Measuring systemic risk in the European banking sector: A Copula CoVaR approach

Emmanouil N. Karimalis* Nikos Nomikos†

This version: March 31, 2014

Abstract

We propose a new methodology based on copula functions to estimate *CoVaR*, the *Value-at-Risk* (*VaR*) of the financial system conditional on an institution being under financial distress. Our Copula *CoVaR* approach provides simple, closed-form expressions for various definitions of *CoVaR* for a broad range of copula families. The proposed framework is flexible as it allows the *CoVaR* of an institution to have time-varying exposure to its *VaR*. We also extend this approach to estimate other "co-risk" measures such as Conditional Expected Shortfall (*CoES*). We estimate systemic risk contribution for a portfolio of large European banks and examine the existence of common market factors triggering systemic risk episodes. Finally, we analyse the extent to which bank-specific characteristics such as size, leverage, and equity beta are associated with institutions' contribution to systemic risk and highlight the importance of liquidity risk at the outset of the financial crisis in summer 2007.

Keywords: Value-at-Risk, Systemic Risk, Risk Spillovers, Copula functions.

JEL classification: G11, G18, G20, G21, G32, G38

*Cass Business School, City University London, 106 Bunhill Row, London, EC1Y 8TZ. Corresponding author. E-mail address: Emmanouil.Karimalis.1@cass.city.ac.uk

†Cass Business School, City University London, 106 Bunhill Row, London, EC1Y 8TZ. E-mail address: N.Nomikos@city.ac.uk

1 Introduction

The ongoing financial crisis has highlighted with the most prominent way the importance for prudent monitoring and assessment of systemic risk. Systemic risk can be seen as the adverse consequence for the financial system and the broader economy, of the fact that a financial institution is in distress. The failure of large credit institutions can not only threat the stability of the financial system but can also have dramatic effects on the real economy. It is well-documented that conditional correlations between asset returns are much stronger in periods of financial distress (see e.g. Longin and Solnik 2001; Ang and Chen 2002) and typically arise from exposure to common shocks, although, amplifications of financial shocks are also associated with balance sheet channels and liquidity spirals (see e.g. Brunnermeier 2009b; Adrian and Shin 2010). As a result, losses tend to spread across financial institutions during stress times, amplifying the risk of systemic contagion.

Assessing the level of contribution to systemic risk of the so-called systemically important financial institutions (SIFIs) and designing a regulatory framework capable of ensuring financial stability is the foremost objective of international financial regulatory institutions. *Value-at-Risk* (*VaR*), the most widely-used risk measure by financial institutions is not capable of capturing the systemic nature of risk since it focuses on the risk of an individual institution when viewed in isolation. As a result, there has been recently a growing interest in developing alternative risk measures that reflect systemic risk and avoid the shortcomings of *VaR*.

One such measure of systemic risk is the *Conditional Value-at-Risk* (*CoVaR*) of Adrian and Brunnermeier (2011) that attempts to capture risk spillovers among financial institutions and has attracted a great attention by the regulatory and academic community, especially after the financial crisis in summer 2007. The general framework of *CoVaR* depends on the conditional distribution of a random variable $R_{s,t}$ representing the entire financial system given that another financial institution i, represented by a random variable $R_{i,t}$, is in distress. Currently, there are two alternative definitions of *CoVaR* in the literature. In the original definition by Adrian and Brunnermeier (2011), *CoVaR* is defined as the conditional distribution of $R_{s,t}$ given that $R_{i,t} = VaR_t^i$, while in the modified definition of *CoVaR*, proposed by Girardi and Ergün (2013), the conditioning event is $R_{i,t} \leq VaR_t^i$. In other words, the former definition represents the *VaR* of the system assuming that institution i is *exactly* at its *VaR* level whereas the latter definition of *CoVaR* represents the same risk metric assuming that institution i is *at most* at its *VaR* level. This change in the *CoVaR* definition has some appealing properties. First of all, it considers more severe distress events for institution i that are further in the tail of the loss distribution (below VaR_t^i level) in contrast to the the highly selective and over-optimistic scenario $R_{i,t} = VaR_t^i$. Moreover, the *CoVaR* estimates based on $R_{i,t} \leq VaR_t^i$ can be tested for statistical accuracy and independence using modified versions of the standard Kupiec (1995) and Christoffersen (1998) tests respectively. Finally, and perhaps most importantly, Mainik and Schaanning (2012) show that conditioning on $R_{i,t} \leq VaR_t^i$ has great advantages for dependence modelling.

In this study, we propose a new methodology based on copula functions to estimate *CoVaR* under both definitions. We derive simple closed-form expressions for a broad range of copula families that allow modelling various forms of dependence, while focusing on extreme co-movements of financial system-institution returns, which is, in practice, the main concern of all systemic risk measures. Given the distinctive characteristics of copula families, our modelling approach enables the separation of dependence from marginal distributions providing greater flexibility and eliminating misspecification biases. A dynamic version of the model is also proposed capable of incorporating time-varying correlation into *CoVaR* calculations. Through counterexamples, we show that *CoVaR* measures generated by our modelling approach share the dependence consistency properties found in Mainik and Schaanning (2012). In addition, we extend the Copula *CoVaR* methodology to other "co-risk" measures. In this respect, we derive expressions for *Conditional Expected Shortfall* (*CoES*) under both definitions. Furthermore, we show that our approach can be easily employed by financial regulators as a useful stress testing tool for assessing the impact of extreme market conditions on the stability of financial system.

Focusing on a portfolio of large European banks, we measure the contribution of each individual bank to systemic risk using both *CoVaR* and *CoES* systemic risk metrics. We show that distribution assumptions are extremely important for accurate modelling of systemic risk. In this respect, we show that the ordering of systemically important institutions and the magnitude of corresponding systemic risk measures are substantially affected by the alternative distribution assumptions both in marginals and dependence but are robust across different systemic risk measures with the same assumptions. In a cross-country comparison, we find that banks from Spain and France have, on average, the highest contribution to systemic risk. Moreover, we investigate whether common market factors or institution specific characteristics are important determinants of systemic risk. We show that liquidity risk is an important determinant of systemic risk contribution. The large impact of funding liquidity in the pre-crisis period partly explains the "liquidity spirals" occurred after the break out of financial crisis in summer 2007. Its relative impact is reduced in the post-crisis period due to the coordinated intervention of European Central Bank (ECB) and Federal Reserve in the interbank market. We also find that size and leverage are the most robust determinants of systemic risk contribution implying that bigger and more leveraged financial institutions can be harmful for the overall stability of financial system.

The rest of the paper is organised as follows: Section 2 discusses briefly the relevant literature, while Section 3 formally defines the *CoVaR* and *CoES* measures and presents the Copula *CoVaR* methodology. Derivation of closed-form expressions both for *CoVaR* and *CoES* systemic risk measures are also presented in this section. Section 4 describes the data we use in the empirical part of this study and Section 5 presents the computation of systemic risk measures. Section 6 reports the results of individual contribution to systemic risk. This section analyses also the determinants of systemic risk and discusses their implications for the stability of the financial system. Section 6 concludes.

2 Related Literature

Our study builds on the $CoVaR$ methodology initially proposed by Adrian and Brunnermeier (2011) and subsequently modified by Girardi and Ergün (2013) to address the shortcomings of original $CoVaR$ definition. Recently, a number of papers have extended the $CoVaR$ methodology and applied it to different financial sectors. For example, Wong and Fong (2011) analyse interconnectivity among economies using sovereign credit default swap (CDS) spreads of 11 Asia-Pacific economies. Gauthier et al. (2012) estimate systemic risk exposures for the Canadian banking system and set macro-prudential capital requirements equal to institution's contribution to systemic risk using $\Delta CoVaR$ as a risk allocation mechanism. Recently, Lòpez-Espinosa et al. (2012, 2013) use the $CoVaR$ methodology to analyse the impact of bank-specific factors on an institution's solvency risk and its contribution to systemic risk in a portfolio of large international banks.

There also exists a growing literature that has proposed a number of alternative to $CoVaR$ quantitative measures of systemic risk using different approaches and data. For instance, Goodhart and Segoviano (2009) work with credit default swap (CDS) data and develop bank stability measures that assess banks' contribution to systemic risk within a multivariate framework. Huang et al. (2009) propose a systemic risk indicator measured by the price of insurance against systemic financial distress based on *ex-ante* measures of default probabilities of individual banks and equity return correlation forecasts. Zhou (2010) assesses the systemic importance of financial institutions within a multivariate Extreme Value Theory (EVT) framework and suggests two measures of systemic risk: the systemic impact index (SII) that measures the size of the systemic impact if one bank fails, and the vulnerability index (VI) that measures the impact on a particular bank when the other part of the system is in financial distress.

In addition, Acharya et al. (2010) use equity returns of financial institutions to calculate systemic expected shortfall (SES), which represents the propensity of a financial institution to be undercapitalised when the financial system as a whole is undercapitalised, and marginal expected shortfall (MES), which denotes an institution's average loss when the financial system is in its left tail. Systemic expected shortfall measures are calculated as the weighted average of institution's MSE and its leverage. Alternatively, Nicolò and Lucchetta (2011) use a dynamic factor model on quarterly time-series sets of indicators of financial and real activity for the G-7 economies and obtain joint forecasts and stress-tests of indicators of systemic real risk and systemic financial risk. More recently, Brownlees and Engle (2012) introduce the SRISK index, the expected capital shortage of a firm conditional on a substantial market decline, as an alternative measure of systemic risk. The SRISK index is a function of leverage, size and marginal expected shortfall (MES) of an institution. Finally, Billio et al. (2012) propose several econometric measures to capture the connectedness among financial institutions based on principal components analysis and Granger-causality networks and apply them to monthly returns of financial institutions belonging to different sectors. An extensive survey of the main quantitative measures of systemic risk proposed over the past several years in the literature can be found in Bisias et al. (2012).

3 *CoVaR* Methodology

3.1 Definition of *CoVaR*

Consider a random variable $R_{i,t}$ that represents the returns of financial institution i at time t $(i = 1, \ldots, N\,;\, t = 1, \ldots, T)$. The *Value-at-Risk* (*VaR*)[1] of the random variable $R_{i,t}$ at the confidence level $\alpha \in (0, 1)$, $VaR^i_{\alpha,t}$, is defined as the α-quantile of the return distribution

$$VaR^i_{a,t} = F^{-1}_{i,t}(\alpha), \tag{1}$$

where $F^{-1}_{i,t}$ is the generalised inverse distribution function of the return distribution $F_{i,t}$, i.e., $F^{-1}_{i,t}(\alpha) := \inf\{r_{i,t} \in \mathbb{R} \,:\, F_{i,t}(r_{i,t}) \geq \alpha\}$. Equivalently, equation (1) can be also written as

$$Pr(R_{i,t} \leq VaR^i_{a,t}) = \alpha. \tag{2}$$

Two alternative definitions of *Conditional Value-at-Risk* (*CoVaR*) appear in the literature using different conditioning events. The notion of $CoVaR^{=}_{\alpha,\beta,t}$ denotes the original definition, introduced by Adrian and Brunnermeier (2011), representing the β-quantile of the returns of financial system $R_{s,t}$ conditional on $R_{i,t} = VaR^i_{\alpha,t}$, while the notion of $CoVaR_{\alpha,\beta,t}$ denotes the alternative definition of *CoVaR*, proposed by Girardi and Ergün (2013), where the conditioning event is $R_{i,t} \leq VaR^i_{\alpha,t}$. Formally, $CoVaR^{=}_{\alpha,\beta,t}$ and $CoVaR_{\alpha,\beta,t}$ are defined as the β-quantiles of the following conditional distributions

$$Pr(R_{s,t} \leq CoVaR^{=}_{\alpha,\beta,t} | R_{i,t} = VaR^i_{\alpha,t}) = \beta, \tag{3}$$

$$Pr(R_{s,t} \leq CoVaR_{\alpha,\beta,t} | R_{i,t} \leq VaR^i_{\alpha,t}) = \beta, \tag{4}$$

where $s \neq i$. The confidence levels α and β are decided *ex-ante* by the financial regulator. Typical values are 1% or 5%. In most of the studies a common confidence level for α and β is used, i.e., $\alpha = \beta$, however, working with different confidence levels, i.e., $\alpha \neq \beta$, is also feasible.

Adrian and Brunnermeier (2011) employ linear quantile regressions to obtain $CoVaR^{=}_{\alpha,\beta,t}$ estimates[2]. The $CoVaR^{=}_{\alpha,\beta,t}$ estimates derived from this procedure, however, do not have a time-varying exposure to institution's $VaR^i_{\alpha,t}$. On the other hand, Girardi and Ergün (2013) follow a three-step procedure based on univariate GARCH-type models and the bivariate DCC model of Engle (2002) to estimate $CoVaR_{\alpha,\beta,t}$. As a result, time-varying correlation is incorporated into their $CoVaR_{\alpha,\beta,t}$ estimates. Their approach, however, requires numerical integration where, depending on the distributional assumptions between $R_{s,t}$ and $R_{i,t}$, can be computationally intensive

[1] It is common to present downside risk statistics, such as *VaR*, in positive values. In this paper, we do not follow this sign convention and maintain the original (negative) sign of conditional quantile for all downside risk measures reported in the subsequent sections, such as *VaR*, *CoVaR*, Δ*CoVaR*, *CoES*, Δ*CoES*.

