table 1) and $\left(D_{0,t}^*, D_{1,t}^*\right) = (t, 1)$ (restricted linear trend, excluding a quadratic trend in Y_t, see H_l in Table 1).

In Table 1 on the next page the two columns under 'processes' refers to what type of process the variables and the cointegrating vectors follow, respectively. When describing the process for the variable as 'quadratic' we mean that the variables follow a process with an quadratic trend (i.e. as in equation (4) with an additional quadratic trend); by the description 'linear' we imply that they follow a linear trend (i.e. as in (4)); and 'constant' refers to a situation where the variables does not follow any trend (i.e. as in (4) where the trend is removed).[1]

In econometric analysis it is recommended to start with system H_l or H_c. If there are trends in the data H_l is recommended, and in systems without trends H_c is recommended.

In a Monte Carlo simulation study Doornik et al. (1998) show that if the system is misspecified by not including a restricted trend, we may not identify all the cointegrating vectors. This is because the deterministic trends will be represented by a stochastic trend (p. 133). However, to erroneously include the trend restricted has a very low cost (p. 133).

Let us assume that the process in (1) is generated by hypothesis H_l in Table 1 on the next page. The system grows at the unconditional rate $E_t[\Delta Y_t] = \gamma$ with long run (cointegration) means $E_t[\beta'(Y_t - \gamma t)] = E_t[\beta' Y_t - \rho t] = \mu$, where

$$\rho \equiv \beta' \gamma, \tag{2}$$

[1]Doornik et al. (1998) refer to the last model in Table 1 on the following page as H_z, and the process of Y is described with 'zero' (and not 'constant'). However, the absence of an intercept and a trend does not imply that ι in (4) is zero. We therefore use the notation H_{cz}, to describe that the variables may have a non-zero initial value (but the cointegrating vectors have zero-means).

Table 1: Deterministic part - normal representation

	$\delta_0 D^*_{0,t} + \delta_1 D^*_{1,t}$		Processes		PcGive formulation	
	$D^*_{0,t} =$	$D^*_{1,t} =$	Y	$\beta' Y$	constant	trend
H_{ql}	$\emptyset$	$(1, t)$	quadratic	linear	unrestricted	unrestricted
H_l	t	1	linear	linear	unrestricted	restricted
H_{lc}	$\emptyset$	1	linear	constant	unrestricted	absent
H_c	1	$\emptyset$	constant	constant	restricted	absent
H_{cz}	$\emptyset$	$\emptyset$	constant	zero	absent	absent

are the trends in the cointegrating vectors.

The system can be rewritten as

$$\Delta Y_t - \gamma = \alpha \left(\beta' Y_{t-1} - \mu - \rho \left(t-1\right)\right) + \sum_{i=1}^{p-1} \Gamma_i \left(\Delta Y_{t-i} - \gamma\right) + \varepsilon_t. \tag{3}$$

Condition 2.1 *Assume that $n - r$ of the roots of the characteristic polynomial*

$$A\left(z\right) = \left(1-z\right) I_n - \alpha\beta' z - \sum_{i=1}^{p-1} \Gamma_i \left(1-z\right) z^i$$

are equal to 1 and the remaining roots are outside the complex unit circle.

Theorem 1 (Granger's representation theorem) *Under Condition 2.1, Y_t has the representation*

$$Y_t = C \sum_{i=1}^{t} \varepsilon_t + \iota + \gamma t + B_t, \tag{4}$$

where $C = \beta_\perp \left(\alpha'_\perp \Gamma \beta_\perp\right)^{-1} \alpha'_\perp$ with $\Gamma = I_n - \sum_{i=1}^{p-1} \Gamma_i$. The process B_t is stationary with zero expectation. The growth rate parameters can be expressed as

$$\gamma = C\delta_1 - \left(C\Gamma - I_n\right) \overline{\beta}\rho, \tag{5}$$

whereas the level coefficients ι depends on initial values in such a way that

$$\mu = \beta'\iota = \overline{\alpha}' \left(\Gamma C - I_n\right) \delta_1 - \overline{\alpha}'\Gamma \left(C\Gamma - I_n\right) \overline{\beta}\rho - \rho. \tag{6}$$

Proof. For proof see Theorem 4.2. in Johansen (1995) or Theorem 2.1 in Johansen et al. (2000). ■

In GRaM a slightly different representation of the cointegration system in (1) will be presented. The representation builds on (3), and has two advantages. First, it will be easier to interpret. Second, it will be easier to represent the system with structural breaks.

Let D_t be a vector of q deterministic variables, such as trend and seasonally dummies. The system can then be written as

$$\Delta Y_t - \gamma \Delta D_t = \alpha \left(\beta' \left(Y_{t-1} - \gamma D_{t-1}\right) - \mu\right) + \sum_{i=1}^{p-1} \Gamma_i \left[\Delta Y_{t-i} - \gamma \Delta D_{t-i}\right] + \varepsilon_t, \tag{7}$$

where γ is now an $n \times q$ matrix.

In (3), $D_t = t$. This is the case with linear trend in the variables, i.e. H_l in Table 1 on the preceding page.

In the case where there are no trends in the variables, D_t vanishes from (7), and the system can be written as

$$\Delta Y_t = \alpha \left(\beta' Y_{t-1} - \mu\right) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t. \tag{8}$$

In both these cases there is a one-to-one correspondence between the system written in the normal way, as in (1), and in the alternative way, as in (7). If the system is estimated as in (1), we can always identify the coefficients in (7) by applying Theorem 1.

Also when seasonal dummies are included, (1) and (7) are two alternative ways of writhing the same system. Generally, however, there is no such one-to-one relationship between the formulations in (1) and (7).

An alternative way to write the system, is to write the system where the deterministic components are removed. Let Y^d be defined as Y where the deterministic components are removed, i.e.

$$Y_t^d = Y_t - \gamma D_t$$

with D_t as the vector of deterministic variables and γ as the corresponding matrix of coefficients. The system can therefore alternatively be written as

$$\Delta Y_t^d = \alpha \left(\beta' Y_{t-1}^d - \mu\right) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i}^d + \varepsilon_t. \tag{9}$$

2.3 The estimation problem without restrictions**

Here a conditional estimators for γ and β^* (respectively) are presented. Next, the estimation problem is formulated.

First, consider the estimator for γ. By defining $Y_t^* = (Y_t', 1)'$ and $\beta^{*\prime} = (\beta', -\mu)$ the system can be written as

$$\begin{aligned} & \Delta Y_t - \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} \\ = \; & \alpha \beta^{*\prime} Y_{t-1}^* + \left(I_n, -\Gamma_1^* - \alpha\beta', -\Gamma_2^*, ..., -\Gamma_p^*\right) \left(I_{p+1} \otimes \gamma\right) \begin{pmatrix} D_t \\ D_{t-1} \\ \vdots \\ D_{t-p} \end{pmatrix} + \varepsilon_t \end{aligned} \tag{10}$$

where $\Gamma_1^* = \Gamma_1 + I_n$, $\Gamma_i^* = \Gamma_i - \Gamma_{i-1}$ $(i = 2, 3, ..., p-1)$ and $\Gamma_p^* = -\Gamma_{p-1}$.

Define $Z_t = vec\left(\Delta Y_t, ..., \Delta Y_{t-p+1}\right)$, $D_t^v = vec\left(D_t, ..., D_{t-p}\right)$, $\Phi = \left(I_n, -\Gamma_1, ..., -\Gamma_{p-1}\right)$, and $\Phi^* = \left(I_n, -\Gamma_1^* - \alpha\beta', -\Gamma_2^*, ..., -\Gamma_p^*\right)$. The cointegrated VAR becomes

$$\Phi Z_t = \alpha\beta^{*\prime} Y_{t-1}^* + \Phi^*\left(I_{p+1} \otimes \gamma\right) D_t^v + \varepsilon_t. \tag{11}$$

Define

$$\begin{aligned} S_{DD} &= T^{-1} \sum D_t^v D_t^{v\prime}, \\ S_{ZD} &= T^{-1} \sum Z_t D_t^{v\prime}, \\ S_{YD} &= T^{-1} \sum Y_{t-1}^* D_t^{v\prime}, \end{aligned}$$

and similarly for S_{ZZ} and $S_{YZ} = S_{ZY}'$. Furthermore, implicitly define the matrix M with dimension $q\left(p+1\right)^2 n \times qn$ by[2]

$$vec\left(I_{p+1} \otimes \gamma\right) = M vec\gamma'. \tag{13}$$

Theorem 2 *With given estimates of α, β^*, $\Gamma_1, ..., \Gamma_{p-1}$ and Ω, the conditional estimator for γ becomes*

$$\begin{aligned} \widehat{vec}\gamma'\left(\alpha, \beta^*, \Gamma_1, ..., \Gamma_{p-1}, \Omega\right) &= \left[M'\left(S_{DD} \otimes \Phi^{*\prime}\Omega^{-1}\Phi^*\right) M\right]^{-1} \\ &\quad \times \left[M' vec\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD} - \Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime} S_{YD}\right)\right], \end{aligned}$$

and corresponding variance matrix

$$V\left(\widehat{vec}\gamma'\right) = T^{-1}\left[M'\left(S_{DD} \otimes \Phi^{*\prime}\Omega^{-1}\Phi^*\right) M\right]^{-1}.$$

Proof. To prove the theorem we use $tr\left(AB\right) = tr\left(BA\right) = \left(vecA'\right)' vecB$ and $vec\left(AXB\right) = \left(B' \otimes A\right) vecX$ where tr is the trace operator. The log-likelihood function (when a constant is disregarded) can be written as

$$\log L = -\frac{T}{2}\left[\begin{array}{c} tr\Omega^{-1}\Phi S_{ZZ}\Phi' + tr\Omega^{-1}\alpha\beta^{*\prime} S_{YY}\beta^*\alpha' \\ -tr\Omega^{-1}\Phi S_{ZY}\beta^*\alpha' - tr\Omega^{-1}\alpha\beta^{*\prime} S_{YZ}\Phi' \\ +N + tr\Omega^{-1}\Phi^*\left(I_{p+1} \otimes \gamma\right) S_{DD}\left(I_{p+1} \otimes \gamma'\right)\Phi^{*\prime} + N' \end{array}\right], \tag{14}$$

[2] The explicit form of this matrix is given as

$$M = \left[\left(\left(I_{p+1} \otimes K_{q,p+1}\right)\left(vecI_{p+1} \otimes I_q\right)\right) \otimes I_n\right] K_{q,n}, \tag{12}$$

where $K_{q,n}$ is the commutation matrix, see e.g. Magnus and Neudecker (1988, pp. 46-48). This is defined as $K_{q,n} = \sum_{i=1}^{q}\sum_{j=1}^{n}\left(J_{i,j} \otimes J_{i,j}'\right)$, where $J_{i,j}$ is an $q \times n$ matrix with all elements equal to zero except the (i, j)'th element, which is unity (see e.g. Lütkepohl, 1996, p. 116).

where

$$\begin{aligned}
N &= -tr\Omega^{-1}\Phi S_{ZD}\left(I_{p+1}\otimes\gamma'\right)\Phi^{*\prime}+tr\Omega^{-1}\alpha\beta^{*\prime}S_{YD}\left(I_{p+1}\otimes\gamma'\right)\Phi^{*\prime} \\
&= -tr\left(I_{p+1}\otimes\gamma'\right)\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD}-\Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime}S_{YD}\right) \\
&= -\left(vec\left(I_{p+1}\otimes\gamma\right)\right)' vec\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD}-\Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime}S_{YD}\right) \\
&= -\left(vec\gamma'\right)' M' vec\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD}-\Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime}S_{YD}\right).
\end{aligned}$$