[2] They also show in the appendix of their study that $CoVaR^{=}_{\alpha,\beta,t}$ can be estimated using *GARCH*-type models, providing closed-form expressions for *CoVaR* estimation in the bivariate Gaussian framework.

and time expensive. In addition, the marginal specification in their framework is restricted and needs to result by the choice of the bivariate distribution of $R_{s,t}$ and $R_{i,t}$ respectively. In practice, the distributional characteristics of $R_{s,t}$ and $R_{i,t}$ can differ substantially and hence restricting the marginal specification may introduce misspecification bias in the computation of $CoVaR_{\alpha,\beta,t}$.

3.2 Copula *CoVaR* Methodology

In this section we show how the *Conditional Value-at-Risk* (*CoVaR*) can be estimated using copula functions. We provide simple analytical expressions for a broad range of copula families for both *CoVaR* definitions. In this respect, our Copula *CoVaR* approach overcomes the burden of numerical integration and also incorporates the time-varying dependence between $R_{s,t}$ and $R_{i,t}$ into the computation of systemic risk measure through the copula parameter(s), which is allowed to vary through time as a function of lagged information. Furthermore, from the intrinsic property of copula functions to allow the isolation of dependence from marginal distributions, Copula *CoVaR* approach provides greater flexibility in the specification of the marginals and dependence structure, i.e., the marginal specification is not restricted by the choice of bivariate copula distribution, eliminating in this way potential misspecification bias in the computation of risk measures. This modelling setting enables also the decomposition of systemic risk into three main components: 1) the dependence structure, 2) the magnitude of dependence and 3) the marginal series. As a result, we can assess the relevant contribution of any of those three components to systemic risk.

The joint distribution function of bivariate random variables (Y, X) is

$$F_{YX}(y, x) = Pr(Y \leq y, X \leq x).$$

The famous theorem of Sklar (1959) gives the connection between marginals and copulas to the joint distribution. Let F_{YX} represent a bivariate cumulative distribution function with marginal distributions F_Y and F_X, then there exists a two dimensional copula cumulative distribution function C on $[0, 1]^2$, such that for all $(y, x) \in \overline{\mathbb{R}}^2$

$$F_{YX}(y, x) = C(F_Y(y), F_X(x)),$$

holds. For continuous F_Y and F_X, C is uniquely determined by

$$C(u, v) = F_{YX}(F_Y^{-1}(u), F_X^{-1}(v)),$$

where random variables $u = F_Y(y)$ and $v = F_X(x)$ (i.e., obtained by the probability integral transform) are uniformly distributed on $[0, 1]$, while $F_Y^{-1}(u)$ and $F_X^{-1}(v)$ are the generalised inverse distribution functions of the marginals.

It can be shown[3], that the conditional probability distribution $Pr(Y \leq y|X = x)$ can be expressed in terms of a copula function as

$$Pr(Y \leq y|X = x) = \frac{\partial C(u, v)}{\partial v}. \tag{5}$$

In contrast, the conditional probability distribution $Pr(Y \leq y|X \leq x)$ can be expressed in terms of a copula function as

$$Pr(Y \leq y|X \leq x) = \frac{Pr(Y \leq y, X \leq x)}{Pr(X \leq x)} = \frac{C(F_Y(y), F_X(x))}{F_X(x)} = \frac{C(u, v)}{v}. \tag{6}$$

The class of Archimedean copulas has found wide usage in the economics and finance literature recently, because of their simple closed-form cumulative distribution functions and their appealing properties for modelling the dependence between random variables. The Archimedean copulas can capture a broad range of asymmetric tail dependence that we know is extremely important for modelling many relationships between financial asset returns. Bivariate Archimedean copulas are defined as

$$C(u, v) = \varphi^{-1}\left[\varphi(u) + \varphi(v)\right],$$

where $\varphi : [0, 1] \to [0, \infty)$ is a continuous strictly decreasing convex function such that $\varphi(1) = 0$ and φ^{-1} is the inverse of φ. The function φ is called *generator function* of the copula C (see Nelsen (2006), for further details).

We begin with the presentation of $CoVaR^{=}_{\alpha,\beta,t}$ in terms of Archimedean copulas and provide general solutions through their corresponding generator functions[4]. From the general result in equation (5) we have

$$Pr(Y \leq y|X = x) = \frac{\partial C(u, v)}{\partial v} = \frac{\varphi'(v)}{\varphi'(C(u, v))} = \frac{\varphi'(v)}{\varphi'\left(\varphi^{-1}\left[\varphi(u) + \varphi(v)\right]\right)}. \tag{7}$$

Assuming that random variables Y and X above represent system $R_{s,t}$ and institution's $R_{i,t}$ returns with distribution functions $F_{s,t}$ and $F_{i,t}$ respectively, the conditional distribution $Pr(R_{s,t} \leq CoVaR^{=}_{\alpha,\beta,t}|R_{i,t} = VaR^{i}_{\alpha,t})$ can be equivalently expressed in terms of a copula generator function as follows

$$Pr(R_{s,t} \leq CoVaR^{=}_{\alpha,\beta,t}|R_{i,t} = VaR^{j}_{\alpha,t}) = \frac{\varphi'(v)}{\varphi'\left(\varphi^{-1}\left[\varphi(u) + \varphi(v)\right]\right)} = \beta.$$

Solving for u, under the general condition that $\partial/\partial v\, C(u, v)$ is partial invertible in its first argument u, we obtain the copula conditional quantile

$$u^{=} \equiv u = \varphi^{-1}\left[\varphi\left(\varphi'^{-1}\left(\frac{1}{\beta}\varphi'(v)\right)\right) - \varphi(v)\right]. \tag{8}$$

[3]See proof in Bouyè and Salmon (2009, p. 726).

[4]In Appendix A we also provide general solutions for elliptical copula families, i.e., Gaussian and Student-t copulas. Even though these particular families do not have copula distributions in closed-form, explicit solution for $CoVaR^{=}_{\alpha,\beta,t}$ can be derived. Unfortunately, there is no explicit solution for $CoVaR_{\alpha,\beta,t}$ and hence numerical integration is needed.

Applying the probability integral transform in equation (8), we derive an explicit expression for $CoVaR^{=}_{\alpha,\beta,t}$ for a broad range of Archimedean copula functions, that is

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\Bigg(\varphi^{-1}\Big[\varphi\Big(\varphi'^{-1}\Big(\frac{1}{\beta}\varphi'\Big(F_{i,t}\Big(VaR^{i}_{\alpha,t}\Big)\Big)\Big)\Big) - \varphi\Big(F_{i,t}\Big(VaR^{i}_{\alpha,t}\Big)\Big)\Big]\Bigg), \tag{9}$$

where $F^{-1}_{s,t}$ is the generalised inverse distribution function of $F_{s,t}$. From the definition of VaR it holds that $v = F_{i,t}(VaR^{i}_{\alpha,t}) = F_{i,t}(F^{-1}_{i,t}(\alpha)) = \alpha$. As such, the expression for $CoVaR^{=}_{\alpha,\beta,t}$ in equation (9) can be simplified further as follows

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\Bigg(\varphi^{-1}\Big[\varphi\Big(\varphi'^{-1}\Big(\frac{1}{\beta}\varphi'(\alpha)\Big)\Big) - \varphi(\alpha)\Big]\Bigg). \tag{10}$$

Alternatively, an analytical expression can be also given for $CoVaR_{\alpha,\beta,t}$ for a wide range of Archimedean copula families. Given the general result in equation (6), the conditional distribution $Pr(R_{s,t} \le CoVaR_{\alpha,\beta,t}|R_{i,t} \le VaR^{i}_{\alpha,t})$ can be equivalently written as

$$Pr(R_{s,t} \le CoVaR_{\alpha,\beta,t}|R_{i,t} \le VaR^{i}_{\alpha,t}) = \frac{\varphi^{-1}\left[\varphi(u) + \varphi(v)\right]}{v} = \beta. \tag{11}$$

Similarly, from the definition of VaR it holds that $v = F_{i,t}(VaR^{i}_{\alpha,t}) = F_{i,t}\Big(F^{-1}_{i,t}(\alpha)\Big) = \alpha$. Therefore, the expression in equation (11) can be expressed as

$$\varphi^{-1}\left[\varphi(u) + \varphi(\alpha)\right] = \alpha \cdot \beta. \tag{12}$$

Finally, after solving for u and applying the probability integral transform, under the general conditions that $C(u, v)$ is partial invertible in its first argument u, $CoVaR_{\alpha,\beta,t}$ has a general representation for Archimedean copulas, that is

$$u^{\le} \equiv u = \varphi^{-1}\left[\varphi(\alpha \cdot \beta) - \varphi(\alpha)\right], \tag{13}$$

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(\varphi^{-1}\left[\varphi(\alpha \cdot \beta) - \varphi(\alpha)\right]\Big). \tag{14}$$

The general representation of *CoVaR* in (10) or (14) implies constant correlation between $R_{s,t}$ and $R_{i,t}$. However, it is known that the dependence structure between financial asset returns is not constant over a long horizon but rather time-varying. Numerous studies have also indicated that correlation between financial series tend to be more pronounced during downturns rather than upturns. A stylised feature that should be considered in the estimation of systemic risk. In this respect, the use of constant correlations may severely affect the risk estimates and lead to incorrect inferences. In this study we follow the specification proposed by Patton (2006) in order to introduce a dynamic version of Copula *CoVaR* model and hence incorporate time-varying correlation into *CoVaR* estimation. Patton (2006) proposed observation driven copula models for which the time-varying dependence parameter(s) of a copula is a parametric function of transformed lagged data. It is essentially an ARMA(1,10)-type process. In Appendix B we derive analytical expressions for $CoVaR^{=}_{\alpha,\beta,t}$ and $CoVaR_{\alpha,\beta,t}$, while in Appendix C we present the time-varying parameter specification for the Clayton, Frank, Gumbel[5] and BB7 copulas respectively. These copula families appear very popular in the literature for modelling the dependence between financial asset returns since they allow for very flexible dependency structures and can capture various forms of tail dependence.

3.3 Extension to *CoES*

The *CoVaR* concept can be easily adopted for other "co-risk" measures. One of them is the *Conditional Expected Shortfall* (*CoES*). We denote by $CoES^{=}_{\alpha,\beta,t}$ the expected shortfall of the financial system conditional on $R_{i,t} = VaR^{i}_{\alpha,t}$ and similarly by $CoES_{\alpha,\beta,t}$ the expected shortfall of the financial system conditional on $R_{i,t} \leq VaR^{i}_{\alpha,t}$. In this respect, *CoES* estimates can be easily obtained for both definitions within our framework as follows

$$CoES^{=}_{\alpha,\beta,t} = \frac{1}{\beta}\int_0^{\beta} CoVaR^{=}_{\alpha,q,t}\, dq, \tag{15}$$

$$CoES_{\alpha,\beta,t} = \frac{1}{\beta}\int_0^{\beta} CoVaR_{\alpha,q,t}\, dq, \tag{16}$$

where $CoVaR^{=}_{\alpha,q,t} = Pr(R_{s,t} \leq F^{-1}_{s,t}(q)|R_{i,t} = VaR^{i}_{\alpha,t})$ and $CoVaR_{\alpha,q,t} = Pr(R_{s,t} \leq F^{-1}_{s,t}(q)|R_{i,t} \leq VaR^{i}_{\alpha,t})$ respectively.

3.4 Systemic Risk Contributor and Dependence Consistency

Adrian and Brunnermeier (2011) define institution's i contribution to systemic risk by

$$\Delta CoVaR^{=}_{\alpha,\beta,t} = CoVaR^{=}_{\alpha,\beta,t} - CoVaR^{=}_{0.5,\beta,t},$$

where $\Delta CoVaR^{=}_{\alpha,\beta,t}$ denotes the difference between the *VaR* of the financial system conditional on $R_{i,t} = VaR^{i}_{\alpha,t}$ and the *VaR* of the financial system conditional on $R_{i,t} = VaR^{i}_{0.5,t}$ (institution i being *exactly* at its median state). Following Adrian and Brunnermeier (2011), we adopt Δ*CoVaR* as measure of institution's i contribution to systemic risk and define also by $\Delta CoVaR_{\alpha,\beta,t}$ the difference between the *CoVaR* of the financial system conditional on $R_{i,t} \leq VaR^{i}_{\alpha,t}$ and on $R_{i,t} \leq VaR^{i}_{0.5,t}$ (institution i being *at most* at its median state) respectively[6], that is

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[5]The $\partial/\partial v\, C(u,v)$ of Gumbel copula is not partial invertible in its u and hence we cannot derive analytical expressions for $CoVaR^{=}_{\alpha,\beta,t}$.

[6]Girardi and Ergün (2013) define the systemic risk contribution of a particular institution i by

$$\Delta CoVaR_{\alpha,\beta,t} = 100 \times (CoVaR_{\alpha,\beta,t} - CoVaR_{b_i,\beta,t})/CoVaR_{b_i,\beta,t},$$

that is the percentage difference of the *VaR* of the financial system conditional on the distressed state of institution i from the *VaR* of the financial system conditional on the benchmark state of institution i. They define the benchmark state b_i as a one-standard deviation about the mean event: $\mu_{i,t} - \sigma_{i,t} \leq R_{i,t} \leq \mu_{i,t} + \sigma_{i,t}$, where $\mu_{i,t}$ and $\sigma_{i,t}$ are the conditional mean and the standard deviation of institution's i returns respectively. However, the *VaR* of the financial system conditional on the benchmark state of institution i can not be derived in explicit form within our framework and thus numerical integration is needed. So far, there is no consensus in the literature regarding the

$$\Delta CoVaR_{\alpha,\beta,t} = CoVaR_{\alpha,\beta,t} - CoVaR_{0.5,\beta,t}.$$

Computation of $CoVaR^{=}_{0.5,\beta,t}$ or $CoVaR_{0.5,\beta,t}$ is straightforward and can be carried out in a similar way as in $CoVaR^{=}_{\alpha,\beta,t}$ or $CoVaR_{\alpha,\beta,t}$ case by simply modifying the stress scenario. We also employ $\Delta CoES$ as a measure of institution's i contribution to systemic risk where the contribution is measured in terms of *CoES*. Therefore, we define

$$\Delta CoES^{=}_{\alpha,\beta,t} = CoES^{=}_{\alpha,\beta,t} - CoES^{=}_{0.5,\beta,t},$$

$$\Delta CoES_{\alpha,\beta,t} = CoES_{\alpha,\beta,t} - CoES_{0.5,\beta,t},$$

where $\Delta CoES^{=}_{\alpha,\beta,t}$ denotes the difference between the *CoES* of the financial system conditional on $R_{i,t} = VaR^{i}_{\alpha,t}$ and on $R_{i,t} = VaR^{i}_{0.5,t}$, while $\Delta CoES_{\alpha,\beta,t}$ denotes the same risk metric with stress scenarios being $R_{i,t} \leq VaR^{i}_{\alpha,t}$ and $R_{i,t} \leq VaR^{i}_{0.5,t}$ respectively.

To investigate whether the alternative representations for measuring contribution to systemic risk, derived within Copula *CoVaR* framework, encompass the dependence consistency properties reported in Mainik and Schaanning (2012), we compare $\Delta CoVaR$ estimates for the bivariate distribution with a Clayton copula[7]. Figure 3.1 presents $\Delta CoVaR^{=}_{\alpha,\beta,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ measures as a function of the dependence parameter θ for Clayton copula with Student-t marginals with three degrees of freedom in three different confidence levels, i.e., 1%, 5% and 10% respectively. The behaviour of risk measures in these two models confirms the results in Mainik and Schaanning (2012). Initially, $\Delta CoVaR^{=}_{\alpha,\beta,t}$ is increasing with respect to dependence parameter, however, after a certain threshold counter-intuitively starts decreasing. In other words, $\Delta CoVaR^{=}_{\alpha,\beta,t}$ fails to detect dependence when it becomes more pronounced. On the other hand, $\Delta CoVaR_{\alpha,\beta,t}$ is increasing with respect to dependence parameter. Therefore, conditioning on $R_{i,t} \leq VaR^{i}_{\alpha,t}$ gives a much more consistent response to dependence than conditioning on $R_{i,t} = VaR^{i}_{\alpha,t}$[8].

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definition of systemic risk contribution. For example, Adrian and Brunnermeier (2011) changed twice the definition of $\Delta CoVaR$ in two earlier versions of their paper. Nevertheless, Mainik and Schaanning (2012) show that the primarily deficiency of $\Delta CoVaR$ is due to the underlying stress scenario $R_{i,t} = VaR^{i}_{\alpha,t}$. As such, we decided to adopt $\Delta CoVaR$, as defined in Adrian and Brunnermeier (2011), as measure of institution's contribution to systemic risk but modified the distress events to ensure that is continuous function of dependence parameter.

[7]We have also compared $\Delta CoVaR$ for the bivariate distribution with a Frank copula. The dependence consistency properties are in line with the results reported for the bivariate distribution with a Clayton copula.

[8]Similar dependence consistency results are obtained when $\Delta CoES$ is employed for the same stochastic models.

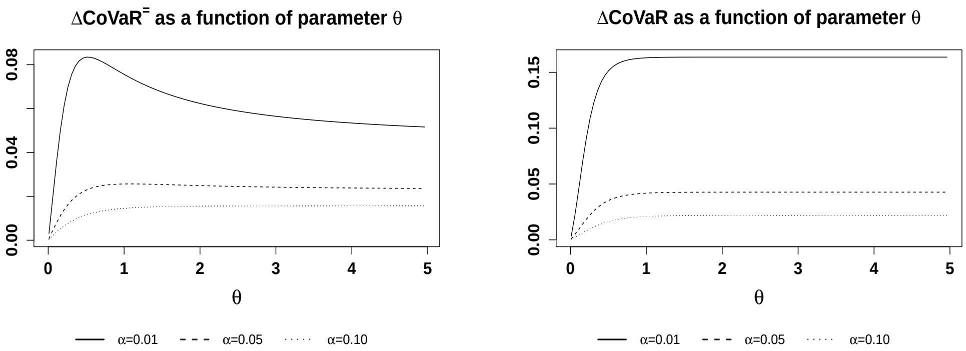

Figure 3.1: Clayton with Student-t margins with 3 degrees of freedom: $\Delta CoVaR^{=}_{\alpha,\beta,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ as a function of θ.

4 Data

We focus on the STOXX Europe 600 Banks Index that consists of 46 large European banks from 15 European countries, characterised by large market capitalisation, international activity, cross-country exposure and representative size in the local market. The STOXX Europe 600 Banks Index is a component of the STOXX Europe 600 Index that represents large, mid and small capitalisation companies across 18 countries of the European region. It is the largest in market capitalisation sector index of STOXX Europe 600 Index (€748.5 billion as of June, 2013), which indicates in this way the relative importance and size of banking sector in Europe. We exclude 4 institutions from the initial sample because the history of their corresponding datasets is narrow and does not cover the time period we want to analyse. Therefore, the resulting sample is formed by a total of 42 European banks, starting in 01/04/2002 and ending in 31/12/2012. This time period provides a good platform to assess the level of contribution to systemic risk of the systemically important financial institutions in Europe since it includes a number of significant events (e.g. the U.S subprime mortgage crisis, the Lehman Brothers collapse, the European sovereign-debt crisis etc.). We assign the Q3 2007 - Q4 2012 as crisis period because the majority of these events occurred during this time window.

We obtain weekly equity adjusted prices to account for capital operations (i.e., splits, dividends etc.) from the Datastream database and form weekly log returns. There are 562 weekly returns for each institution in our sample. Appendix D lists these institutions. For each bank, an equally-weighted average of the returns of the remaining banks in the sample is used as a proxy for financial system. In this way, the resulting system return portfolios can be considered as representative of European financial system allowing the study of possible spillover effects between a stressed institution and financial system. Moreover, this approach rules out any spurious correlation that may be induced due to sizeable disparity in the composition of financial system proxy. For example, HSBC has a total contribution of 20.5% in the composition of STOXX Europe 600 Banks Index. As a result, if the corresponding index is used as a proxy for the financial system, systemic risk estimates generated conditional on HSBC will be severely affected by the simultaneous presence and large scale factor of HSBC in financial system's portfolio proxy.

5 Copula *CoVaR* Estimation

Computation of *CoVaR* or *CoES* requires estimation of the parameter(s) of the marginal densities and copula function that captures the dependence between $R_{s,t}$ and $R_{i,t}$. Assume a vector of system and institution returns $R_t = (R_{s,t}, R_{i,t})'$, $t = 1, \ldots, T$. Given that a copula function and the marginals are continuous, their joint probability density function can be expressed in terms of the copula density function, $c(\cdot, \cdot\,; \theta_t)$, and the univariate marginal densities, $f_{s,t}(R_{s,t}; \phi_s)$ and $f_{i,t}(R_{i,t}; \phi_i)$, as follows

$$f(R_{s,t}, R_{i,t}) = c(u_t, v_t\,; \theta_t) \cdot f_{s,t}(R_{s,t}\,; \phi_s) \cdot f_{i,t}(R_{i,t}\,; \phi_i), \tag{17}$$

where θ_t denotes the copula parameter while ϕ_s and ϕ_i denote the parameters for system and institution's i marginal distributions. In the above expression $u_t = F_{R_{s,t}}(R_{s,t}\,; \phi_s)$ and $v_t = F_{R_{i,t}}(R_{i,t}\,; \phi_i)$ are the uniformly transformed marginal series.

The log-likelihood function of equation (17) is given by

$$L(\theta_t, \phi_s, \phi_j) = \sum_{t=1}^{T} [\log c(u_t, v_t\,; \theta_t) + \log f_{s,t}(R_{s,t}\,; \phi_s) + \log f_{i,t}(R_{i,t}\,; \phi_i)] \tag{18}$$

The marginal densities $f_{s,t}(R_{s,t};\, \phi_s)$ and $f_{i,t}(R_{i,t}\,; \phi_i)$ can be conditional densities and the series $R_{s,t}$ and $R_{i,t}$ are usually modelled by a GARCH-type model, whose residuals are treated as *i.i.d* random variables. Under this setting, full maximum likelihood estimates (MLE) can be obtained by maximising equation (18) with respect to the parameters $(\theta_t, \phi_s, \phi_i)$. In general, the full MLE estimation would be the first choice of estimation due to the well-known optimality properties of the maximum likelihood. However, the Inference Functions for Margins (IFM) method is usually preferred to full MLE due to its computational tractability and comparable efficiency. The IFM method (see Joe and Hu (1996); Joe (1997), for further details) is a multi-step optimisation technique. It divides the parameter vector into separate parameters for each marginal and parameters for the copula model. Therefore, one may break up the optimisation problem into two parts. In this study we adopt the IFM method to estimate the parameters of the marginal distributions and copula function and subsequently obtain *CoVaR* and *CoES* estimates.

When modelling the distribution of financial asset returns, the critical issues are dynamic volatility and the modelling of asymmetries. It is well-documented that asset return distributions are skewed and fat-tailed. Moreover, the volatility of asset returns is not constant, it is mean-reverting and tends to cluster. Another important stylised fact of asset returns volatility is that a large negative price shock increases volatility by much more than a positive price shock of the same magnitude, which is also known as "leverage-effect". To address these features we assume that the returns of financial system and institution's i at time t, $R_t = (R_{s,t}, R_{i,t})'$, follow an AR(1)-GJR-GARCH(1,1) model[9].

[9]The asymmetric GJR-GARCH model is developed in Glosten et al. (1993).

Therefore for $j \equiv s, i$ and time $t = 1, \dots, T$ we estimate

$$R_{j,t} = \mu_{j,t} + \varepsilon_{j,t} = \phi_{j,0} + \phi_{j,1} R_{j,t-1} + \sigma_{j,t} z_{j,t}, \tag{19}$$

$$\sigma^2_{j,t} = \omega_j + \alpha_j \sigma^2_{j,t-1} + \beta_j \varepsilon^2_{j,t-1} + \gamma_j \varepsilon^2_{j,t-1} + \xi_j I_{t-1} \varepsilon^2_{j,t-1}, \tag{20}$$

where I_{t-1} is an indicator function equals to 1 if $\varepsilon_{j,t-1} < 0$, and 0 otherwise. We assume that the distribution of the innovations $z_{j,t}$ is a white noise process with zero mean, unit variance and distribution function given by $F_{z_j,t}$. To allow for asymmetry in the marginal distributions, we assume that the distribution of the innovations follows the skewed-t distribution, as introduced in Fernández and Steel (1998). It is very common, however, when modelling asset returns to assume normality. In this respect, we also estimate the time-series models in (19) and (20) based on the assumption of normal distributed innovation.[10]. We denote the cumulative distribution functions of financial system and institution's i innovations by $u_t \equiv F_{z_s,t}(z_{s,t})$ and $v_t \equiv F_{z_i,t}(z_{i,t})$, respectively[11]. The dependence parameter is then estimated by maximising the log-likelihood function in (18), given the estimated parameters of marginal series.

In this respect, *CoVaR* estimates can be obtained by evaluating the analytical expressions derived in section 3.2. Note that the conditional quantiles implied by equations (8) and (13) correspond to the conditional quantiles of innovations. To obtain time-varying *CoVaR* measures, we rescale $CoVaR^{=}_{\alpha,\beta,t}$ or $CoVaR_{\alpha,\beta,t}$ estimates with the fitted conditional mean $\mu_{s,t}$ and standard deviation $\sigma_{s,t}$ of $R_{s,t}$, obtained from estimated equations (19) and (20), as follows

$$CoVaR^{=}_{\alpha,\beta,t} = \mu_{s,t} + \sigma_{s,t}\, F^{-1}_{z_s,t}(u^{=}_t),$$

$$CoVaR_{\alpha,\beta,t} = \mu_{s,t} + \sigma_{s,t}\, F^{-1}_{z_s,t}(u^{\leq}_t),$$

where $F^{-1}_{z_s,t}$ is the generalised inverse of financial system's innovation distribution function and $u^{=}_t$ and $u^{\leq}_t$ are the conditional quantiles of the general solutions in (8) and (13) respectively[12]. Note also that the conditional quantiles in (8) and (13) correspond to a static model (i.e., θ is constant). However, the dynamic version of the model (i.e., θ_t is time-varying) implies that conditional quantiles have also time-varying exposure. As such, we use the subscript t in $u^{=}_t$ and $u^{\leq}_t$ to distinguish between the dynamic and static model.