The derivative of (14) with respect to $vec\gamma'$ is

$$\begin{aligned}
&\frac{\partial \log L}{\partial vec\gamma'} \\
&= TM'\left[vec\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD}-\Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime}S_{YD}-\Phi^{*\prime}\Omega^{-1}\Phi^{*}\left(I_{p+1}\otimes\gamma\right)S_{DD}\right)\right].
\end{aligned} \tag{15}$$

Disregarding the T, the last part of the right hand side of (15) can be written as

$$\begin{aligned}
M' vec\left(\Phi^{*\prime}\Omega^{-1}\Phi^{*}\left(I_{p}\otimes\gamma\right)S_{DD}\right) &= M'\left(S_{DD}\otimes\Phi^{*\prime}\Omega^{-1}\Phi^{*}\right)vec\left(I_{p}\otimes\gamma\right) \\
&= \left[M'\left(S_{DD}\otimes\Phi^{*\prime}\Omega^{-1}\Phi^{*}\right)M\right]vec\gamma', \qquad (16)
\end{aligned}$$

and combining (15) and (16) yields

$$\begin{aligned}
\frac{\partial \log L}{\partial vec\gamma'} &= T'M'\left[vec\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD}-\Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime}S_{YD}\right)\right] \\
&\quad -T\left[M'\left(S_{DD}\otimes\Phi^{*\prime}\Omega^{-1}\Phi^{*}\right)M\right]vec\gamma'. \qquad (17)
\end{aligned}$$

Setting this equal to zero and solving for $vec\gamma'$ yields the estimator, and taking the derivative of (17) yields the negative of the 'variance' matrix. ■

Let $Z_{0,t}(\gamma) = \Delta Y_t - \gamma\Delta D_t$, $Z_{1,t}(\gamma) = \left[\left(Y_{t-1}-\gamma D_{t-1}\right)',1\right]'$ and $Z_{2,t}(\gamma) = vec\left(\Delta Y_{t-1} - \gamma D_{t-1},...,\Delta Y_{t-p+1}-\gamma D_{t-p+1}\right)$. The system becomes

$$Z_{0,t} = \alpha\beta^{*\prime}Z_{1,t} + \Theta Z_{2,t} + \varepsilon, \tag{18}$$

where $\Theta = \left(\Gamma_1,...,\Gamma_{p-1}\right)$.

An estimator for β in the equation above could be found by applying reduced rank regression, see e.g. Anderson (1951) or Johansen (1995). However, here only a conditional estimator for β is considered.

The equation above can be estimated by considering the concentrated log-likelihood function, which up to a constant term is given by

$$logL = -\frac{T}{2}tr\Omega^{-1}\left(S_{00}-S_{01}\beta^{*}\alpha'-\alpha\beta^{*\prime}S_{10}+\alpha\beta^{*\prime}S_{11}\beta^{*}\alpha'\right), \tag{19}$$

where

$$S_{ij} = T^{-1}\left(Z_i'Z_i - Z_i'Z_2\left[Z_2'Z_2\right]^{-1}Z_2'Z_j\right), i,j = 0,1, \tag{20}$$

with Z_0, Z_1 and Z_2 the data matrices of $Z_{0,t}$, $Z_{1,t}$ and $Z_{2,t}$, respectively.

Theorem 3 *The conditional estimator for β is*

$$\beta^* = \left[\alpha'\Omega^{-1}\alpha \otimes S_{11}\right]^{-1}\left[\alpha'\Omega^{-1} \otimes I_{n+1}\right] vecS_{10}.$$

Proof. The derivative of (19) (with respect to β) is

$$\frac{\partial \log L}{\partial vec\beta^*} = Tvec\left[S_{10}\Omega^{-1}\alpha - S_{11}\beta\alpha'\Omega^{-1}\alpha\right] \tag{21}$$

Setting this equal to zero, and solving for β yields the (conditional) estimator in the theorem. ■

Now, consider how to estimate the system. First, suppose β^* and γ were known. Then the remaining coefficients could be estimated by applying ordinary least square in (7). Let $l\left(\widehat{\alpha}\left(\beta^*,\gamma\right),\beta^*,\gamma',\widehat{\Gamma}_1\left(\beta^*,\gamma\right),...,\widehat{\Gamma}_{p-1}\left(\beta^*,\gamma\right),\widehat{\Omega}\left(\beta^*,\gamma\right)\right)$ be the corresponding likelihood value.

Problem 2.1 *The maximum likelihood estimates for β^* and γ are the coefficient matrices that solves*

$$\max_{\beta^*,\gamma}\left\{l\left(\widehat{\alpha}\left(\beta^*,\gamma\right),\beta^*,\gamma',\widehat{\Gamma}_1\left(\beta^*,\gamma\right),...,\widehat{\Gamma}_{p-1}\left(\beta^*,\gamma\right),\widehat{\Omega}\left(\beta^*,\gamma\right)\right)\right\}.$$

The solution to this problem must imply that (the empirical counterparts to) (15) *and* (21) *are equal to zero, i.e. that the first order conditions are satisfied. [Something about the second order condition...]*

Problem 2.1 describes how GRaM considers the estimation problem. The program tries to maximise the log-likelihood, and applies the two first order conditions (15) and (21) to increase the speed and probability to reach the maximum.

3 GRaM tutorial

3.1 Introduction

To start GRaM you need to start OxPack and choose Gram under 'Packages'. (If you can not find Gram under this menu you have not installed GRaM properly. See Section 1.2 for how to install GRaM.)

In order to follow this tutorial you need to load the data file lutkepohlDet.in7. (You can find this file ... Where?) Lütkepohl and Wolters (1998) and Saikkonen and Lütkepohl (2000) use a data set covering the unification to estimate a model for money demand in Germany.[3]

First we look at the data. Choose 'Graphics...' under the 'Tools' menu in GiveWin. (Alternatively you could click on the graphics button or press Alt+G.) Highlight the following data series: m, y, r and Dp. Then press '<<' and choose 'Apply separate actual values plots' in the top left part of the window. Now you should get a graph which looks something like Figure 1 on the next page.

The four variables are; (log of) real money M3 (m), (log of) real GNP (y), an opportunity cost of money (r) and inflation (Dp). The opportunity cost of money is defined as the difference between long term interest rate and the own rate on M3.

In the data series for money and income there is a significant shift in the level form 1990q3. This corresponds to the monetary (re-) unification of Germany, which took place July 1, 1990.[4] For the opportunity cost of money or the inflation rate there is no obvious shift in level or trend.

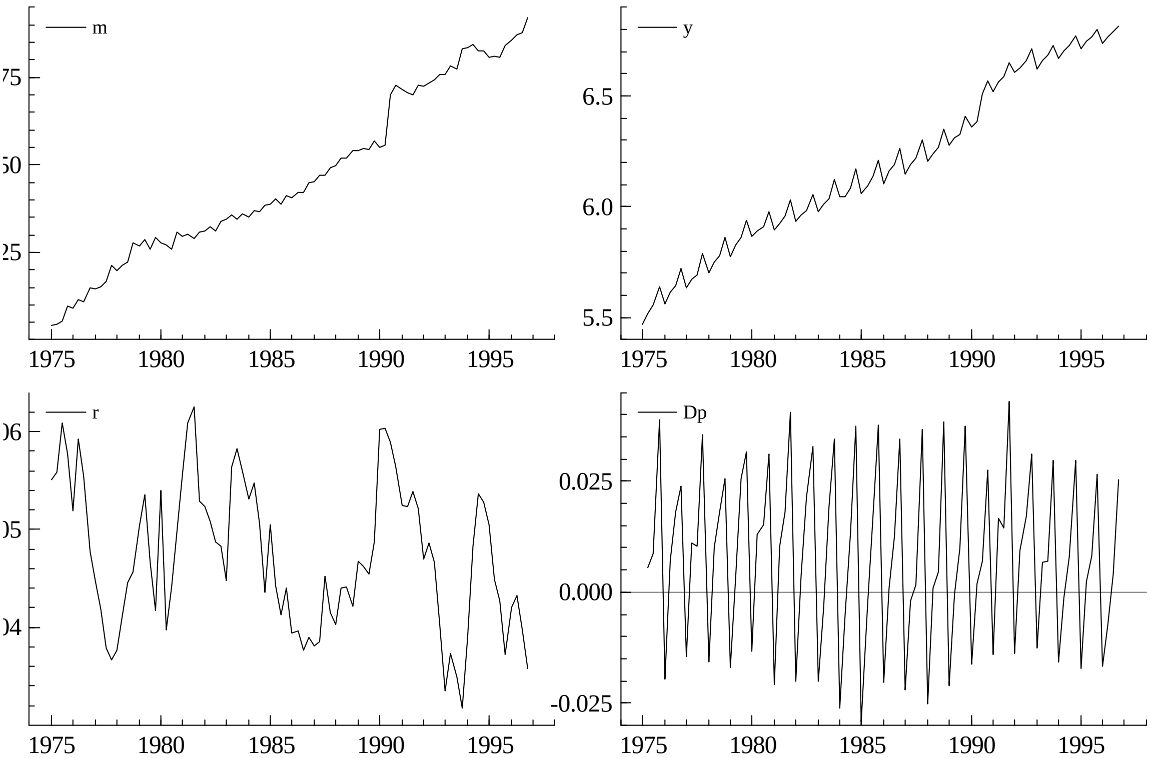
To take account for the shift in level for money and income a shift dummy is constructed. This shift dummy, D1990q3, is zero until 1990q2 and unity thereafter. A corresponding broken trend, defined as the cumulate of the shift dummy, is also included to allow for tests shifts in the growth rates, see Hungnes (2004).

There is (possibly) a seasonal pattern in the income and inflation rate series. Therefore, some centred seasonal dummies are constructed; CS0, CS1 and CS2. [Change to CS1 - CS3!!, (where CSi is defined as 0.75 in the i'th quarter and -0.25 in the remaining quarters.)]

[3]The data are available at: ftp://141.20.100.2/pub/econometrics/germanm3.zip. Source: Lütkepohl and Wolters (1998).

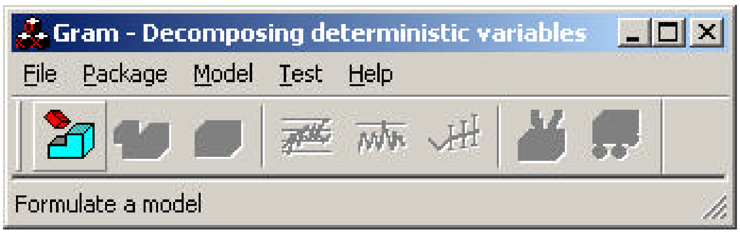
[4]The official date of unification is October 3, 1990. However, the monetary unification took place about three months earlier.

Figure 1: German money demand data



The money stock (m), (real) income/gdp (y), real interest rate (r) and inflation quarter/quarter (Dp) in Germany (West Germany until 1990q2 and unified Germany thereafter).

Figure 2: GRaM in OxPack



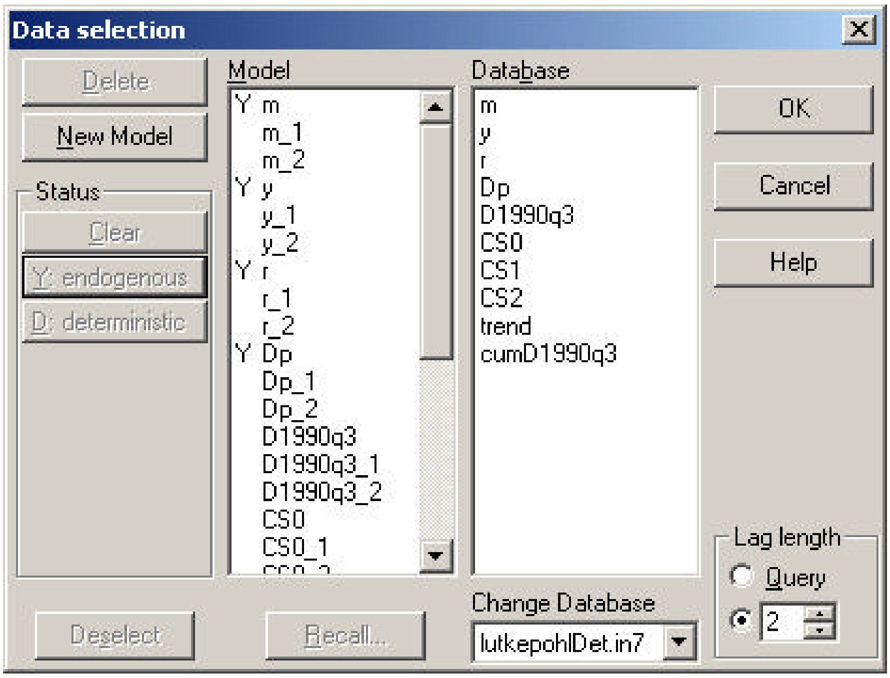
3.2 Formulation

Now, let us turn to the estimation of this data set.

If you have started GRaM, OxPack will look something like Figure 2. Choose 'Formulate...' under 'Model' (or click on the highlighted 'Formulate' button, or press CTL+Y). You will then get to the 'Data selection' menu.

In the 'Data selection' menu you choose the data series to be included in the model. First

Figure 3: Data selection



choose the number of lags; here 2 lags. Then choose the variables to be included (both endogenous and deterministic variables); here all variables in the 'Database'. Finally, choose which variables to be endogenous and which to be deterministic. GRaM automatically suggests the first variable only to be endogenous, and the remaining being deterministic. Her we want the first four variables (money, income, opportunely cost of money, and the interest rate) to be endogenous. Therefore, highlight the the variables form 'y' to 'Dp' in the 'Model', and click the button 'Endogenous'.[5] The 'Data selection' menu should now look something like Figure 3.

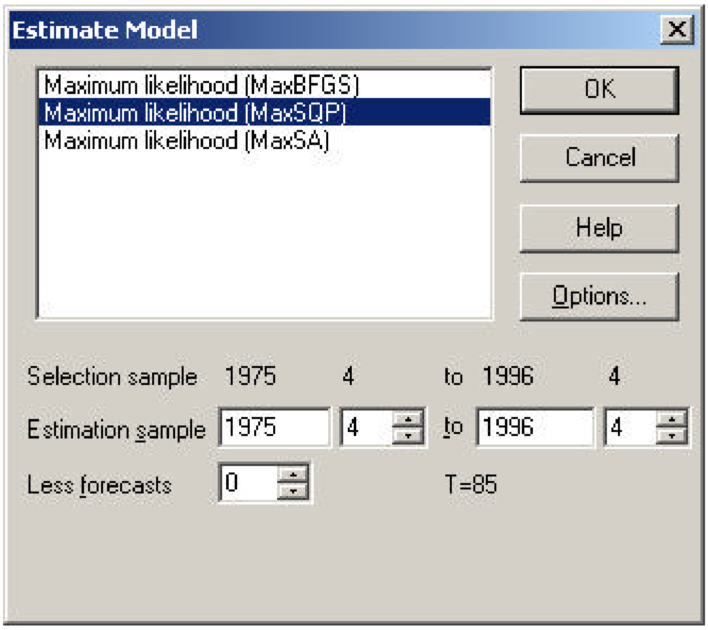
The next menu is the 'Estimate model' menu, see Figure 4 on the following page. Here you can choose different types of simulation algorithms, or you can adjust the estimation period. The different types of simulation algorithms will be explained in more details later. Now, use 'Maximum likelihood (MaxSQP)'.

In the 'General restrictions' menu (Figure 5 on page 17) you choose the number cointegrating vectors, and the number of restrictions on the different coefficient matrixes. Choose '2' for 'Cointegrating rank', and zero for all the restrictions. (In the next subsections you will learn about imposing restrictions.)

After you click on 'OK' on the 'General restrictions' menu, the program starts to simulate in order to try to find the maximum likelihood value and the corresponding estimates. The program will (probably) write some intermedin results. After a few seconds (depending on

[5] You do not have to highlight the deterministic variables and choose 'Deterministic'; if a variable is not chosen to be 'Endogenous', GRaM will assume it is 'Deterministic'.

Figure 4: Estimate Model



how fast your computer is), you will get the final result, see Figure 6 on page 18. (Note that the figure is over tow pages!)

The first few lines reports intermediate results from the simulation. We will ignore this part of the output here.

The next lines reports the estimated coefficient matrixes, with the corresponding standard deviations. First, γ' (gamma') is reported (with standard deviations), followed by α (alpha), β (beta) and ρ' (rho). (The standard deviation for β can not be computed when no restrictions are imposed.)

The eigenvalues of the companion matrix are also reported. These should be either on or inside the unit circle, i.e. the modulus should be unity or less. (The number of unit roots should equal the number of endogenous variable minus the cointegrating rank, see Condition 2.1 on page 8.) These eigenvalues are reported to evaluate the reliability of the results; if one or more of the eigenvalues are outside the unit circle results are not reliable (and the corresponding estimated system is explosive).

Finally, the likelihood value and some other information about the system are reported. Also the results form the simulation algorithm is reported. The output here is similarly to the output in 'Multiple-Equation Dynamic Modelling...' package in PcGive, see Doornik and Hendry (2001).

The estimated γ' is also reported in Table 2 on the following page. The second last row, labelled 'trend', reports the trend in the different variables in the system. Since both money

Figure 5: General restrictions

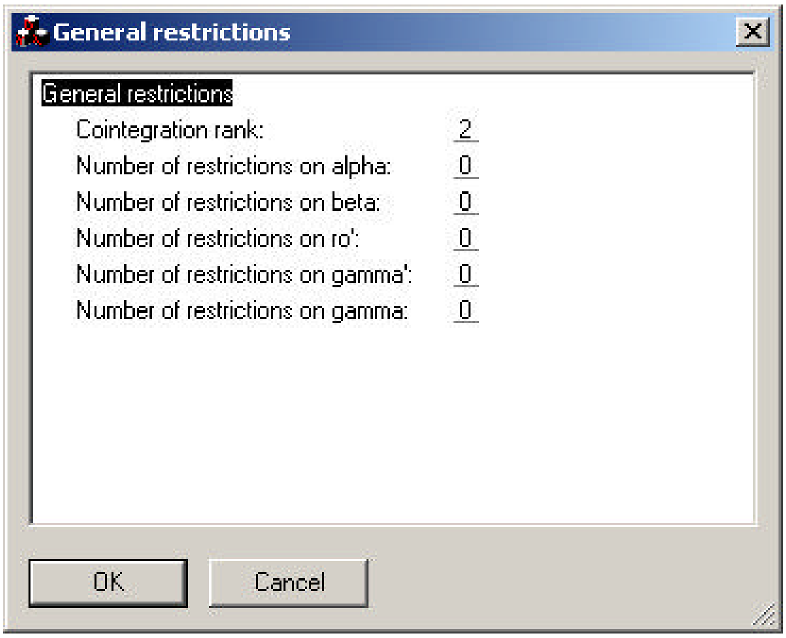


Table 2: Estimated γ

	m	y	r	Dp
D1990q3	0.14638	0.10704	0.00007	0.00313
CS0	-0.01208	-0.09169	0.00113	-0.05218
CS1	-0.00853	-0.06954	0.00246	-0.02969
CS2	-0.02143	-0,05574	0.00207	-0.02035
trend	0.00529	0.01373	0.00009	0.00004
cumD1990q3	0.00592	-0.00301	-0.00112	-0.00050

and income are measured in logs, the corresponding coefficients can be interpreted as growth rates. The growth rate for money is therefore 0.53 per cent each quarter. This corresponds to an annual growth rate of 2.13.[6] The estimated quarterly growth rate for income is 1.37 per cent, which corresponds to an annual growth rate of 5.61 per cent.

The coefficients for the trend in the variables for opportunity cost and inflation rate are approximately zero, implying no growth in these variables. (Based on the t-values, i.e. the ratio between the estimated coefficient and its corresponding standard deviation, the null hypothesis of each of these two growth rates to be zero would not be rejected.)

In the last line of Table 2 the corresponding coefficients for $cum1990q3$ are reported. Since

[6]The annual growth rate can be calculated as $\gamma_a = (1+\gamma_q)^4 - 1$, where γ_a is the annual growth rate and γ_q is the quarterly growth rate. However, when the growth rate is close to zero, the simplification $\gamma_a \approx 4 \cdot \gamma_q$ works well.

Figure 6: GiveWin output

```
---- GiveWin 2.20 session started at 10:50:37 on Tuesday 18 May 2004 ----

Ox version 3.30 (Windows) (C) J.A. Doornik, 1994-2003
GRaM package version 0.7, (c) H. Hungnes, 2003-2004, object created on 18-05-2004
lutkepohlDet.in7 loaded from H:\MineDokumenter\Dg.prosjekt\GRaM\lutkepohlDet.in7

it0    f=      39.82048 e2=6.209e+004 step=1
it106  f=      40.14324 e2=2.054e-007 step=1
Strong convergence

fAB0:f = 40.14324 diff: 0
fAB1:f = 40.14324 diff: 0
LogLikf - LogLikm = 2.13163e-014 >= 0
Sw0:  f = 40.14324    ->  1223.6487411012531
Sw2:  f = 40.14324    ->  1223.6487411012533
fAB0:f = 40.14324 diff: 0
fAB1:f = 40.14324 diff: 0


gamma'
                         m             y             r            Dp
D1990q3            0.14638       0.10704   6.5850e-005     0.0031323
CS0              -0.012079     -0.091688     0.0011336     -0.052181
CS1             -0.0085290     -0.069543     0.0024576     -0.029688
CS2              -0.021426     -0.055743     0.0020736     -0.020349
trend            0.0052940      0.013730   8.6271e-005   3.6537e-005
cumD1990q3       0.0059167    -0.0030122    -0.0011189   -0.00050455

st.d.
D1990q3          0.0084791     0.0019897     0.0025453     0.0019685
CS0              0.0013092     0.0026274      0.012040     0.0030763
CS1              0.0032858     0.0030466     0.0012990     0.0024518
CS2              0.0040269    0.00088277     0.0010042    0.00087032
trend           0.00037854    0.00075454     0.0017932     0.0016195
cumD1990q3       0.0019274     0.0016218    0.00013691    0.00026426

alpha
            m             y             r            Dp
     0.016918      0.013817     0.0040258     -0.031468
     -0.17772    -0.0056879      0.015973     0.0054317

st.d.
    0.0067470      0.011693     0.0037834     0.0041502
     0.029110      0.050452      0.016324      0.017906

beta
m                   1.8448       0.75655
y                  -3.6756       0.58589
r                   3.4627        2.9908
Dp                  46.721      -0.42769
Constant            15.948       -5.0432
```

*cum*1990*q*3 is defined as the cumulated value of the step dummy *D*1990*q*3, this variable is zero until 1990q2, and increases by one unit each quarter thereafter. The corresponding coefficients therefore picks up the shift in the trend in the different variables. Therefore, the quarterly growth rate for money is 1.12 per cent after 1990q2 (i.e. 0.53 + 0.59). The reported coefficient for the trend is therefore the trend (or growth rate) in the variable before 1990q3.