[10]Note that given the innovations distributional assumptions we can easily obtain time-varying *VaR* estimates for each institution i (see Duffie and Pan (1997); Jorion (2001) among others, for further details)

[11]The innovations can be also transformed to uniformly distributed data non-parametrically by their corresponding empirical distribution functions without assuming any particular parametric distribution for the marginals. This semi-parametric (SP) method is also known as pseudo maximum likelihood (PML) method (see Genest et al. (1995), for further details). The semi-parametric (SP) method can tackle the marginal misspecification problem since treats the marginal distributions as unknown functions.

[12]To obtain time-varying *CoES* measures, the same process as in the computation of *CoVaR* is followed, however, the copula conditional quantiles $u^{=}_t$ and $u^{\leq}_t$ are obtained from the corresponding expressions in (15) and (16).

6 Results

6.1 Computing *CoVaR* and *CoES* measures

In this section we present results based on Girardi and Ergün (2013) representation of *CoVaR*[13]. In our search for the best copula model that can sufficiently describe the dependence between financial system and institution returns, we consider four alternative copula functional forms: Clayton, Frank, Gumbel and BB7. Each of these copula families allows for positive dependence but implies a different type of tail dependence between the variables. For example, Clayton copula allows for negative tail dependence only and hence would fit best if negative changes in financial system and institution returns are more highly correlated than positive changes. In contrast, Gumbel copula allows for positive tail dependence only, while Frank copula does not allow for tail dependence. Finally, the BB7 copula allows for asymmetric upper and lower tail dependence. In practice, *CoVaR* focuses on the joint tail distribution of financial system-institution pair returns and thus tail dependence is a rather important concept for *CoVaR* computation.

We estimate dynamic $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ measures for each institution i. We employ two alternative distributional assumptions for the marginal series: Gaussian and Skewed-t. The selection of the best-fitting copula model for each system-institution pair is based on Akaike Information Criterion (AIC) (Akaike, 1974)[14]. All risk measures ($VaR^i_{\alpha,t}$, $CoVaR_{\alpha,\beta,t}$, $CoES_{\alpha,\beta,t}$) are computed at the same confidence level, i.e., $\alpha = \beta = 5\%$. We also evaluate $CoVaR_{\alpha,\beta,t}$ estimates for statistical accuracy and independence using modified versions of the standard Kupiec (1995) and Christoffersen (1998) tests (see Girardi and Ergün (2013), for further details on the implementation of the modified tests). Figure 6.1 shows time-series average $VaR^i_{\alpha,t}$, $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ measures, while Figure 6.2 shows time-varying average Kendall's τ correlations implied by the estimated bivariate copula families, across all financial system-institution pairs with skewed-t marginals. The light blue shaded area in the graphs corresponds to Q3 2007 - Q4 2012 crisis period. It is clear that $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ estimates are higher in absolute value during this period. This is partly due to the increasing correlation between financial system-institution returns as shown in Figure 6.2.

[13]As discussed earlier, *CoVaR* under this definition is dependent consistent measure of systemic risk, which is an essential property for a well-specified model. Another attractive characteristic is that *CoVaR* estimates generated under this framework can be statistically evaluated, providing a distinctive opportunity to assess the statistical adequacy of systemic risk model. In Appendix E, we present a graphical comparison for *CoVaR* and *CoES* measures under both definitions conditional on HSBC returns. Similar results are also obtained for the rest of the pairs analysed.

[14]The Bayesian Information Criterion (BIC) of Schwarz (1978) was also employed in the selection procedure for the best-fitting copula model, however, the results remained unaffected.

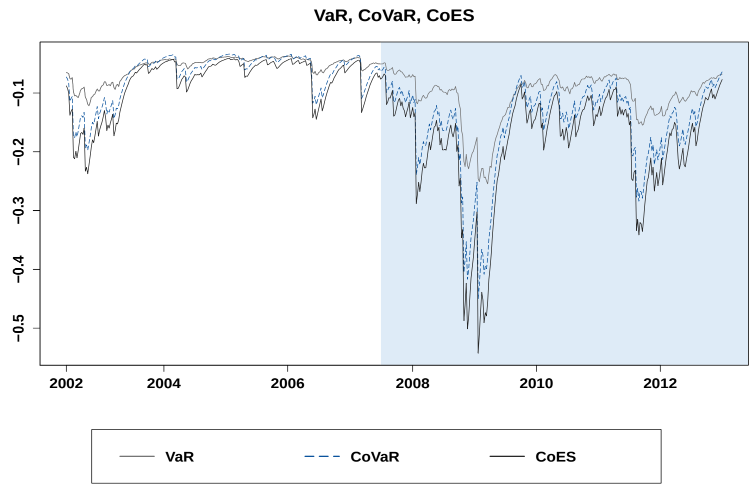

Figure 6.1: This figure shows time-series average values of weekly $VaR^i_{\alpha,t}$, $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ measures across all financial system-institution pairs. All risk measures are generated under the assumption of skewed-t margins and computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

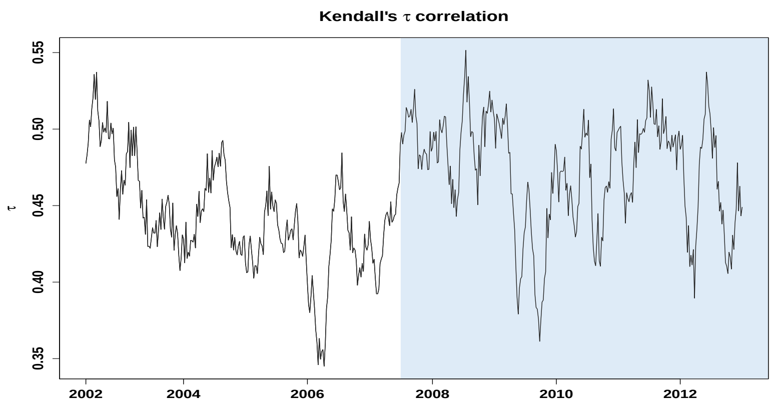

Figure 6.2: This figure shows time-series average Kendall's τ correlation estimates implied by estimated copula families across all financial system-institution pairs. All risk measures are generated under the assumption of skewed-t margins and computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

Nevertheless, the time-varying correlation results cannot fully support the empirical findings in Longin and Solnik (2001) and Ang and Chen (2002) indicating that conditional correlations between financial asset returns are much stronger in downturns than in upturns. The time-varying Kendall's τ correlations are slightly more pronounced during the crisis period than in the pre-crisis period for most of the pairs; the average value being 0.44 in the pre-crisis period and 0.47 in the crisis period for all pairs under consideration. Figures 6.1 and 6.2 also indicate the importance of consistency of systemic risk measures with respect to dependence. It is clear from the two graphs that high values of Kendall's τ correlation are associated with high in absolute value systemic risk estimates. Therefore, a systemic risk measure that provides an inconsistent response to dependence may fail to detect systemic risk when it is more pronounced, i.e., during periods of financial distress, and lead financial system regulators to undertake inappropriate policy decisions. Therefore, the notion of consistency of systemic risk measure with respect to dependence becomes rather significant during the crisis periods.

Figure 6.3 displays a cross-section plot of institution's average $VaR^i_{\alpha,t}$ and its contribution to systemic risk, measured by average $\Delta CoVaR_{\alpha,\beta,t}$. It is clear that there is a weak relationship between institution's $VaR^i_{\alpha,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ in the cross-section. Similar findings are also reported in Adrian and Brunnermeier (2011) and Girardi and Ergün (2013) leading to the conclusion that regulating the risk of financial institutions in isolation, through institutions' *VaR*, might not be the optimal policy for protecting the financial sector against systemic risk. Figure 6.4 plots the time-series average of $VaR^i_{\alpha,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ over time. It is evident that $VaR^i_{\alpha,t}$ and $\Delta CoVaR_{\alpha,\beta,t}$ measures have a strong relationship in the time series. Adrian and Brunnermeier (2011) report the same strong relationship, while Girardi and Ergün (2013) confirm a weak relationship for those two risk measures in the time series. Given our findings, we can conclude that the association between these two measures over time is primarily directed by the alternative definitions of $\Delta CoVaR$ and not from the alternative $CoVaR$ definitions[15].

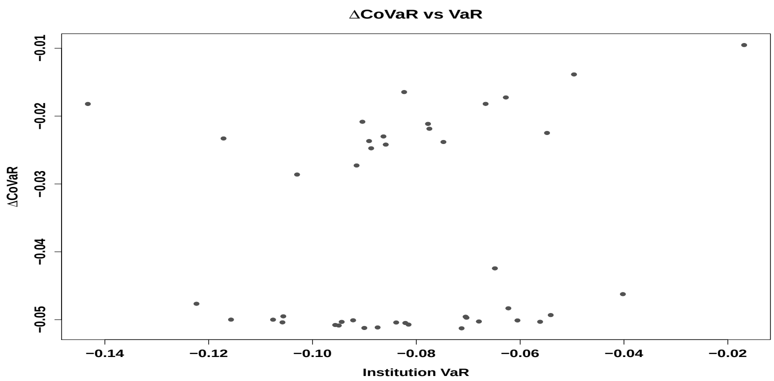

Figure 6.3: This scatter plot shows the cross-sectional link between the time-series average of financial institution's risk in isolation, measured by $VaR^i_{\alpha,t}$, and the time-series average contribution to systemic risk, measured by $\Delta CoVaR_{\alpha,\beta,t}$. All risk measures are generated under the assumption of skewed-t margins and computed at $\alpha = \beta = 5\%$ level.

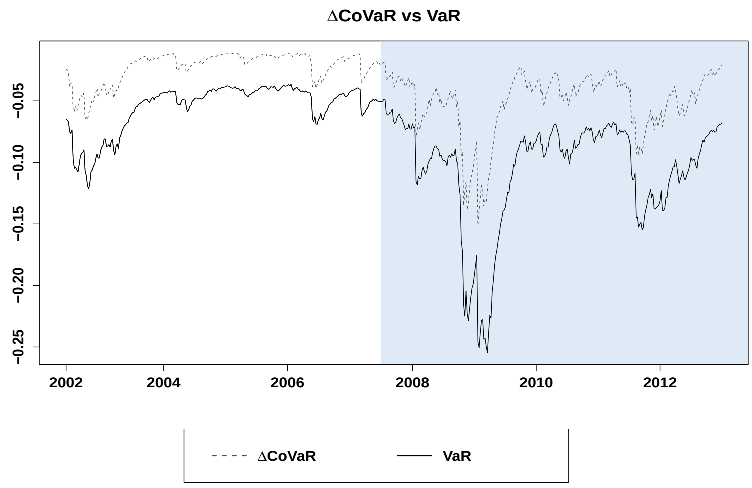

Figure 6.4: This figure shows the time-series average of weekly $\Delta CoVaR_{\alpha,\beta,t}$ and $VaR^i_{\alpha,t}$ measures. All risk measures are generated under the assumption of skewed-t margins and computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

[15] This conclusion results from estimating $CoVaR$ under both stress scenarios $R_{i,t} = VaR^i_{\alpha,t}$ and $R_{i,t} \leq VaR^i_{\alpha,t}$ and employing alternative $\Delta CoVaR$ definitions for three copula models: Clayton, Gumbel and Frank. Numerical integration is used to estimate $CoVaR$ when explicit expressions are not available in our Copula $CoVaR$ framework (see also footnote 6). The weak relationship between $\Delta CoVaR$ and VaR in the time series is supported only when Girardi and Ergün (2013) definition of $\Delta CoVaR$ is used, regardless of alternative $CoVaR$ definitions.

6.2 Systemic risk contribution

Table 6.1 ranks the contribution of each individual bank to overall systemic risk, as measured by the time-series average of $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,t}$ estimates, under the assumption of Gaussian and skewed-t margins respectively. Table 6.1 displays also the selected copula functions and the average value of Kendall's τ correlation coefficients implied by the estimated copula parameters of each financial system-institution pair. The Frank copula is the most preferred functional form for describing the dependence between financial system and institution returns and Gumbel copula is the second most popular choice under the assumption of Gaussian marginals. In contrast, the BB7 copula is the most popular functional form for modelling the dependence under the skewed-t marginals assumption, while the Frank copula is the second most favoured choice. The Clayton copula has not been selected for any of the pairs analysed under both marginal assumptions. It is clear from Table 6.1 that the distribution assumptions in the marginals affect the selection of the best-fitting copula and hence the overall $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ results. As such, special care should be given when specifying marginals since the use of inappropriate marginals do not only deliver biases directly but also affect systemic risk measures indirectly, through copula parameter estimation or copula misspecification[16].