Figure 6 cont.

```
ro'
D1990q3           0.023165      0.17232
CS0                -2.1193    -0.037150
CS1                -1.1387    -0.027150
CS2               -0.77819    -0.033964
trend            -0.038694     0.012292
cumD1990q3      -0.0054609  -0.00041903

st.d. (cond. on beta)
D1990q3           0.063370     0.016028
CS0               0.072067    0.0039203
CS1               0.084296    0.0045164
CS2               0.072361    0.0038820
trend           0.00049735   0.00022161
cumD1990q3       0.0034949    0.0010135


Eigenvalues of companion matrix
         real         imag      modulus
      0.83777      0.00000      0.83777
    -0.038338      0.52297      0.52437
    -0.038338     -0.52297      0.52437
     -0.23264      0.16179      0.28337
     -0.23264     -0.16179      0.28337
     0.059922      0.00000     0.059922
       1.0000      0.00000       1.0000
       1.0000      0.00000       1.0000

log-likelihood     1223.64874  -T/2log|Omega|     1706.08784
no. of observations        85  no. of parameters          54
rank of long-run matrix     2  no. long-run restr.         0
cointegration space is not identified

SQP using analytical derivatives (eps1=1e-005; eps2=0.005):
Strong convergence
```

The first line in Table 2 on page 17 reports the level shift in the different variables. As expected, the shift is significant for both money and income, and the shift is much less for the opportunity cost and the inflation rate.

From the reported γ we can also see how much seasonality there is in the different variables. The reported coefficients indicates that there is most seasonality in income and inflation, which we also guessed based on the graph of the series.

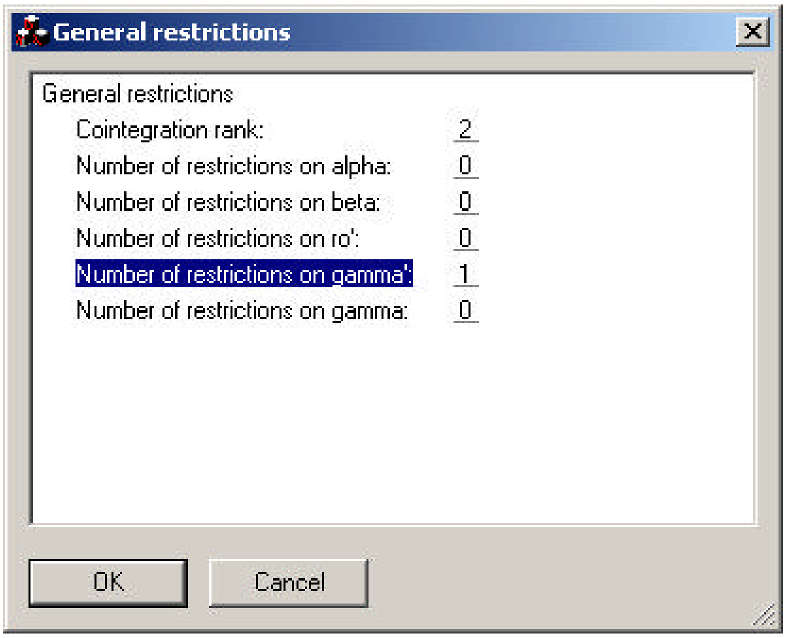
3.3 Imposing restrictions on γ'

3.3.1 Restrictions on all variables

In the previous subsection γ was estimated without restrictions. But we may also want to impose restrictions on γ (or some other matrixes of coefficients). One interesting hypothesis is to test if there is a significant shift in the trend (or the growth rate) in the variables from 1990q3. This implies testing if the coefficients in the last row in Table 2 on page 17 are zero.

In GRaM restrictions on γ' are imposed by applying restriction matrixes. The restrictions

Figure 7: General restrictions (2)



we want to impose on γ' is written as

$$R'_{\gamma'}\gamma' = 0. \tag{22}$$

The hypothesis that the coefficients for the trend shift are zero can be tested with the restriction matrix $R' = (0, 0, 0, 0, 0, 1)$, since

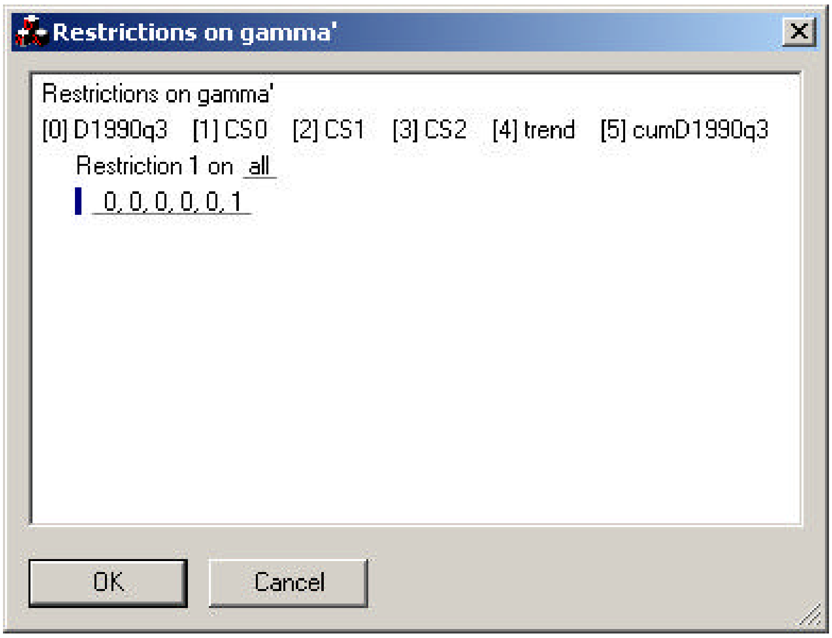
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma_{m,D1990q3} & \gamma_{y,D1990q3} & \gamma_{r,D1990q3} & \gamma_{Dp,D1990q3} \\ \gamma_{m,CS0} & \gamma_{y,CS0} & \gamma_{r,CS0} & \gamma_{Dp,CS0} \\ \gamma_{m,CS1} & \gamma_{y,CS1} & \gamma_{r,CS1} & \gamma_{Dp,CS1} \\ \gamma_{m,CS0} & \gamma_{y,CS2} & \gamma_{r,CS2} & \gamma_{Dp,CS2} \\ \gamma_{m,trend} & \gamma_{y,trend} & \gamma_{r,trend} & \gamma_{Dp,trend} \\ \gamma_{m,cumD90q3} & \gamma_{y,cumD90q3} & \gamma_{r,cumD90q3} & \gamma_{Dp,cumD90q3} \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$$

implies $\gamma_{i,cumD1990q3} = 0$, $i = \{m, y, r, Dp\}$.

Since you have already formulated the model, you do not have to do that again. Therefore, choose 'Estimate...' under 'Model' (or click on the highlighted 'Estimate' button, or press CTL+L). You will then get to the 'Estimate Model' menu. Press 'OK'.

In the 'General restrictions' menu changes the number of restrictions on γ' in the second

Figure 8: Restrictions on γ'



last line to 1, see Figure 7 on the preceding page. (We choose 1 because we only impose one restriction on the γ' matrix.) Press 'OK'.

Now you come to the menu 'Restrictions on gamma''. For each restrictions, there are two choices. The first choice is wether you want the restrictions be be a restrictions on all the (endogenous) variables in your system or on only one of them. If you press on the underlined all, a roll down menu appears. Here you can chose between *'all'*, *'m'*, *'y'*, *'r'*, and *'Dp'*. We want to impose the restriction on all the variables in our system, and we therefore choose *'all'* (which is the default).

The second choice is the restriction you want to impose. To make the specification of the restriction vector simpler, GRaM generates a row of equally many zeros as elements in the restriction vector. In addition, in the upper part of the menu, a row of the corresponding deterministic variables are reported. This should make it easier to to construct the restriction vectors.

We want to restrict the coefficients for $cumD1990q3$ to be zero for all the variables in the system. According to the line in the upper part of the menu, the variable $cumD1990q3$ corresponds to the last element in the restriction vector. Therefore, we changes the last element in the restriction vector to unity (or any non-zero number), see Figure 8. Press 'OK', and GRaM starts to simulate. After a few seconds, the results are reported to the screen. The last part of the print out should look something like Figure 9 on the following page.

In the system we have estimated now 4 independent coefficient restrictions where im-

Figure 9: GiveWin output (part of)

```
log-likelihood     1221.55518  -T/2log|Omega|     1703.99428
no. of observations        85  no. of parameters          54
rank of long-run matrix     2  no. long-run restr.         4
cointegration space is not identified

SQP using analytical derivatives (eps1=1e-005; eps2=0.005):
Strong convergence
```

posed, i.e. one restriction for each of the four endogenous variables. This is also reported in the output (`no. long-run restr.: 4`). Since this system is a restricted version of the previous, the log likelihood value is less (1221.56 vs. 1223.65). We can apply a likelihood ratio test to test the (null) hypothesis that there is no trend-break in the (endogenous) variables in the system. The likelihood ratio is twice the difference between the likelihood values, i.e. $2 \cdot (1223.65 - 1221.56) = 4.19$. To find the corresponding probability value, we choose 'Progress...' under 'Model'. In the 'Progress' menu, just choose 'OK'. The program reports information about the general and restricted system. It also reports an χ^2 test of the restriction. (We assume the ratio is χ^2 distributed, see Section 4.6 on page 37.) The reported probability value is 0.381, and with this probability value you would not reject the null hypothesis (that there is no shift in the trend - or 'growth rates' - in the variables). [NB: Need to change the program here, so it calculates the number of coefficients p correctly.]