The average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates with skewed-t margins are much higher in absolute value than those generated under the assumption of Gaussian margins. The size differences in systemic risk measures, however, result not only from the alternative marginal assumptions but also from the characteristics of copula functions that model the dependence for each pair. The dominant copula function when assuming Gaussian margins is Frank, while BB7 is the most popular copula family under skewed-t marginals. As explained, Frank copula does not imply tail dependence, while the BB7 copula allows for asymmetric tail dependence. In this regard, the general dependence structure, and especially the dependence structure in extremes, affect substantially the computation of $CoVaR_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$. This is also confirmed by the implied Kendall's τ estimates reported in Table 6.1. It is clear from Table 6.1 that for those copula families that do not imply lower tail dependence, such as Frank or Gumbel copulas, the average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates are primarily driven by the degree of dependence.

The stronger the dependence between financial system-institution returns the higher the average values of $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ respectively. In contrast, when the dependence between financial system-institution's returns is modelled by an asymmetric BB7 copula, the average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates are not monotonic functions of Kendall's τ correlation estimates but their values are also affected by the degree of tail dependence. Figure 6.5 shows the average time-varying upper (λ^U) and lower (λ^L) tail dependence indices estimated from those pairs modelled by a BB7 copula under the assumption of skewed-t margins. There are clear evidences of asymmetric tail dependence.

[16] The effects on the computation of the VaR when there is a misspecification in the marginals and in the copulas has been investigated by Fantazzini (2009).

The average value of upper and lower tail dependence indices are 0.45 and 0.50 respectively, leading to the conclusion that joint negative extremes occur more often than joint positive extremes. To investigate more the impact of asymmetries in the tails in the computation of systemic risk metrics, we compute non-parametric (N-P) estimates (average of non-parametric estimates in Dobrić and Schmid (2005)) for upper (λ^U) and lower (λ^L) tail dependence coefficients and sample Kendall's τ correlation coefficients for each financial system-institution pair of standardised residuals, obtained from the fit of the univariate time-series models in section 5. Table 6.3 reports average $\Delta CoVaR_{\alpha,\beta,t}$, sample Kendall's τ and non-parametric tail dependence indices for each pair. It is not surprising that banks having high coefficients of lower tail dependence appear among the most systemic financial institutions, indicating in this way the importance of asymmetries in systemic risk modelling.

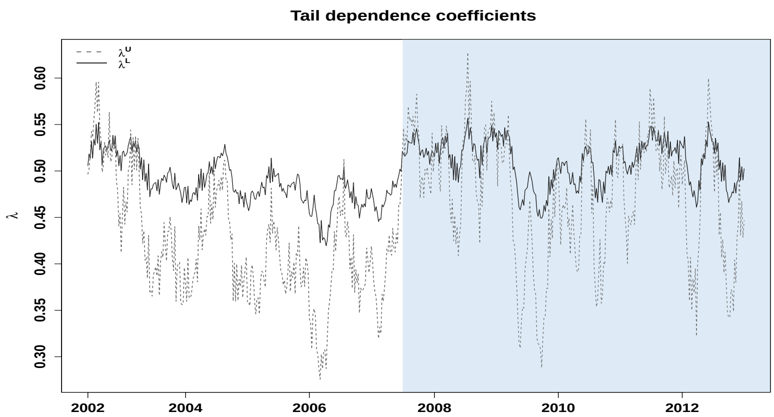

Figure 6.5: This figure shows time-varying average values of upper (λ^U) and lower (λ^L) tail dependence coefficients implied by BB7 copulas under the assumption of skewed-t margins. All risk measures are computed at $\alpha = \beta = 5\%$ level. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

The ranking of the systemically important financial institutions in Table 6.1 varies significantly across different marginal distributional assumptions but is more consistent across alternative systemic risk measures within the same marginal distributional assumptions. For example, Santander bank is ranked as the 2nd most systemic financial institution according to its average contribution to systemic risk, measured by $\Delta CoVaR_{\alpha,\beta,t}$, under the assumption of Gaussian margins, while it is ranked in the 7th place when skewed-t margins are assumed instead. Moreover, BNP Paribas is ranked as the 3rd most systemic bank based on its average $\Delta CoES_{\alpha,\beta,t}$ measure under normality but under the assumption of skewed-t margins is ranked in the 26th place. Nevertheless, the hierarchy of systemic banks across $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ does not differ significantly under the same marginal distribution assumptions implying that qualitative results depend more on the underlying distribution assumptions in the margins and dependence structure and less on the systemic risk measures *per se*.

From the ranking results in Table 6.1 and market capitalisation values of financial institutions reported in Table D.1 in the Appendix D, it can be also shown that banks which are large in size with strong cross-country exposure and international activity appear among the most systemic financial institutions under both distribution assumptions. For instance, banks such as BBVA, UBS, Deutsche Bank, Credit Suisse or BNP Paribas are placed among those institutions. Table 6.2 displays a cross-country comparison of systemic risk contribution measured by the average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ of financial firms belonging to the same country. Financial institutions from France and Spain appear as the most systemic according to their average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates under Gaussian and skewed-t marginal distribution assumptions respectively. In contrast, banks from Portugal, Ireland or Greece are classified among the least systemic financial institutions in our sample. One may consider this classification as an economic paradox since banks, belonging to national economies that have suffered the most by the European sovereign debt crisis, and at the same time the market value of their corresponding share prices have declined significantly during crisis appear among the least systemic financial institutions in the cross-country comparison. However, banks from those particular countries are typical commercial banks with substantial presence in the local market but limited international activity and cross-country exposure. As such, the implied correlation and more importantly the dependence in extreme events between those banks and financial system is typically reduced generating in this way lower in absolute value systemic risk estimates. This is also confirmed by the fact that Frank copula which does not allow for tail dependence is the preferred copula functional form for most of those particular pairs.

These findings should not be considered as weakness of *CoVaR* model but rather than a merit. According to Brunnermeier (2009a), a systemic risk measure should identify the risk to the system by individually "systemically important" institutions, which are highly interconnected and large that they can cause negative spill over effects on others, as well as by small institutions that are "systemic" when acting as part of a herd. In this respect, the relative size and interconnectedness of each particular financial institution should be considered in systemic risk measurement. The *CoVaR* methodology incorporate implicitly institution size and interconnectedness into systemic risk estimation though correlation and dependence on extreme events. In our study, financial system is represented by components of STOXX Europe 600 Banks Index that includes the largest banks by market capitalisation in Europe. It is a portfolio of 42 financial institutions from 15 different European countries. The majority and largest in size of those financial institutions come from countries such as Germany, France, Spain, Italy and Great Britain. As such, the implied dependence between each of those particular institutions and financial system is stronger by construction due to within-country dependence (e.g increased commonalities for institution returns from same country) and the dependence that arise from their large size and dominant position in the European market. This may partly explain why banks from those particular countries are listed among the most systemic financial institutions in our study. The results in Table 6.4 support this argument. Table 6.4 reports average sample Kendall's τ and non-parametric upper (λ^U) and lower (λ^L) tail dependence estimates for each country. It is evident that Kendall's τ correlations and non-parametric tail dependence coefficients are much stronger for those particular countries implying a stronger dependence and dependence in the tails of joint distribution and consequently higher, on average, systemic risk estimates.

Table 6.1: This table ranks the average contribution to systemic risk for each individual institution.

ΔCoVaR results								ΔCoES results							
Normal Margins				Skewed-t Margins				Normal Margins				Skewed-t Margins			
Bank	Copula	ΔCoVaR	τ	Bank	Copula	ΔCoVaR	τ	Bank	Copula	ΔCoES	τ	Bank	Copula	ΔCoES	τ
BBVA	Gumbel	-2.741	0.62	POP	BB7	-5.127	0.46	BBVA	Gumbel	-2.549	0.62	POP	BB7	-5.236	0.46
SCH	Gumbel	-2.709	0.61	DBK	BB7	-5.122	0.55	SCH	Gumbel	-2.522	0.61	DBK	BB7	-5.227	0.55
BNP	Gumbel	-2.691	0.60	UBSN	BB7	-5.114	0.53	BNP	Gumbel	-2.507	0.60	UBSN	BB7	-5.226	0.53
CRDA	Gumbel	-2.585	0.57	CSGN	BB7	-5.086	0.51	CRDA	Gumbel	-2.417	0.57	CSGN	BB7	-5.186	0.51
UCG	Gumbel	-2.524	0.55	CRDA	BB7	-5.078	0.53	UCG	Gumbel	-2.365	0.55	CRDA	BB7	-5.181	0.53
SEA	Gumbel	-2.488	0.54	SEA	BB7	-5.073	0.50	SEA	Gumbel	-2.336	0.54	LLOY	BB7	-5.178	0.46
NDA	Gumbel	-2.476	0.54	SCH	BB7	-5.049	0.56	NDA	Gumbel	-2.326	0.54	BSAB	BB7	-5.168	0.38
BARC	Gumbel	-2.450	0.54	BBVA	BB7	-5.041	0.57	BARC	Gumbel	-2.302	0.54	SEA	BB7	-5.160	0.50
LLOY	Gumbel	-2.334	0.50	LLOY	BB7	-5.039	0.46	LLOY	Gumbel	-2.203	0.50	SVK	BB7	-5.155	0.41
SGE	Frank	-1.934	0.64	UCG	BB7	-5.033	0.50	SGE	Frank	-1.746	0.64	SCH	BB7	-5.125	0.56
DBK	Frank	-1.883	0.63	BSAB	BB7	-5.031	0.38	DBK	Frank	-1.700	0.63	UCG	BB7	-5.124	0.50
KB	Frank	-1.802	0.60	NDA	BB7	-5.027	0.50	KB	Frank	-1.626	0.60	BP	BB7	-5.123	0.47
UBSN	Frank	-1.798	0.60	SVK	BB7	-5.011	0.41	UBSN	Frank	-1.623	0.60	DNB	BB7	-5.121	0.38
CSGN	Frank	-1.798	0.59	BP	BB7	-5.009	0.47	CSGN	Frank	-1.622	0.59	NDA	BB7	-5.118	0.50
CBK	Frank	-1.713	0.57	KB	BB7	-5.000	0.52	CBK	Frank	-1.545	0.57	DAB	BB7	-5.111	0.38
ISP	Frank	-1.704	0.56	CBK	BB7	-4.999	0.49	ISP	Frank	-1.537	0.56	BBVA	BB7	-5.110	0.57
BMPS	Frank	-1.664	0.55	DNB	BB7	-4.971	0.38	BMPS	Frank	-1.500	0.55	BARC	BB7	-5.085	0.48
KNF	Frank	-1.660	0.53	DAB	BB7	-4.956	0.38	KNF	Frank	-1.497	0.53	CBK	BB7	-5.078	0.49
MB	Frank	-1.652	0.56	BARC	BB7	-4.951	0.48	MB	Frank	-1.490	0.56	KB	BB7	-5.075	0.52
RBS	Frank	-1.645	0.54	SYD	BB7	-4.932	0.33	RBS	Frank	-1.483	0.54	SYD	BB7	-5.073	0.33
POP	Frank	-1.594	0.53	JYS	BB7	-4.832	0.34	POP	Frank	-1.437	0.53	JYS	BB7	-5.033	0.34
PMI	Frank	-1.587	0.53	ETE	BB7	-4.766	0.35	PMI	Frank	-1.430	0.53	BPSO	BB7	-4.888	0.29
HSBA	Frank	-1.580	0.52	BPSO	BB7	-4.624	0.29	HSBA	Frank	-1.424	0.52	ETE	BB7	-4.887	0.35
BP	Frank	-1.575	0.53	BCV	BB7	-4.244	0.27	BP	Frank	-1.419	0.53	BCV	BB7	-4.582	0.27
SWED	Frank	-1.526	0.50	SGE	Frank	-2.862	0.62	SWED	Frank	-1.374	0.50	SGE	Frank	-2.910	0.62
STAN	Frank	-1.492	0.50	BNP	Frank	-2.728	0.61	STAN	Frank	-1.343	0.50	BNP	Frank	-2.769	0.61
ERS	Frank	-1.464	0.50	ISP	Frank	-2.474	0.53	ERS	Frank	-1.318	0.50	ISP	Frank	-2.507	0.53
SVK	Frank	-1.455	0.48	BMPS	Frank	-2.420	0.53	SVK	Frank	-1.310	0.48	BMPS	Frank	-2.460	0.53
DAB	Frank	-1.383	0.46	MB	Frank	-2.382	0.54	DAB	Frank	-1.244	0.46	MB	Frank	-2.422	0.54
BSAB	Frank	-1.357	0.45	KNF	Frank	-2.368	0.50	BSAB	Frank	-1.220	0.45	KNF	Frank	-2.405	0.50
POH	Frank	-1.321	0.45	RBS	Frank	-2.330	0.51	POH	Frank	-1.188	0.45	RBS	Frank	-2.354	0.51
BKIR	Frank	-1.315	0.44	PMI	Frank	-2.300	0.51	BKIR	Frank	-1.183	0.44	PMI	Frank	-2.335	0.51
DNB	Frank	-1.305	0.44	HSBA	Frank	-2.249	0.51	DNB	Frank	-1.173	0.44	HSBA	Frank	-2.285	0.51
BES	Frank	-1.274	0.42	SWED	Frank	-2.187	0.48	BES	Frank	-1.145	0.42	SWED	Frank	-2.216	0.48
JYS	Frank	-1.260	0.39	STAN	Frank	-2.115	0.48	JYS	Frank	-1.132	0.39	STAN	Frank	-2.144	0.48
SYD	Frank	-1.227	0.41	ERS	Frank	-2.084	0.47	SYD	Frank	-1.102	0.41	ERS	Frank	-2.118	0.47
ETE	Frank	-1.217	0.40	BKIR	Frank	-1.821	0.41	ETE	Frank	-1.093	0.40	BKIR	Frank	-1.845	0.41
BCP	Frank	-1.212	0.40	POH	Frank	-1.820	0.42	BCP	Frank	-1.088	0.40	POH	Frank	-1.840	0.42
BPE	Frank	-1.081	0.37	BES	Frank	-1.725	0.39	BPE	Frank	-0.969	0.37	BES	Frank	-1.744	0.39
BPSO	Frank	-1.016	0.34	BCP	Frank	-1.646	0.37	BPSO	Frank	-0.910	0.34	BCP	Frank	-1.659	0.37
BCV	Frank	-1.011	0.33	BPE	Frank	-1.386	0.33	BCV	Frank	-0.905	0.33	BPE	Frank	-1.399	0.33
VATN	Frank	-0.813	0.28	VATN	Frank	-0.953	0.23	VATN	Frank	-0.726	0.28	VATN	Frank	-0.953	0.23

This table reports average $\Delta CoVaR_{\alpha,\beta,t}$, $\Delta CoES_{\alpha,\beta,t}$ and implied Kendall's τ estimates along with the selected copula families of each financial system-institution pair in our sample under two marginals specifications: Normal and Skewed-t. All risk measures are computed at $\alpha = \beta = 5\%$ level.