Two things that should be noted:

- the estimated gamma matrix (elements corresponding to $cumD1990q3$ are zero)
- the estimated rho matrix (elements corresponding to $cumD1990q3$ are zero)

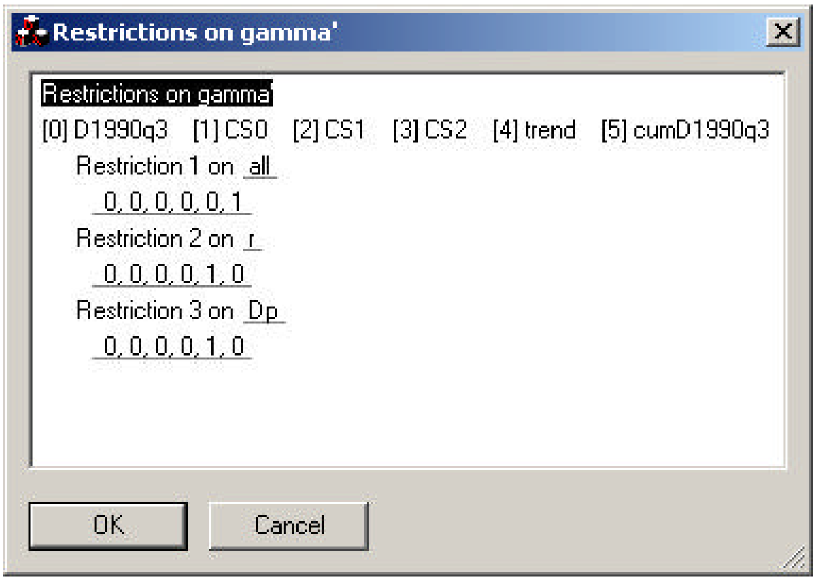
Finally; we would have got the same estimation results if we had deleted $cum1990q3$ as a deterministic variable in our system, and estimated that system without restrictions.

3.3.2 Restriction on one variable only

In the previous 'subsubsection' we imposed the restriction that there were no trend-breaks in any of the endogenous variables. Here we will show how to impose the additional restrictions that there is no trend in the opportunity cost or in the inflation rate. In the 'General restrictions' menu we therefore choose 3 restrictions on gamma' in the second last line.

In GRaM restrictions can also be imposed on one vector in γ'. If we want to impose the

Figure 10: Restrictions on γ' (2)



restriction that there is no trend in the opportunity cost, this can be written as

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma_{r,D1990q3} \\ \gamma_{r,CS0} \\ \gamma_{r,CS1} \\ \gamma_{r,CS2} \\ \gamma_{r,trend} \\ \gamma_{r,cumD90q3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$$

In the 'Restrictions on gamma'' menu we shall now formulate 3 restrictions. The first restriction is the same as above. Since GRaM remembers the restrictions, we do not have to change the first restriction now.

The second restriction is to restrict the trend for the opportunity cost to be zero. Change therefore $'all'$ to $'r'$ in the line 'Restriction 2 on ..'. The restriction vector should read $R' = (0,0,0,0,1,0)$. Similarly, for the third restriction we change $'all'$ to $'Dp'$ and replaces the second last zero with unity, see Figure 10. [Note: There is something wrong in the present version of GRaM!!]

In the output GRaM reports there are now 6 independent coefficient restrictions, and the likelihood is now 1221.28. Comparing it to the unrestricted system yields $\chi^2 = 4.9[0.56]$, or comparing it to the system with restrictions only on the broken trend yields $\chi^2 = 0.7[0.70]$.

3.4 Imposing other (coefficient) restrictions

3.4.1 Restrictions on γ

Not all interesting restrictions on γ can be imposed by the formulation above. For example, in the system we look at, we might want to test if the 'growth rates' for money and income are equal. To allow for restrictions between the coefficients for the deterministic variables in γ, GRaM also allows for restrictions of the form

$$R'_{\gamma}\gamma = 0, \tag{23}$$

if the restriction is to hold for all deterministic variables, or

$$R'_{\gamma,i}\gamma_i = 0, \tag{24}$$

if it refers to only one deterministic variable. (Here γ_i is the i'th (column) vector in γ.)

The restriction that the 'growth rates' for money and income are equal, can therefore be imposed by the formulation

$$\begin{pmatrix} 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{m,trend} \\ \gamma_{y,trend} \\ \gamma_{r,trend} \\ \gamma_{Dp,trend} \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix},$$

which implies $\gamma_{m,trend} = \gamma_{y,trend}$. This is an example of restrictions of the form in (24).

To impose this restriction, go to the 'General restrictions' menu. Now, change the number of restrictions on γ in the last line to 1 (and keep the 3 restrictions on γ'). After you click on 'OK', you get to the menu for 'Restrictions on gamma''. Do not changes these restrictions, but press 'OK'.

Now you get to the menu 'Restrictions on gamma'. The restriction we want to impose is on the 'trend' variable only. Therefore, change '*all*' to '*trend*'. The restriction vector we want to impose on the trend is $R' = (1, -1, 0, 0)$. Therefore, change the first element in the row of zeros to 1 and the second to -1. The restriction that the 'growth rates' for money and income are equal is now imposed, and you can let GRaM simulate by clicking 'OK'.

The resulting likelihood value is 1214.45, and the likelihood ratio of this additional restriction is 13.66 with an probability value of 0.00 (with one degree of freedom). Therefore, the hypothesis of equal 'growth rates' for money and income is rejected.

NOTE: The restrictions for no trend in 'r' and 'Dp' could alternatively been imposed here!

3.4.2 Restrictions on ρ'

Restrictions on γ involves restrictions on the variables in the system. Restrictions on ρ involves restrictions on the cointegrating vectors in the system. We will now consider restrictions on the deterministic variables in the cointegrating vectors. The test we will impose, is if there is a significant shift in the trend (or the 'growth rates') in the two cointegrating vectors from 1990q3.

The restrictions are imposed similarly to the restrictions on γ, that is of the form

$$R'_{\rho}\rho' = 0 \tag{25}$$

if the restriction is to be imposed on 'all' cointegrating vectors, or γ, that is of the form

$$R'_{\rho,i}\rho'_i = 0 \tag{26}$$

if it is to be imposed on the i'th cointegrating vector only.

The hypothesis that the coefficients for the trend shift are zero in both cointegrating vectors can be tested with the restriction matrix $R'_{\rho} = (0, 0, 0, 0, 0, 1)$, since

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_{CI1,D1990q3} & \rho_{CI2,D1990q3} \\ \rho_{CI1,CS0} & \rho_{CI2,CS0} \\ \rho_{CI1,CS1} & \rho_{CI2,CS1} \\ \rho_{CI1,CS0} & \rho_{CI2,CS2} \\ \rho_{CI1,trend} & \rho_{CI2,trend} \\ \rho_{CI1,cumD90q3} & \rho_{CI2,cumD90q3} \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

implies $\gamma_{i,cumD1990q3} = 0$, $i = \{CI1, CI2\}$. (Here $CI1$ refers to the first cointegrating vector and $CI2$ refers to the second cointegrating vector.)

In the 'General restrictions' choose 1 in the row for 'Restrictions on rho'', and choose 0 for the restrictions on γ' and γ. (I.e. now we are only considering restrictions on ρ.)

In the 'Restrictions on rho'' menu, keep 'all' after 'Restriction 1 on ..', since we will impose the restriction on all (or both, since there are only two cointegrating vectors) cointegrating vectors. Next, change the last 0 to 1, so the restriction vector becomes $R'_{\rho} = (0, 0, 0, 0, 0, 1)$. Press 'OK'.

The likelihood value with this restriction is 1223.03, which yields $\chi^2 = 1.23[0.54]$. The restriction is therefore not rejected. This we would expect, since the restriction of no trend break in any of the variables (which is a stronger restriction) was not rejected either.

We can test for co-breaking when we test restrictions on ρ. The concept of co-breaking was introduced by Hendry and Mizon (1998). If deterministic breaks in a system of equation can be removed by taking linear combination of the system variables, the variables are said to co-break. With GRaM we can test if the co-breaking vectors coincides with the cointegrating vectors. For example, the restriction of no level shift in the variables ($R'_{\gamma'} = (0, 0, 0, 0, 0, 1)$) are rejected in our system. However, if the restriction of no level shift in the two cointegrating vectors ($R'_{\rho} = (0, 0, 0, 0, 0, 1)$) is not rejected, the (space spanned by the) cointegarting vectors equals the (space spanned by the) co-breaking vectors. Then we can say that the cointegrating space also represents a co-breaking space.

3.4.3 Restrictions on β^*

As other programs, GRaM allows for restrictions among the endogenous variables in the cointegrating space, β. In addition, GRaM allows for restrictions on the intercept of the cointegrating vectors, i.e. μ in (3) on page 8. These coefficients (μ) may be interpreted as the cointegration means, at least if $\rho = 0$. The coefficient restrictions on β and μ may be formalised as

$$R'_{\beta}\beta^* = c_{\beta}, \tag{27}$$

where $\beta^* = (\beta, -\mu')$. In (27) we have included a level variable, c_{β}, which makes it possible to normalise the cointegrating vectors.

Of course, if there are more than one cointegrating vector, it is possible to impose restrictions on them separately;

$$R'_{\beta,i}\beta^*_i = c_{\beta,i} \tag{28}$$

[Something about rotation of the cointegrating space... Also important for rho!]

[An example...]

3.5 Graphical output

With GRaM you can also get some graphical output of your estimation results. Estimate the system with cointregation rank 2 and without restrictions. Choose 'Graphical Analysis' under the 'Test' menu. You will then get a graphical output similarly to Figure 11 on the next page. In the first column the true time series are plotted together with one period predictions ('fitted'). In the second column the residuals are plotted. In the last column the 'de-trended' variables are plotted. The 'de-trended' variables are defined by

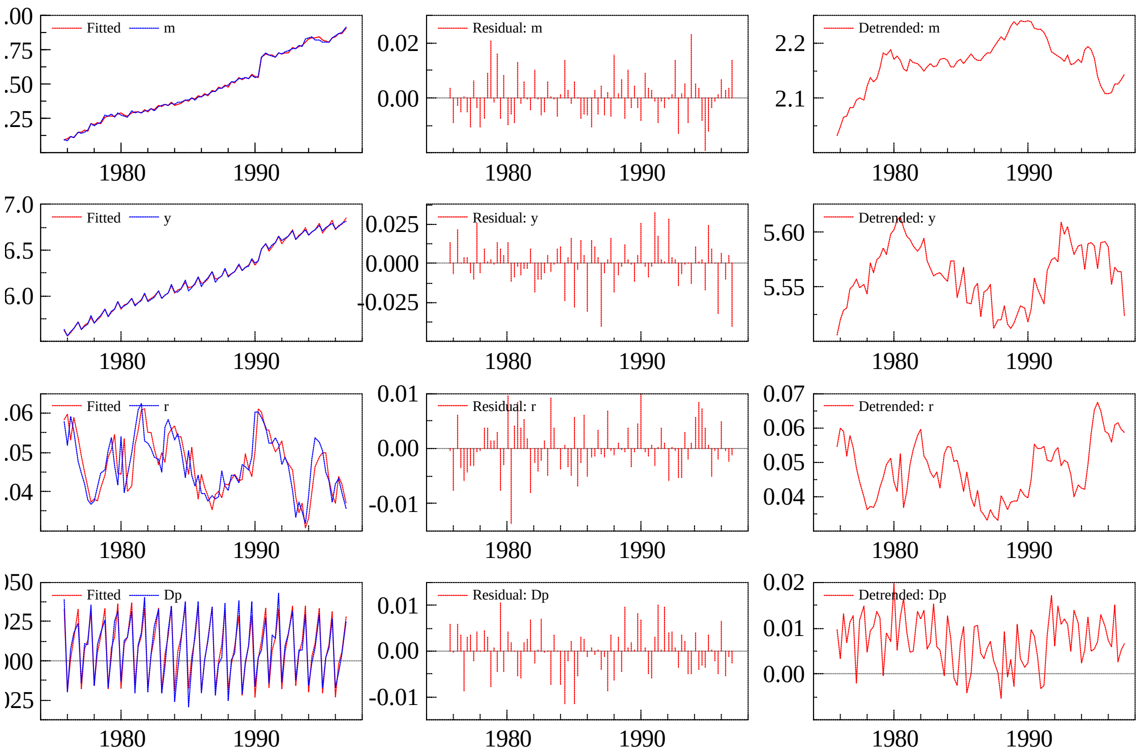
$$Y_t^{det} = Y_t - \gamma \cdot D_t, \tag{29}$$

i.e. where the deterministic components are removed from the series. In these 'de-trended' data series there should be no deterministics such as trend and seasonality. (If there is, this indicates that you have not included all necessary deterministic variables in the system.) From the graph we see that the trending is removed in the series for money and income. And in the graph for 'de-tended' inflation much of the variation is removed, which indicates that much of the variation in inflation were seasonal related.