Table 6.2: This table ranks the average contribution to systemic risk by country.

ΔCoVaR results				ΔCoES results			
Normal Margins		Skewed-t Margins		Normal Margins		Skewed-t Margins	
Country	ΔCoVaR	Country	ΔCoVaR	Country	ΔCoES	Country	ΔCoES
France	-0.0222	Spain	-0.0506	France	-0.0204	Spain	-0.0516
Spain	-0.0210	Germany	-0.0505	Spain	-0.0193	Germany	-0.0515
Sweden	-0.0199	Belgium	-0.0499	Sweden	-0.0184	Norway	-0.0512
Great Britain	-0.0190	Norway	-0.0497	Great Britain	-0.0175	Belgium	-0.0507
Belgium	-0.0180	Denmark	-0.0490	Belgium	-0.0163	Denmark	-0.0507
Germany	-0.0180	Greece	-0.0476	Germany	-0.0162	Greece	-0.0489
Italy	-0.0160	Sweden	-0.0432	Italy	-0.0145	Sweden	-0.0441
Austria	-0.0146	Swiss	-0.0384	Austria	-0.0132	Swiss	-0.0399
Swiss	-0.0136	Great Britain	-0.0333	Swiss	-0.0122	Great Britain	-0.0341
Finland	-0.0132	France	-0.0326	Finland	-0.0119	France	-0.0332
Ireland	-0.0132	Italy	-0.0320	Ireland	-0.0118	Italy	-0.0328
Norway	-0.0131	Austria	-0.0208	Norway	-0.0117	Austria	-0.0212
Denmark	-0.0129	Ireland	-0.0182	Denmark	-0.0116	Ireland	-0.0185
Portugal	-0.0124	Finland	-0.0182	Portugal	-0.0112	Finland	-0.0184
Greece	-0.0122	Portugal	-0.0168	Greece	-0.0109	Portugal	-0.0170

This table reports average $\Delta CoVaR_{\alpha,\beta,t}$ and $\Delta CoES_{\alpha,\beta,t}$ estimates for each country in our sample under two marginal specifications: Normal and Skewed-t. All risk measures are computed at $\alpha = \beta = 5\%$ level.

Table 6.3: Dependence and tail dependence estimates.

Bank	Copula	ΔCoVaR	τ	λ^L	λ^U
POP	BB7	-5.127	0.51	0.46	0.24
DBK	BB7	-5.122	0.62	0.65	0.42
UBSN	BB7	-5.114	0.57	0.59	0.44
CSGN	BB7	-5.086	0.56	0.49	0.41
CRDA	BB7	-5.078	0.57	0.46	0.54
SEA	BB7	-5.073	0.54	0.62	0.40
SCH	BB7	-5.049	0.61	0.59	0.44
BBVA	BB7	-5.041	0.63	0.62	0.53
LLOY	BB7	-5.039	0.50	0.41	0.42
UCG	BB7	-5.033	0.56	0.48	0.28
BSAB	BB7	-5.031	0.43	0.42	0.24
NDA	BB7	-5.027	0.54	0.48	0.51
SVK	BB7	-5.011	0.47	0.30	0.34
BP	BB7	-5.009	0.51	0.55	0.29
KB	BB7	-5.000	0.57	0.56	0.33
CBK	BB7	-4.999	0.55	0.57	0.24
DNB	BB7	-4.971	0.42	0.45	0.21
DAB	BB7	-4.956	0.43	0.38	0.26
BARC	BB7	-4.951	0.54	0.54	0.43
SYD	BB7	-4.932	0.37	0.34	0.16
JYS	BB7	-4.832	0.37	0.31	0.27
ETE	BB7	-4.766	0.38	0.34	0.23
BPSO	BB7	-4.624	0.31	0.30	0.11
BCV	BB7	-4.244	0.28	0.19	0.21
SGE	Frank	-2.862	0.63	0.60	0.50
BNP	Frank	-2.728	0.61	0.44	0.52
ISP	Frank	-2.474	0.53	0.42	0.33
BMPS	Frank	-2.420	0.54	0.50	0.24
MB	Frank	-2.382	0.53	0.36	0.19
KNF	Frank	-2.368	0.50	0.37	0.26
RBS	Frank	-2.330	0.51	0.47	0.41
PMI	Frank	-2.300	0.51	0.50	0.24
HSBA	Frank	-2.249	0.51	0.33	0.32
SWED	Frank	-2.187	0.48	0.46	0.31
STAN	Frank	-2.115	0.48	0.36	0.23
ERS	Frank	-2.084	0.47	0.38	0.26
BKIR	Frank	-1.821	0.41	0.29	0.15
POH	Frank	-1.820	0.42	0.20	0.22
BES	Frank	-1.725	0.38	0.25	0.18
BCP	Frank	-1.646	0.37	0.41	0.00
BPE	Frank	-1.386	0.33	0.13	0.17
VATN	Frank	-0.953	0.24	0.17	0.12

This table reports average $\Delta CoVaR_{\alpha,\beta,t}$, sample Kendall's τ correlations and non-parametric upper (λ^U) and lower (λ^L) tail dependence estimates (average of non-parametric estimates in Dobrić and Schmid (2005)) of each financial system-institution pair in our sample. $\Delta CoVaR_{\alpha,\beta,t}$ estimates are obtained under the assumption of skewed-t margins. All risk measures are computed at $\alpha = \beta = 5\%$ level.

Table 6.4: Dependence and tail dependence estimates by country.

Country	τ	λ^L	λ^U
Germany	0.58	0.61	0.33
Belgium	0.57	0.56	0.33
Spain	0.54	0.52	0.37
France	0.58	0.47	0.46
Sweden	0.51	0.46	0.39
Norway	0.42	0.45	0.21
Great Britain	0.51	0.42	0.36
Italy	0.48	0.41	0.23
Austria	0.47	0.38	0.26
Swiss	0.41	0.36	0.30
Greece	0.38	0.34	0.23
Denmark	0.39	0.34	0.23
Portugal	0.38	0.33	0.09
Ireland	0.41	0.29	0.15
Finland	0.42	0.20	0.22

This table reports average sample Kendall's τ correlations and non-parametric upper (λ^U) and lower (λ^L) tail dependence estimates (average of non-parametric estimates in Dobrić and Schmid (2005)) for each country in our sample.

6.3 Backtesting and Stress testing *CoVaR*

A well-specified risk model should satisfy the appropriate theoretical statistical properties. As such, the proportion of exceedances should approximately equal the confidence level, while the exceedances should not occur in clusters but instead independently. Table 6.5 reports the average p-values from modified Kupiec (1995) and Christoffersen (1998) statistical tests for unconditional coverage, independence and conditional coverage of $CoVaR_{\alpha,\beta,t}$ estimates under both Gaussian and skewed-t distribution assumptions computed at $\alpha = \beta = 5\%$ level.

The null hypotheses of unconditional and conditional coverage are rejected at the 5% level of significance under Gaussian assumption. On the other hand, the average p-values from statistical tests under the assumption of skewed-t margins are pretty high and hence the null hypotheses cannot be rejected at any conventional level of significance. Thus, it seems that a combination of copula functions that allow for asymmetries in the tails with asymmetric margins is a better candidate for systemic risk modelling. Our test results are in line with the *CoVaR* backtest results in Girardi and Ergün (2013) and the *VaR* test results in the vast literature that reject the underlying normality assumption of risk models suggesting alternative distribution assumptions which allow for asymmetries.

Table 6.5: Statistical test results

Test	Normal Margins	Skewed-t Margins
Unconditional coverage	0.0206	0.3633
Independence	0.5649	0.8802
Conditional coverage	0.0286	0.5410

This table reports average p-values of statistical tests for unconditional, independence and conditional coverage properties for $CoVaR_{\alpha,\beta,t}$ estimates. All risk estimates are computed at $\alpha = \beta = 5\%$ level.

On the other hand, stress testing exercises are useful for financial regulators to gauge potential implications of extreme market conditions for the stability of the financial system as a whole. Before the outset of financial crisis, financial stability stress tests were largely focused on implications of the system-wide macroeconomic shocks and rarely consider idiosyncratic shocks such as the failure of a single large firm. Recently, there has been a growing interest in such systemic stress testing exercises by central banks and financial regulators. Our modelling framework can be easily employed as part of the tool-kit for financial stability assessment. Stress testing exercises under this framework can simulate scenarios that are absent from historical data or are more likely to occur than historical observation suggests, as well as simulate shocks that reflect permanent structural breaks or temporal dependence breakdowns.

Figures 6.6 displays a scenario analysis example for HSBC and crystallise its influence on systemic risk as measured by $CoVaR_{\alpha,\beta,t}$ under certain scenarios. In particular, Figure 6.6 plots the implied $CoVaR_{\alpha,\beta,t}$ measures generated by Clayton, Gumbel, Frank and BB7 copulas for $\beta = 0.01$ to 0.80, $\alpha = 0.05$ and dependence parameter(s) estimated for each particular copula family assuming skewed-t margins[17]. As such, the discrepancies in $CoVaR_{\alpha,\beta,t}$ measures are subject to alternative copula models and do not arise from margin specifications. The implied $CoVaR_{\alpha,\beta,t}$ results in Figure 6.6 have an appealing interpretation. For instance, we are 99% confident, given that HSBC is at most as its 95% *VaR* level, that the financial system will not experience a distress event worse than −16.84% according to Clayton copula. For the same confidence level, $CoVaR_{\alpha,\beta,t}$ estimates implied by Gumbel, Frank and BB7 copulas are −15.08%, −13.56% and −16.74% respectively.[18].

Given the unique characteristic of copula functions enabling the separation of dependence from marginal distributions, we are able to quantify the potential effects on the stability of the financial system from risks associated with marginal distribution assumptions or from risks related to dependence structure. For example, a scenario that implies a structural break in correlation between financial system and institution's returns can be analysed by modifying the level of Kendall's τ parameter, while a change in dependence structure can be studied through alternative copula functional forms. Similarly, a scenario that implies high volatility or severe equity price declines can be examined though alternative marginal specifications. Complex stress test exercises that combine all the above scenarios can also be analysed simultaneously providing in this way a powerful tool for systemic risk assessment.

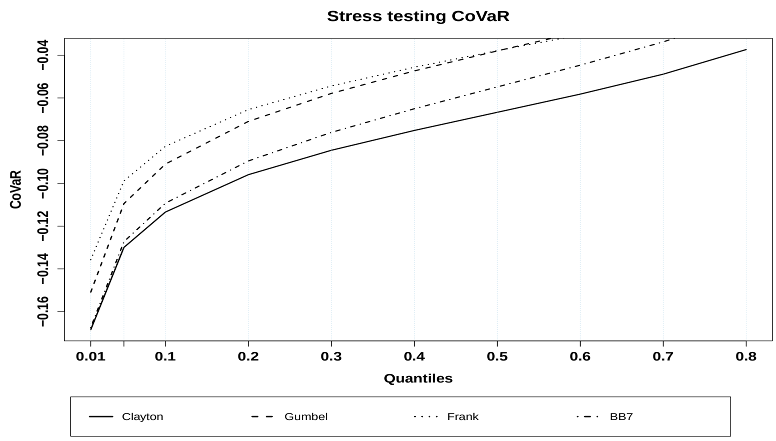

Figure 6.6: This figure shows the implied $CoVaR_{\alpha,\beta,t}$ estimates of financial system conditional on HSBC returns generated by Clayton, Gumbel, Frank and BB7 copulas with skewed-t marginals across different quantile levels ($\beta = 0.01$ to 0.8 and $\alpha = 0.05$). The Kendall's τ sample correlation parameter is equal to 0.51, i.e., $\tau = 0.51$.