3.6 Using GRaM together with PcGive and PcGets

In GRaM you can estimate the long run properties of your system, and construct the corresponding 'de-trended' variables, see (29). These 'de-trended' variables can be saved in the database and then be used in other programs such as PcGive or PcGets. To save the 'de-trended' variables choose 'Store in database...' under the 'Test' menu, and cross off for 'De-trended variables'. Then choose names for your 'de-trended' variables. (GRaM suggests a

Figure 11: Graphical output



...

prefix 'Det' before the original labels of your data series.)

...

4 Technical documentation

4.1 The estimation problem with restrictions

In this section the estimation problem with restrictions is described. Two main versions of the maximising problem will be formulated: one where all linear restrictions among the coefficients in the different coefficient matrices are allowed for (Section 4.1.1), and one where only a subset of such restrictions can be imposed (Section 4.1.2). GRaM uses both these problem formulations to solve the maximising problem.

For each of the main formulations of the problem, two alternative reformulations are presented. In these reformulations the restriction in the maximising problem is incorporated directly, allowing for simulation algorithms such as BFGS and SA.

4.1.1 Restrictions in vector form

Introduction: We allow for linear restrictions between all the elements in each of the coefficient matrices γ, β and ρ.

Restrictions on gamma: Restrictions on γ can be written as

$$R_{\gamma}^{v\prime} vec\gamma' = 0. \tag{30a}$$

The restrictions on γ can alternatively be written as

$$vec\gamma' = H_{\gamma}^{v} \cdot \phi_{\gamma}^{v}, \tag{30b}$$

where $H_{\gamma}^{v} = \left(R_{\gamma}^{v}\right)_{\perp}$.

Restrictions on beta: We define $\beta^{*} = (\beta', -\mu)'$ and $X_t^{*} = (X_t', 1)'$, so restrictions on the cointegration means can also be imposed. The restrictions on the cointegrating space can be written as

$$R_{\beta}^{v\prime} vec\beta^{*} = c_{\beta}, \tag{31a}$$

where each column in R_{β} represents a restriction on $vec\beta^{*}$. The inclusion of the vector c_{β} makes it possible to impose normalisation restrictions on β. Equivalently, we can write

$$vec\beta^{*} = H_{\beta}^{v} \cdot \phi_{\beta}^{v} + h_{\beta}, \tag{31b}$$

where $H^v_\beta = \left(R^v_\beta\right)_\perp$ and $h_\beta = \overline{R_\beta} \cdot c_\beta$.

Restrictions on rho: If we want to test for different types of structural breaks in the cointegrating space, we have to impose restrictions on ρ directly. Let these restrictions be written as

$$\begin{aligned} R^{v\prime}_\rho vec\rho' &= 0 \\ &\Updownarrow \\ R^{v\prime}_\rho vec\left(\gamma'\beta\right) = R^{v\prime}_\rho \left(I_r \otimes \gamma' J\right) vec\beta^* &= R^{v\prime}_\rho \left(\beta' \otimes I_q\right) vec\gamma' = 0, \end{aligned} \tag{32}$$

where $J = (I_n, 0_{n\times 1})$.

Now, consider how to estimate the system. First, suppose ϕ^v_β and ϕ^v_γ (and therefore β^* and γ) were known. Then the remaining coefficients could be estimated by applying OLS in (7) on page 9. Let $l\left(\widehat{\alpha}\left(\phi^v_\beta, \phi^v_\gamma\right), H^v_\beta\phi^v_\beta, H^v_\gamma\phi^v_\gamma, \widehat{\Gamma}_1\left(\phi^v_\beta,\phi^v_\gamma\right), ..., \widehat{\Gamma}_{p-1}\left(\phi^v_\beta,\phi^v_\gamma\right), \widehat{\Omega}\left(\phi^v_\beta,\phi^v_\gamma\right)\right)$ be the corresponding log likelihood value.

Problem 4.1 *(General problem with vector restrictions) The maximum likelihood estimates for β, γ and ρ can be found by* (30b), (31b) *and $\rho = \beta'\gamma$ respectively, where we use the coefficient matrices that solves*

$$\max_{\phi^v_\beta,\phi^v_\gamma} \left\{ l\left(\widehat{\alpha}\left(\phi^v_\beta,\phi^v_\gamma\right), H^v_\beta \cdot \phi^v_\beta, H^v_\gamma \cdot \phi^v_\gamma, \widehat{\Gamma}_1\left(\phi^v_\beta,\phi^v_\gamma\right), ..., \widehat{\Gamma}_{p-1}\left(\phi^v_\beta,\phi^v_\gamma\right), \widehat{\Omega}\left(\phi^v_\beta,\phi^v_\gamma\right)\right) \right.$$
$$\left. \text{subject to } R^{v\prime}_\rho vec\left[\gamma\left(\phi^v_\gamma\right)' \cdot J \cdot H^m_\beta \cdot \phi^m_\beta\right] = 0 \right\},$$

where $J = (I_n, 0_{n\times 1})$.

The solution must imply that (the empirical counterparts to) the following first order conditions holds:

$$\begin{aligned} H^{v\prime}_\gamma M' \left[vec\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD} - \Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime}S_{YD} - \Phi^{*\prime}\Omega^{-1}\Phi^*\left(I_{p+1}\otimes\gamma\right)S_{DD}\right)\right] \\ -\lambda'\left[R^{v\prime}_\rho\left(\beta'\otimes I_q\right)H^{v\prime}_\gamma\right] \\ = 0 \end{aligned} \tag{33a}$$

$$\begin{aligned} H^{v\prime}_\beta \left[\left(\alpha'\Omega^{-1}\otimes I_{n+1}\right) vecS_{10} - \left(\alpha'\Omega^{-1}\alpha \otimes S_{11}\right)\left(H^v_\beta\phi^v_\beta + h_\beta\right)\right] \\ -\lambda'\left[R^{v\prime}_\rho\left(I_r\otimes\gamma' J\right)H^{v\prime}_\beta\right] &= 0 \end{aligned} \tag{33b}$$

$$R^{v\prime}_\rho vec\left(\gamma'\beta\right) = 0 \tag{33c}$$

where γ and β^* are given by (30b) *and* (31b) *respectively, and λ is the vector of Lagrange multipliers. [Something about the second order condition...]*

To apply Solution 4.1 we need an algorithm that allows restrictions. Alternatively, we could transform the restrictions on ρ to restrictions on γ or β.

Remark 4.1 *Restrictions on ρ can be transformed to restrictions on γ by applying* (32). *The joint set* *of restrictions imposed on γ is therefore*

$$\begin{pmatrix} R_{\gamma}^{v\prime} \\ R_{\rho}^{v\prime}\left(\beta\otimes I_q\right) \end{pmatrix} vec\gamma' = 0, \tag{34}$$

and we can apply Solution 4.1 (without the restriction) with the modification that H_{γ}^{v} is the orthogonal compliment to the transposed matrix in the parenthesis in (34).

Remark 4.2 *Restrictions on ρ can be transformed to restrictions on β by applying* (32)*. The joint set of restrictions imposed on β is therefore*

$$\begin{pmatrix} R_{\beta}^{v\prime} \\ R_{\rho}^{v\prime}\left(I_r\otimes\left(\gamma'\cdot J\right)\right) \end{pmatrix} vec\beta^* = \begin{pmatrix} c_{\beta} \\ 0 \end{pmatrix}, \tag{35}$$

where $J = (I_n, 0_{n\times 1})$. The system can now be maximised by applying Solution 4.1 (without the restriction) where H_{β}^{v} is the orthogonal compliment to the transposed matrix in the first parenthesis in (35)*, and h_{β} constructed based on the last parenthesis.*

4.1.2 Restrictions on matrix form

Sometimes it is possible to reformulate the restrictions to restrictions on the space spanned by the matrixes β and ρ. That is, the restrictions can be written as

$$R_{\beta}^{m\prime}\beta^* = 0 \Leftrightarrow \beta^* = H_{\beta}^{m}\cdot\phi_{\beta}^{m}, \tag{36}$$

$$R_{\rho}^{m\prime}\rho' = 0 \Leftrightarrow R_{\rho}^{m\prime}\gamma'\beta = 0, \tag{37}$$

whereas restrictions on γ still are written as (30a).

Problem 4.2 *(General problem with matrix restrictions) The maximum likelihood coefficients for β, γ and ρ can be found by* (30b), (36) *and $\rho = \beta'\gamma$ respectively, where we use the coefficient matrices that solves*

$$\max_{\phi_{\beta}^{m},\phi_{\gamma}^{v}} \Big\{ l\left(\widehat{\alpha}\left(\phi_{\beta}^{m},\phi_{\gamma}^{v}\right), H_{\beta}^{m}\cdot\phi_{\beta}^{m}, H_{\gamma}^{v}\cdot\phi_{\gamma}^{v}, \widehat{\Gamma}_1\left(\phi_{\beta}^{m},\phi_{\gamma}^{v}\right), ..., \widehat{\Gamma}_{p-1}\left(\phi_{\beta}^{m},\phi_{\gamma}^{v}\right), \widehat{\Omega}\left(\phi_{\beta}^{m},\phi_{\gamma}^{v}\right)\right) \\ \text{subject to } R_{\rho}^{v\prime}vec\left[\gamma\left(\phi_{\gamma}^{v}\right)'\cdot J\cdot H_{\beta}^{m}\cdot\phi_{\beta}^{m}\right] = 0 \Big\}.$$

The solution must imply that (the empirical counterparts to) the following first order conditions holds:

$$\begin{aligned} H_{\gamma}^{v\prime}M'\left[vec\left(\Phi^{*\prime}\Omega^{-1}\Phi S_{ZD} - \Phi^{*\prime}\Omega^{-1}\alpha\beta^{*\prime}S_{YD} - \Phi^{*\prime}\Omega^{-1}\Phi^{*}\left(I_{p+1}\otimes\gamma\right)S_{DD}\right)\right] \\ -\lambda\left[\beta^{*\prime}J'H_{\gamma}^{v\prime}\otimes R_{\rho}^{m\prime}\right] \\ = 0 \end{aligned} \tag{38a}$$

$$vec\left[H_{\gamma}^{m\prime}S_{10}\Omega^{-1}\alpha+H_{\gamma}^{m\prime}S_{11}\gamma\alpha'\Omega^{-1}\alpha\right]$$
$$-\lambda'\left[I_r\otimes R_{\rho}^{v\prime}\gamma' J H_{\beta}^{m}\right] = 0 \tag{38b}$$

$$R_{\rho}^{m\prime}\left(\gamma'\beta\right)=0 \tag{38c}$$

where λ is the vector of Lagrange multipliers.