[17] We could also set the parameters for Clayton, Gumbel and Frank copulas to a pre-specified value such as the Kendall's τ sample correlation coefficient because there is an one-to-one relationship between those particular one-parameter copula families and Kendall's τ. Such a relationship, however, does not exist for the two-parameter BB7 copula. To maintain the consistency of implied systemic risk estimates, we use the estimated parameter(s) for each particular copula family instead.

[18] Similar stress testing exercises can be also obtained using $CoES_{\alpha,\beta,t}$ as a measure of systemic risk.

Figure 6.6 provides also a distinct graphical way to illustrate the importance of tail dependence in systemic risk computation and facilitate the interpretation of results in Tables 6.1 and 6.2. It is clear from Figure 6.6 that Clayton and BB7 copulas which allow for lower tail dependence produce much larger in absolute value $CoVaR_{\alpha,\beta,t}$ measures compared to the corresponding measures generated by Frank or Gumbel copulas, which do not allow for lower tail dependence. As already explained, the average $\Delta CoVaR_{\alpha,\beta,t}$ or $\Delta CoES_{\alpha,\beta,t}$ measures reported in Tables 6.1 and 6.2 do not differ in size only due to alternative distribution assumptions in margins but differ also due to the distinctive characteristics of alternative copula functional forms employed. Therefore, copula misspecification may critically affect the systemic risk estimates and as such dependence modelling should proceed with caution.

6.4 Systemic risk determinants

In this section, we investigate the main drivers of systemic risk in the European banking system. The analysis is split into two main parts. In the first part, we investigate whether there are common market factors explaining an institution's contribution to systemic risk. We also investigate how and towards which direction these factors affect systemic risk. As explained, systemic risk measures can be decomposed within Copula *CoVaR* framework due to the unique property of copula functions that enable the separation of dependence from marginal distributions. Thus, *CoVaR* can be viewed as an increasing non-linear function of financial system-institution's i correlation and financial system's volatility. In this respect, we assess the impact of market factors on those variables and analyse their importance for the stability of the financial system. In the second part, we study the relationship between individual institution characteristics and institution's contribution to systemic risk as measured by $\Delta CoVaR_{\alpha,\beta,t}$ estimates obtained in section 6.2. All results are based on skewed-t marginal distribution assumptions[19].

The employment of linear quantile regressions is a very popular technique for generating time-varying systemic risk measures. In general, estimation is carried out using few market variables as conditioning variables in the linear quantile specification. As Adrian and Brunnermeier (2011) explain, these variables should not be interpreted as systemic risk factors but rather as conditioning variables that scale the conditional mean and variance of risk measures. In this study, however, we investigate whether such market variables are important in explaining the variation of institution's contribution to systemic risk and extract information useful for macro-prudential policy and regulation. Therefore, we regress weekly $\Delta CoVaR_{\alpha,\beta,t}$, Kendall's τ correlations and financial system's volatility estimates, on lagged values of the following market factors:

(i) *Vix*, which is a proxy for the implied volatility in the stock market reported by the Chicago Board Options Exchange (CBOE).

(ii) *Liquidity Spread*, which is a short term "liquidity spread" defined as the difference between the three-month interbank offered rate and the three-month repo rate. This spread is a

[19] We also analysed the same relationships based on the results from Gaussian margins. Moreover, we employed $\Delta CoES_{\alpha,\beta,t}$ as alternative measure of institution's contribution to systemic risk. The qualitative results, however, remained unchanged.

common proxy for short-term funding liquidity risk. We use the three-month Euribor rate and the three-month Eurepo rate, both reported by European Banking Federation (EBF).

(iii) *Change Euribor*, which is the change in the three-month Euribor rate.

(iv) *Change Slope*, which is the change in the slope of the yield curve, measured by the spread between the German ten-year government bond yield and the German three-month Bubill rate.

(v) *Change Credit Spread*, which is the change in credit spread between the ten-year Moody's seasoned BAA-rated corporate bond and the German ten-year government bond.

(vi) *S&P*, which is the S&P 500 Composite Index returns and used as a proxy for equity market returns.

The data have been obtained from Bloomberg and are sampled weekly. Table 6.6 reports the summary statistics of market variables. Almost all extreme values of those variables occur during stress periods. It is also evident that the distributions of the variables are highly skewed.

Table 6.6: Market variables summary statistics

	Vix	Liquidity Spread	Change Euribor	Change Slope	Credit Credit Spread	S&P
Mean	21.834	35.642	−0.706	−0.085	0.086	0.070
Median	19.090	20.000	0.000	−0.700	0.000	0.138
Std. dev	10.219	34.721	6.198	14.261	11.892	3.125
Min	9.970	7.300	−51.100	−50.200	−41.100	−15.723
Max	80.060	188.800	21.100	119.500	126.200	13.604
Skewness	1.973	1.700	−2.555	1.560	2.498	−0.318
Kurtosis	5.246	2.995	15.362	11.278	27.012	4.834

This table reports summary statistics for the weekly market variables. Vix denotes the CBOE implied volatility. Liquidity Spread represents the difference between the 3-month Euribor rate and 3-month Eurepo rate. Change Euribor denotes the change in 3-month Euribor rate. Change Slope denotes the change in the yield slope between the 10-year and 3-month German government bond rates. Change Credit Spread represents the change of yield spread between ten-year Moody's seasoned BAA-rated corporate bond and the German ten-year government bond. S&P is the market equity returns. The spreads and spreads changes are expressed in basis points, returns and the Vix in percentage points.

Table 6.7 reports bank fixed-effect panel regression estimates for $\Delta CoVaR_{\alpha,\beta,t}$, Kendall's τ and financial's system volatility estimates on the above lagged market variables[20]. To investigate the impact of financial crisis and the response of market variables in the pre-crisis and crisis periods, we also include in the regression specification dummy variables (I_{crisis}) that take the value of zero in the pre-crisis period and one in the period we designate as crisis period. Across both sub-periods, the lagged values of the Vix, Liquidity Spread and Change Euribor variables appear highly significant in explaining the variation in $\Delta CoVaR_{\alpha,\beta,t}$ at conventional significance levels. In particular, higher lagged values of implied market volatility are associated with more negative $\Delta CoVaR_{\alpha,\beta,t}$ measures in the pre-crisis period. In contrast, the impact of lagged S&P Return, Credit Spread and Change Slope variables on $\Delta CoVaR_{\alpha,\beta,t}$ do not appear statistically significant in this period (Change Slope is significant only at 10% level).

[20]We also regressed the market variables on time-series average $\Delta CoVaR_{\alpha,\beta,t}$, Kendall's τ and financial system's volatility estimates. The statistical significance of market variables remained unaffected.

The results in Table 6.7 highlight also the importance of funding liquidity on systemic risk contribution. Banks typically raise short-term funding in the unsecured interbank market or through over-the-counter collateralised repurchase agreements (repos). In times of uncertainty, banks charge higher rates for unsecured loans and thus interbank offered rates increase. The spread between the Eurepo and Euribor rates measures the difference in interest rates between short-term funding of different risk. As Figure 6.7 shows, this spread had shrunk to historical low levels during the pre-crisis period but it begun to surge upward during crisis period. The positive impact of funding liquidity on $\Delta CoVaR_{\alpha,\beta,t}$ in the pre-crisis period is confirmed by the results in Table 6.7. The coefficient of Liquidity Spread in this period is negative and rather significant in magnitude. On average, an 1% increase in Liquidity Spread, which indicates a worsening of funding liquidity, contributes almost 13.7% to systemic risk as measured by $\Delta CoVaR_{\alpha,\beta,t}$.

Table 6.7: Panel regression results.

Variables	ΔCoVaR	Kendall's τ	Volatility
Vix (lag)	−0.149***	0.004***	0.140***
Liquidity Spread (lag)	−13.739***	−0.313***	13.997***
Change Euribor (lag)	−3.691***	0.057	3.453***
Change Slope (lag)	0.658*	−0.024	−0.607*
Credit Spread (lag)	−0.089	−0.027***	0.101
S&P Return (lag)	−0.021	0.001	0.019
Vix (lag) · I_{crisis}	0.044***	−0.003***	−0.044***
Liquidity Spread (lag) · I_{crisis}	11.866***	0.337***	−12.234***
Change Euribor (lag) · I_{crisis}	5.951***	−0.035	−5.494**
Change Slope (lag) · I_{crisis}	−0.301	0.017	0.276
Credit Spread (lag)· I_{crisis}	−0.932***	−0.014***	0.947***
S&P Return (lag) · I_{crisis}	0.032	−0.002	−0.029
Adj. R^2	0.770	0.219	0.876

This table displays results from bank fixed-effects panel data methodology (within estimator). The columns ΔCoVaR, Kendall's τ and Volatility report estimated coefficients from regressions of weekly $\Delta CoVaR_{\alpha,\beta,t}$ measures, Kendall's τ correlation and financial system's volatility estimates on the same lagged values of market variables: Vix, Liquidity Spread, Change Euribor, Change Slope, Change Credit Spread and S&P Returns. The I_{crisis} is a crisis dummy that takes the value of 0 for the Q2 2002 - Q2 2007 pre-crisis period and 1 for the Q3 2007 - Q4 2012 crisis period. Estimated coefficients for spreads, yield changes, Vix and market returns correspond to percent changes. The results are based on weekly data from Q2 2002 - Q4 2012. All $\Delta CoVaR_{\alpha,\beta,t}$ measures are estimated at 5% level. Kendall's τ correlations are obtained after transforming the time-varying copula parameters for each financial system-institution i pair to theoretical Kendall's τ values. Financial system's volatility estimates are obtained by an univariate asymmetric AR(1)-GJR-GARCH(1,1) model for each financial system portfolio. Following Thompson (2011), we compute standard errors that cluster by both firm and time. *** denotes significant at 1%, ** denotes significant at 5% and * denotes significant at 10%.

The results are in line with a large number of theoretical and empirical research papers that associate market declines with liquidity dry-ups in triggering systemic episodes (see e.g. Brunnermeier 2009b; Adrian and Shin 2010; Brunnermeier and Pedersen 2009; Hameed et al. 2010, and references therein). The burst of crisis in summer 2007, caused two "liquidity spirals". Financial institutions' capital eroded due to the initial decline in asset prices and increasing wholesale funding cost. Consequently, both events triggered fire-sales pushing asset prices further down and increased the uncertainty in the interbank lending market. As such, European banks that relied excessively on short-term funding were particularly exposed to a dry-up in liquidity. In this respect, the large size of the pre-crisis Liquidity Spread coefficient estimate partly explains why the sudden dry-up in liquidity had such a severe impact on the stability of the financial system.

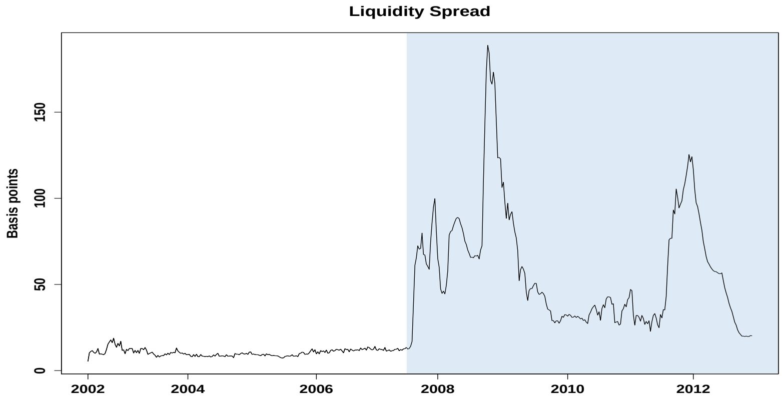

Figure 6.7: This figure shows the short-term Liquidity Spread between the 3-months Euribor rate and 3-months Eurepo rate measured in basis points. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

The regression results in Table 6.7 for the Change Euribor variable are also of great interest. As explained, the Euribor rate represents the unsecured rate at which a large panel of European banks borrow funds from one another. An increase in short-term rates implies a higher borrowing cost for banks. In this respect, banks relying on short-term funding are more vulnerable to liquidity risk. The pre-crisis coefficient estimate of the change in the three-month Euribor rate variable indicates the negative relation between changes in the short-term rates and systemic risk contribution. On average, an increase by 1% in the change of three-month Euribor rate adds up an additional 3.7% on $\Delta CoVaR_{\alpha,\beta,t}$.

In contrast, the signs of almost all estimated coefficients have been switched in the crisis period indicating an asymmetric response of market factors on systemic risk in those sub-periods. In particular, the coefficient estimates of Liquidity Spread and Change Euribor variables have been switched from negative in the pre-crisis to positive in the crisis period. One of the main reasons behind this behaviour is the coordinated intervention of central banks in both United States and Europe in response to the freezing up of the interbank market. To alleviate the liquidity crunch, the European Central Bank (ECB) and the Federal Reserve (Fed) reduced the interest rates they charge financial institutions to borrow from them; they also expanded their balance sheets by broadening the type of collateral that banks could use, as well as increased the maturity of their loans to the banks (see Giannone et al. (2012), for further details). Figure 6.8 shows average $CoVaR_{\alpha,\beta,t}$ estimates and a timeline of key events and actions taken by European Central Bank (ECB) to provide liquidity and restore financial stability over the recent financial crisis. It is clear from Figure 6.8 that the highest $CoVaR_{\alpha,\beta,t}$ measures (in absolute value) are reported after Lehman Brothers collapse in September 2008. Figure 6.8 also depicts the actions taken by European Central Bank (ECB) in response to liquidity crunch and overall financial market turmoil. It can be shown that the systemic risk measures returned to lower levels whereas the initial liquidity dry-up in the interbank market calmed down and the short-term interbank rates returned to lower levels, as Figure 6.7 and Figure 6.9 display respectively.