To apply Solution 4.2 we need an algorithm that allows restrictions. Alternatively, we could transform the restrictions on ρ to restrictions on γ or β.

Remark 4.3 *Restrictions on ρ can be transformed to restrictions on γ by applying* (37)*. The joint set of restrictions imposed on γ is therefore*

$$\begin{pmatrix} R_{\gamma}^{m\prime} \\ \beta'\otimes R_{\rho}^{m\prime}\end{pmatrix}\gamma'=0, \tag{39}$$

and we can apply Solution 4.2 (without the restriction) with the modification that H_{γ}^{v} is the orthogonal compliment to the transposed matrix in the parenthesis in (39)*.*

Remark 4.4 *Restrictions on ρ can be transformed to restrictions on β by applying* (32)*. The joint set of restrictions imposed on β is therefore*

$$\begin{pmatrix} R_{\beta}^{m\prime} \\ R_{\rho}^{m\prime}\gamma' J\end{pmatrix}\beta^{*}=0. \tag{40}$$

where $J=(I_n,0_{n\times 1})$. The system can now be maximised by applying Solution 4.1 (without the restriction) where H_{β}^{v} is the orthogonal compliment to the transposed matrix in the parenthesis in (40)*.*

4.2 Starting values

In GRaM you can choose between different methods for computing starting values. The default method is descried in Section 4.2.3.

Some, but not all, coefficient restrictions are taken into account when the starting values are chosen. All restrictions imposed on γ are taken into account. This means that we are really considering starting values of ϕ_{γ}^{v}, and the corresponding starting values for γ is therefore given by (30b).

When choosing starting values for β not all restrictions are imposed. Only the restrictions on β that involves restrictions on the cointegration space are imposed, see (36).

Restrictions on ρ are not taken into account when starting values are computed if the chosen simulation algorithm is MaxSQP. However, if MaxBFGS or MaxSA are used as starting values, the restrictions on ρ that are formed as (37) are transformed into restrictions on the cointegration space. Therefore the joint set of restrictions that are taken into account when the starting values for β are calculated are given by (36), where H_{β}^{m} is given by (**??**).

4.2.1 Zeros (or almost zero)

Here, the starting values of γ are zero or close to zero.[7] (If restrictions are imposed on γ, the elements in ϕ_γ^v are set equal to (or close to) zero, and (30b) is used to calculate the corresponding starting values for γ.)

Based on these starting values for γ, GRaM apply (29) on page 26 to de-trend the data series. With these de-trended variables estimates for β are identified by normal reduced rank. Restrictions on β formed as in (36) are imposed when estimating starting values for β. Depending on the simulation algorithm that is used, also restrictions via ρ may be imposed.

4.2.2 As for rank = 0

Here GRaM calculates starting values in two steps. In the first step the starting values to γ are set equal to (or close to, se above) zero (but no starting values for β are calculated. In the second step GRaM use the MaxBFGS algorithm to simulate for new starting values for γ in the system where cointegration rank equal to zero is imposed. (Therefore are restrictions on β and ρ not taken into account. However, the restrictions on γ are imposed.)

Based on the starting values for γ from the first step, starting values for β are computed by reduced rank. Her the restrictions on the cointegration space (i.e. equation (36)) or the joint set of restrictions on β and ρ given by Remark **??** are imposed, depending on which simulation algorithm is used. (The former is used when MaxSQP is chosen and the latter is used when MasBFGS or MaxSA are chosen.)

4.2.3 OLS - Static regression (gamma restrictions imposed)

Here the starting values of γ are found by estimating

$$Y_t = \gamma \cdot D_t + u_t \tag{41}$$

when the restrictions given by (30a) are imposed on γ. Only a first round estimate is found, where the identity matrix is used as a proxy for the covariance matrix to the errors.

4.2.4 OLS - Static regression (rho restrictions imposed)

Also here starting values of γ are found by estimating (41), but here the restrictions on γ are combined with restrictions on the space of ρ, i.e.

$$\begin{pmatrix} R_\gamma^{v\prime} \\ I_n \otimes R_\rho^{m\prime} \end{pmatrix} vec\gamma' = 0. \tag{42}$$

[7] Sometimes it can be problematic to use starting values of γ equal to zero. When there are restrictions on ρ, these restrictions are allays fulfilled when γ consists of zeros only, see (32). Therefore, when γ equals zero restrictions on ρ will not involve any restrictions on β. However, when the simulation algorithm tries to change value of the elements in γ slightly, the restrictions on ρ will imply restrictions on β. The algorithm may therefore not move from the situation where all the elements in γ equals zero. This problem is solved by choosing starting values not equal (but close to) zero.

Since we are imposing so strong restrictions on γ, we only need to impose the restrictions on the cointegration space (see equation (36)) independent of which simulation algorithm we use. The restrictions implied by (37) are already imposed on γ.

4.2.5 JMN/SL - Johansen, Mosconi, Nilsen and Saikkonen, Lütkepohl

The system in (7) on page 9 can be rewritten as

$$\Delta y_t = \alpha \left(\beta' Y_{t-1} - \mu - \rho D_{t-1}\right) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \left[\gamma \Delta D_t - \sum_{i=1}^{p-1} \Gamma_i \gamma \Delta D_{t-i}\right] + \varepsilon_t \qquad (43)$$

Use $\beta^* = H_\beta^m \cdot \phi_\beta^m$, $\gamma' = H_\gamma^m \cdot \phi_\gamma^m$, and $\rho' = H_\rho^m \cdot \phi_\rho^m$, where $H_\beta^m = \left(R_\beta^m\right)_\perp$, $H_\gamma^m = \left(R_\gamma^m\right)_\perp$, and $H_\rho^m = \left(\left(R_\rho^{m\prime}, R_\gamma^{m\prime}\right)'\right)_\perp$. Note that the special construction of H_ρ^m secures that all restrictions on γ also are imposed on ρ, i.e. $span\left(H_\rho^m\right) \subset span\left(H_\gamma^m\right)$.[8]

With these restrictions the system can be rewritten as

$$\begin{aligned}
\Delta Y_t &= \alpha \left(\phi_\beta^{m\prime}, \phi_\rho^{m\prime}\right) \begin{pmatrix} H_\beta^{m\prime} Y_{t-1}^* \\ \left[H_\rho^{m\prime} D_{t-1}\right] \end{pmatrix} \\
&\quad + \left(\Gamma_1, ..., \Gamma_{p-1}, \Psi_0, \Psi_1, ..., \Psi_{p-1}\right) \begin{pmatrix} \Delta Y_{t-1} \\ \vdots \\ \Delta Y_{t-p+1} \\ \begin{bmatrix} H_\gamma' \Delta D_t \\ H_\gamma' \Delta D_{t-1} \\ \vdots \\ H_\gamma' \Delta D_{t-p+1} \end{bmatrix} \end{pmatrix} + \varepsilon_t,
\end{aligned}$$

with some restrictions on the coefficient matrices Ψ_i (which are ignored when computing the starting values).

When the coefficient restrictions in the Ψ's are ignored, the system above may include linear dependent variables. Therefore, let $[...]_0$ only include linearly independent variables (i.e. exclude variables that are linearly dependent). Furthermore, in $[...]_1$, exclude all variables that are linearly dependent of other variables in $[...]_0$ or $[...]_1$.

Now α, β, Γ_i $(i = 1, ..., p-1)$ and Ω can be estimated by applying reduced rank technics, see e.g. Johansen (1995, pp. 89-93). Based on these estimates, γ can be estimated by[9]

$$\begin{aligned}
\widehat{vec\gamma}' \left(\alpha, \beta^*, \Gamma_1, ..., \Gamma_{p-1}, \Omega\right) &= H_\gamma^v \left[H_\gamma^{v\prime} M' \left(S_{DD} \otimes \Phi^{*\prime} \Omega^{-1} \Phi^*\right) M H_\gamma^v\right]^{-1} \\
&\quad \times \left[H_\gamma^{v\prime} M' vec\left(\Phi^{*\prime} \Omega^{-1} \Phi S_{ZD} - \Phi^{*\prime} \Omega^{-1} \alpha \beta^{*\prime} S_{YD}\right)\right],
\end{aligned}$$

[8] See Johansen et al. (2000, Section 4) for a discussion on the problems when the span restiction is not fullfilled.

[9] To use the equation below one needs to know β^* (and not only β). The present version of GRaM use $\mu = \sum_{t=1}^T \beta' Y_{t-1}$. However, $\mu = \sum_{t=1}^T \beta' Y_{t-1} - \phi_\rho^{m\prime} \left[H_\prime^{m\prime} D_{t-1}\right]$ will probably be a more precise estimate.

where the same notation as in Theorem 2 on page 10 is used. The proof is similar to the proof of Theorem 2.

If MaxSQF is chosen as the maximising algorithm, the estimates of β^* and γ achieved by the procedure above is used as starting values. If MaxBFGS or MaxSA is used as maximising algorithm, β^* is estimated again where also the restrictions on ρ are imposed.

4.3 Simulation

The simulation is divided into two parts.

4.3.1 First simulation - space restrictions imposed

In the first part of the simulation all restrictions on γ are taken into account. However, only a subset of the restrictions imposed on β and ρ are taken into account. The restrictions on β and ρ that is allowed for in this part, are those that involves restrictions on the cointegrating space and their corresponding deterministics. That is, only restrictions that can be formulated as $R'\beta^* = 0$ (see equation (36)) and $R'\rho' = 0$ (see equation (37)) are imposed on β^* and ρ, respectively. On γ we allow for all linear coefficient restrictions, see (30a).

When no restrictions are imposed, the matrix $(\beta^{*\prime}, -\rho)$ is only unique up to a rotation of the space spanned by the matrix. In the first part of the simulation we only include restrictions that restrict this space, but do not involve any restrictions on the rotation of the space. The space can therefore rotate freely in this first part, which increases the speed of reaching the maxima as well as the probability of reaching it.

If MaxSQP is used as the simulation algorithm the log-likelihood function formulated in Problem 4.2 is maximised, and the first order conditions (38a) - (38c) are used to making the algorithm analytical.

If MasBFGS or MaxSA is used as the simulation algorithm, GRaM uses a modification of Problem 4.2 described in the remarks. Remark 4.3 is used if possible (i.e. when there are not so many restrictions on β^* and ρ that the orthogonal complement to the parenthesis (40) is not defined), Remark 4.4 otherwise.

4.3.2 Rotation of cointegration space

After the restrictions on the space of $(\beta^{*\prime}, -\rho)$ are imposed, GRaM tries to rotate this space such that it corresponds as good as possible to all restrictions imposed on it. GRaM applies the method descried in Doornik (1995).

Let

$$vec\widehat{b}_1^* = H_\beta^v \left[\overline{H_\beta^v}' \left(vec\left(\widehat{\beta^*}_{1.step} \right) - h_\beta \right) \right] + h_\beta \tag{44}$$

and

$$vec\widehat{b}_2^{*\prime} = H_\rho^v \cdot \overline{H_\rho^v}' \cdot vec\left(\widehat{\rho}'_{1.step} \right), \tag{45}$$

where $H_\rho^v = \left(R_\rho^v\right)_\perp$ and the subscript *1.step* indicates the estimates achieved from the first simulation step. Define $\langle\cdot\rangle$ as dropping those rows which have no restrictions in them; if this yields less than r rows, the add rows back in, so that the $\langle\cdot\rangle$ matrix is $m \times q$, with $m \geq r$. Then the least square estimator

$$\widehat{A} = \left(\left\langle \begin{array}{c} \widehat{\beta^*}_{1.step} \\ -\widehat{\rho}'_{1.step} \end{array} \right\rangle' \left\langle \begin{array}{c} \widehat{\beta}_{1.step} \\ -\widehat{\rho}'_{1.step} \end{array} \right\rangle\right)^{-1} \left(\left\langle \begin{array}{c} \widehat{\beta^*}_{1.step} \\ -\widehat{\rho}'_{1.step} \end{array} \right\rangle' \left\langle \begin{array}{c} \widehat{b_1} \\ -\widehat{b_2}' \end{array} \right\rangle\right) \tag{46}$$

can be used to derive

$$\widehat{\alpha_0} = \widehat{\alpha}_{1.step} \cdot \left(\widehat{A'}\right)^{-1}, \tag{47}$$

and the corresponding starting value of β^* in the second simulation part is

$$\begin{aligned} vec\widehat{\beta_0^*} &= H_\beta^v \left[H_\beta^{v\prime}\left(\alpha'\Omega^{-1}\alpha \otimes S_{11}\right) H_\beta^v\right]^{-1} \\ &\times \left[H_\beta^{v\prime}\left(\alpha'\Omega^{-1} \otimes I_{n+1}\right) vecS_{10} - H_\beta^{v\prime}\left(\alpha'\Omega^{-1}\alpha \otimes S_{11}\right) h_\beta\right] + h_\beta \end{aligned} \tag{48}$$

4.3.3 Second simulation - vectorised restrictions imposed

In the second part of the simulation all restrictions are taken into account. Then the formulation in Problem 4.1 is used. The formulation is modified according to the remarks if MaxBFGS or MaxSA are used as simulation algorithms.

4.4 Switching

In Section 2.3 it was shown that the system could be written as

$$\Phi Z_t = \alpha\beta^{*\prime} Y_{t-1}^* + \Phi^*\left(I_{p+1} \otimes \gamma\right) D_t^v + \varepsilon_t, \tag{49}$$

where $Z_t = vec\left(\Delta Y_t, ..., \Delta Y_{t-p+1}\right)$, $D_t^v = vec\left(D_t, ..., D_{t-p}\right)$, $\Phi = \left(I_n, -\Gamma_1, ..., -\Gamma_{p-1}\right)$, and $\Phi^* = \left(I_n, -\Gamma_1^* - \alpha\beta', -\Gamma_2^*, ..., -\Gamma_p^*\right)$.

Alternatively, the system could be written as

$$Z_{0,t} = \alpha\beta^{*\prime} Z_{1,t} + \Theta Z_{2,t} + \varepsilon, \tag{50}$$

where $Z_{0,t}(\gamma) = \Delta Y_t - \gamma\Delta D_t$, $Z_{1,t}(\gamma) = \left[(Y_{t-1} - \gamma D_{t-1})', 1\right]'$, $Z_{2,t}(\gamma) = vec\,(\,\Delta Y_{t-1} - \gamma D_{t-1}, ..., \Delta Y_{t-p+1} - \gamma D_{t-p+1}\,)$ and $\Theta = \left(\Gamma_1, ..., \Gamma_{p-1}\right)$.

Furthermore, define

$$\begin{aligned} S_{DD} &= T^{-1}\sum D_t^v D_t^{v\prime}, \\ S_{ZD} &= T^{-1}\sum Z_t D_t^{v\prime}, \\ S_{YD} &= T^{-1}\sum Y_{t-1}^* D_t^{v\prime}, \end{aligned}$$

(and similarly for S_{ZZ} and $S_{YZ} = S'_{ZY}$),

$$M_{ij} = T^{-1} \sum_{t=1} T Z_{it} Z'_{jt} \quad , i,j = 0,1,2,$$
$$S_{ij} = M_{ij} - M_{i2} M_{22}^{-1} M_{2j} \quad , i,j = 0,1,$$

and implicitly define the matrix M with dimension $q\,(p+1)^2\, n \times qn$ by[10]

$$vec\left(I_{p+1} \otimes \gamma\right) = M vec\gamma'.$$

Theorem 4 *The conditional maximum likelihood estimators for β, α, $\Theta \equiv \left(\Gamma_1, ..., \Gamma_{p-1}\right)$, γ and Ω under the restrictions* (30a)*,* (31a) *and* (32) *are given by*

$$\begin{aligned} \widehat{vec\beta^*} &= H_\beta^v \left[H_\beta^{v\prime} \left(\alpha' \Omega^{-1} \alpha \otimes S_{11} \right) H_\beta^v \right]^{-1} \\ &\quad \times \left[H_\beta^{v\prime} \left(\alpha' \Omega^{-1} \otimes S_{11} \right) \left(vec\left(S_{11}^{-1} S_{10} \right) - \left(\alpha \otimes I_{n+1} \right) h_\beta \right) \right] + h_\beta \end{aligned} \tag{51}$$

$$\widehat{vec\alpha'} = \left[\Omega^{-1} \otimes \beta^{*\prime} S_{11} \beta^* \right]^{-1} \left(\Omega^{-1} \otimes \beta^{*\prime} S_{11} \right) vec\left(S_{11}^{-1} S_{10} \right) \tag{52}$$

$$\widehat{\Theta} = M_{02} \left(M_{22} \right)^{-1} - \alpha \beta^{*\prime} M_{12} \left(M_{22} \right)^{-1} \tag{53}$$

$$\begin{aligned} \widehat{vec\gamma'} &= H_\gamma^v \left[H_\gamma^{v\prime} M' \left(S_{DD} \otimes \Phi^{*\prime} \Omega^{-1} \Phi^* \right) M H_\gamma^v \right]^{-1} \\ &\quad \times \left[H_\gamma^{v\prime} M' vec\left(\Phi^{*\prime} \Omega^{-1} \Phi S_{ZD} - \Phi^{*\prime} \Omega^{-1} \alpha \beta^{*\prime} S_{YD} \right) \right] \end{aligned} \tag{54}$$

$$\widehat{\Omega} = S_{00} - \alpha \beta^{*\prime} S_{10} - S_{01} \beta^* \alpha' + \alpha \beta^{*\prime} S_{11} \beta^* \alpha \prime \tag{55}$$

where

$$H_\beta^v = \left(R_\beta^v \right)_\perp \quad and \quad H_\gamma^v = \left(R_\gamma^v, \left(\beta' \otimes I_q \right) R_\rho^v \right)_\perp \tag{I}$$

or

$$H_\beta^v = \left(R_\beta^v, \left(I_r \otimes J'\gamma \right) R_\rho^v \right)_\perp \quad and \quad H_\gamma^v = \left(R_\gamma^v \right)_\perp \tag{II}$$

or

$$H_\beta^v = \left(R_\beta^v, \left(I_r \otimes J'\gamma \right) R_\rho^v \right)_\perp \quad and \quad H_\gamma^v = \left(R_\gamma^v, \left(\beta' \otimes I_q \right) R_\rho^v \right)_\perp \tag{III}$$

with $J = (I_n, 0_{n\times 1})$.

Proof. ... ■

From (I) - (III) it can be seen that the restrictions on ρ are imposed on either β^* or γ, or on both.

The advantage of using (III) in an algorithm is that is secures that the likelihood value increases for each iteration. However, it might reduce the parameter space the algorithm can

[10]See footnote 2 on page 10 for the explicit form of M.

search. Therefore, if the hypothesis restrictions in (III) are imposed, the algorithm might not find the maximum solution.

The advantage of using (I) or (II) in an algorithm is that it seeks a broader parameter space than if (III) is used. However, it might lead to a decrease in the likelihood value.

GRaM use the following algorithm:

Algorithm 4.1 *Estimators for β, α, $\Theta \equiv \left(\Gamma_1, ..., \Gamma_{p-1}\right)$, γ and Ω under the restrictions* (30a), (31a) *and* (32) *may be obtained by the following iterative procedure, starting from a set of initial values* $\{\beta, \alpha, \Theta, \gamma, \Omega\}$*:*

$$
\begin{aligned}
\widehat{vec\beta^*}_i &= \beta\left(\alpha_{i-1}, \gamma_{i-1}, \Omega_{i-1}; H_\beta^v\right) \\
\widehat{vec\alpha'}_i &= \alpha\left(\beta_i, \gamma_{i-1}, \Omega_{i-1}\right) \\
\widehat{\Theta}_i &= \Theta\left(\beta_i, \gamma_{i-1}, \Omega_{i-1}\right) \\
\widehat{vec\gamma'}_i &= \gamma\left(\beta_i, \alpha_i, \Theta_i, \Omega_{i-1}; H_\gamma^v\right) \\
\widehat{\Omega}_i &= \Omega\left(\beta_i, \alpha_, \gamma_i\right) \\
i &= 1, 2, ...
\end{aligned}
$$

Use (I)*. However, if an iteration leads to a decreased likelihood, use the estimates from the previous iteration and use* (III) *in this iteration. In the next iteration, use* (I)*, unless this leads to a decreased likelihood. (And so forth...) Continue until convergence.*

4.5 Reliability (2nd order condition)

...

4.6 Distribution for the likelihood tests

When GRaM tests restrictions, it applies the χ^2-distribution. For restrictions on β it has been shown in e.g. Boswijk (1995) or Boswijk and Doornik (2003) that this is the correct distribution. Boswijk and Doornik (2003) also shows that restrictions on β^* is χ^2 in the H_c case, (i.e D_t is empty, or at least does not include a trend). In the H_l case μ is not included in the cointegration vectors when estimating cointegrated VAR models in its normal form (1), and therefore similar results are not available.

For restrictions on γ ... : Johansen et al. (2000) shows that (at least a subset of) restrictions of the form $R'\gamma'$ are χ^2-distributed.

Restrictions on ρ can be reformulated into restrictions on β or γ; so if restrictions on β and γ can be tested based on a χ^2-distribution, restrictions on ρ can be tested based on the same distribution as well.

In order to apply the χ^2 test one needs to know the degrees of freedom in the test, i.e. how many independent restrictions that are imposed. The number of restrictions on γ is given by $rank\left(R_\gamma^v\right)$.

If there are no restrictions on β^* there are $(n+1-r)\,r$ independent elements i β^*. If restrictions are imposed, Boswijk (1995) shows that the number of independent elements in β^* is given by

$$rank\left[\left(I_r \otimes \beta_{\perp}^{*\prime}\right) H_\beta\right].$$

If both restrictions on β and γ are considered;

$$H_\beta^v = \left(R_\beta^v, \left(I_r \otimes J'\gamma\right) R_\rho^v\right)_\perp.$$

The number of independent restrictions imposed on β^* and ρ is therefore given by $(n+1-r)\,r - rank\left[\left(I_r \otimes \beta_{\perp}^{*\prime}\right) H_\beta\right]$.

The degrees of freedom used in the χ^2 test is the sum of restrictions on γ and the restrictions on β^* and ρ.

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