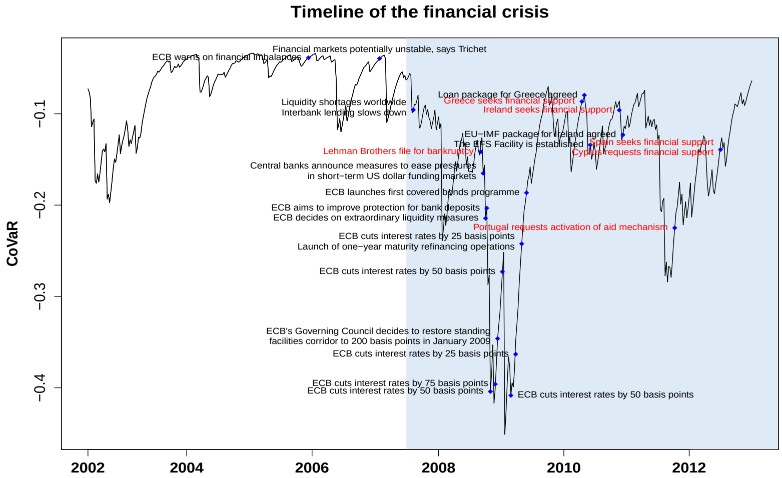

Figure 6.8: This figure shows average $CoVaR_{\alpha,\beta,t}$ estimates, key events (in red) and actions taken by European Central Bank (ECB) to provide liquidity in the interbank market and restore financial stability. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period. Source of timeline events: European Central Bank (ECB), www.ecb.europa.eu.

The overall increase in systemic risk during the crisis period, however, is not only driven by the solvency problems of several Euro-area financial institutions, but also by the sovereign debt crisis for a large number of Eurozone member countries. As Figure 6.8 suggests, systemic risk estimates reached their highest levels after the collapse of Lehman Brothers in Q3 2007, however, high values are also associated with the weakness of several countries in the Euro-zone to repay or refinance their government debt without the assistance of third parties. As Shambaugh (2012) points out, the euro area faced three interdependent crises. That is a combined sovereign debt crisis, a banking crisis and a growth and competitiveness crisis. In this respect, the problems of undercapitalised banks and high sovereign debt are mutually reinforcing, and both are amplified by slow and unequally distributed growth among member countries. Therefore, our regression results and asymmetric response of market factors on systemic risk should be viewed in conjunction with the overall characteristics of the crisis in the Eurozone.

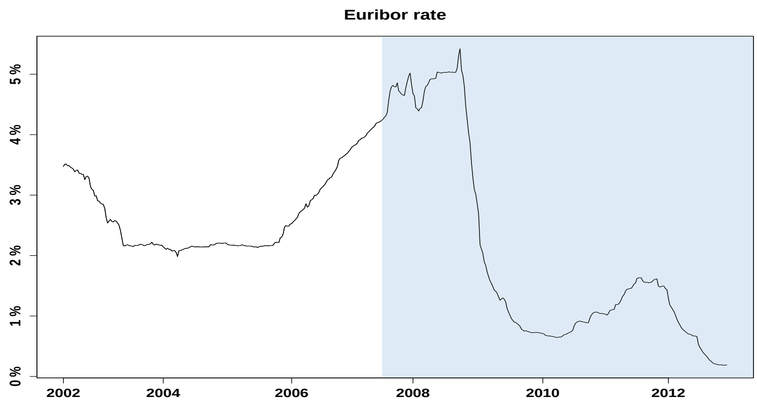

Figure 6.9: This figure shows the 3-month euro interbank offered rate (Euribor), the interest rate at which euro interbank 3-month deposits are offered by one prime bank to another prime bank within the euro area. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

It is also of great interest to investigate the effect of market factors on Kendall's τ correlation and financial system's volatility estimates. Kendall's τ correlation estimates are asymmetrically related to lagged values of the Vix and Liquidity Spread variables, although the magnitude of asymmetries are not large. Interestingly, liquidity shocks (widening of liquidity spread) reduce Kendall's τ correlation in the pre-crisis period, while having a positive impact in crisis period. A widening in Credit Spread suggests also a decrease in Kendall's τ correlation in both periods. The above market factors appear also significant in explaining financial system's volatility and demonstrate the same asymmetric behaviour. Overall, an increase in the Vix, Liquidity or Credit Spread variables increases financial system's volatility and as a consequence the level of systemic risk. The Change Euribor variable is also asymmetrically related to financial system's volatility, however, the degree of asymmetry is pretty high between those sub-periods, with the regression coefficients changing from 3.453 to -5.494. This substantial asymmetric response highlights also the impact of European Central Bank's (ECB) intervention in the interbank market.

An overall increase in the change of three-month Euribor rate, counterintuitively, suggests a reduction in financial system's volatility. This is due to the substantial disparity of estimated coefficients in the pre-crisis and crisis period respectively. However, as shown in Figures 6.8 and 6.9, the actions taken by European Central Bank (ECB) during the crisis period eventually reduced the level of short-term interest rates and, thus, distorted the positive pre-crisis relationship between the change in short-term rates and financial system's volatility. From the results in Table 6.7, it can be also seen that the impact of funding liquidity is primarily transmitted on $\Delta CoVaR_{\alpha,\beta,t}$ though financial system's volatility and not from Kendall's τ correlation. In other words, the sudden-dry up of liquidity in the pre-crisis period reduced the level of correlation among financial institutions but increased considerably the volatility of the financial system. This can be also confirmed after comparing the estimated coefficients of Liquidity Spread variable on $\Delta CoVaR_{\alpha,\beta,t}$ and volatility variables, which are almost identical in absolute value.

In the second part of our analysis, we investigate how individual characteristics of financial institutions contribute to the systemic risk. In this regard, we employ panel regressions and regress quarterly-aggregated $\Delta CoVaR_{\alpha,\beta,t}$ measures on a set of institution-specific variables. In particular, we use the following set of quarterly bank-specific characteristics:

(i) $VaR^i_{\alpha,t-k}$ defined as the quarterly-aggregated *VaR* measures for bank i at quarter $t-k$, calculated by averaging the corresponding weekly measures within each quarter.

(ii) $MtB_{i,t-k}$ defined as the ratio of the market to book value of total equity for bank i at quarter $t-k$ and is a proxy for growth opportunities.

(iii) $Size_{i,t-k}$ defined by the log of total book value of equity for bank i at quarter $t-k$.

(iv) $Leverage_{i,t-k}$ defined as the ratio of the total assets to total book value of equity for bank i at quarter $t-k$ and is a proxy for the solvency of the bank.

(v) $Beta_{i,t-k}$ is the equity market beta for bank i at quarter $t-k$, calculated from weekly equity return data within each quarter.

(vi) $Vol_{i,t-k}$ is the equity return volatility for bank i at quarter $t-k$, calculated from weekly equity return data within each quarter.

The balance-sheet data for each individual bank are obtained from Worldscope database. Table 6.8 below, provides the summary statistics for $\Delta CoVaR_{\alpha,\beta,t}$ and $VaR^{i}_{\alpha,t}$ measures and the bank-specific characteristics at quarterly frequency.

Table 6.8: Bank-specific variables summary statistics

Variable	Mean	Median	Std. dev.
ΔCoVaR	−0.0469	−0.0353	0.0334
VaR	−0.0798	−0.0649	0.0552
MtB	1.2684	1.2406	0.8558
Size	8.4790	8.5084	0.6605
Leverage	21.6602	19.7551	14.7454
Beta	0.7232	0.6433	0.5415
Vol	0.0214	0.0172	0.0151

This table reports summary statistics of quarterly-aggregated bank-specific variables. VaR estimates are obtained by averaging the corresponding weekly measures within each quarter. All risk measures are estimated at 5% level of significance under the assumption of skewed-t marginals.

Table 6.9 reports results from panel regressions, after controlling for bank fixed-effects and, additionally, allowing for bank and time clustered errors. We report results from three different specifications based on the forecast horizon of explanatory variables: one quarter, one and two years. Across forecast periods, Size and Leverage appear as the most robust determinants of systemic risk. The estimated coefficient of Size variable is negative and highly significant, suggesting that bigger institutions contribute more to systemic risk.

Table 6.9: Determinants of systemic risk - Individual institution characteristics.

Variable	1-Quarter	1-Year	2-Year
VaR (lag)	−0.213 49**	0.128 39*	0.004 43
MtB (lag)	0.000 03	−0.010 95*	−0.015 09**
Size (lag)	−0.015 66***	−0.040 46***	−0.038 94***
Leverage (lag)	−0.000 74***	−0.001 28***	−0.000 76***
Beta (lag)	0.008 47	−0.004 08	−0.015 95***
Volatility (lag)	−1.834 96***	0.436 13*	0.341 99
Adj. R^2	0.440	0.288	0.325

This table displays results from bank fixed-effects panel regression methodology (within estimator). The columns report estimated coefficients from regressions of lagged quarterly bank-specific data on quarterly aggregated $\Delta CoVaR_{\alpha,\beta,t}$ measures. The column 1-Quarter corresponds to results based on lagged variables equal to one-quarter, while columns 1-Year and 2-Year corresponds to results based on lagged variables equal to one and two years, respectively. The results are based on quarterly data from Q2 2002 - Q4 2012.All $\Delta CoVaR_{\alpha,\beta,t}$ measures are estimated at 5% level. Following Thompson (2011), we compute standard errors that cluster by both firm and time *** denotes significant at 1%, ** denotes significant at 5% and * denotes significant at 10%.

These findings support the empirical results in Section 6.2. Few of the largest banks in our sample are placed among the most systemic financial institutions based on their average $\Delta CoVaR$ or $\Delta CoES$ measures reported in Table 6.1. Leverage is also negative and significant across all forecasting horizons. As explained, Leverage is used as a proxy for the solvency of the financial institution. The coefficient estimate of Leverage implies that highly leveraged banks contribute more to systemic risk. The VaR of each financial institution and equity return volatility are also statistical significant at the one quarter horizon, whereas equity beta is statistical significant at two year horizon. Overall, our results in Table 6.9 are in line with other studies. Similar to Acharya et al. (2010), Adrian and Brunnermeier (2011) and Girardi and Ergün (2013), we find that size, leverage and equity beta are important determinants of systemic risk. However, we found no statistical support that market to book value of total equity ratio is important in explaining institutions' contribution to systemic risk.

7 Summary

In this study we propose a new way for estimating the *Conditional Value-at-Risk* ($CoVaR$), based on copula functions. Our Copula $CoVaR$ methodology provides simple explicit expressions, under various $CoVaR$ definitions, for a broad range of copula families. In this regard, we avoid the burden of numerical integration in $CoVaR$ computation and offer a dynamic and more flexible approach for systemic risk modelling, eliminating also potential biases that may arise from misspecification in the marginals or joint distribution. Our approach is also extended to other systemic risk measures, such as the *Conditional Expected Shortfall* ($CoES$). The systemic risk measures generated from our framework share the main properties reported in Mainik and Schaanning (2012) for each particular $CoVaR$ specification. Furthermore, we illustrate how the Copula $CoVaR$ methodology can facilitate stress testing exercises, employed by financial regulators to measure the impact of extreme market scenarios to the stability of financial system.

We focus on a portfolio of large European banks and estimate $CoVaR$ and $CoES$ measures using alternative distribution assumptions in the margins and dependence structure. We illustrate the importance of taking asymmetries into account and highlight the threats to accurate systemic risk measurement from misspecification biases in the margins or dependence model. We measure institution's contribution to systemic risk using both $\Delta CoVaR$ and $\Delta CoES$ measures. Banks such as BBVA, UBS, Deutsche Bank, Credit Suisse and BNP Paribas appear among the most systemic European banks, whereas French and Spanish banks generate the highest average $\Delta CoVaR$ and $\Delta CoES$ estimates. We also investigate whether there are common market factors explaining institution's contribution to systemic risk. In principle, lagged values of the implied market volatility, funding liquidity, credit spread and the change in the three month Euribor rate are significant in explaining $\Delta CoVaR$. They appear also important in explaining financial system-institution's correlation and financial system's volatility. The asymmetric behaviour of market factors in the pre-crisis and crisis period is partly attributed to the coordinated intervention of central banks in response to the financial crisis. Finally, we investigate the impact of bank-specific characteristics on systemic risk and regress $\Delta CoVaR$ measures on a set of balance-sheet variables. Across all alternative model specifications considered, size and leverage appears as the most robust determinant of systemic risk, implying that bigger and highly leveraged financial institutions can generate large systemic risk externalities.

Appendices

A *CoVaR* derivation for Normal and Student-t copulas

A.1 Normal Copula

The Normal copula does not have a closed form distribution. Its distribution is given by

$$C(u,v) = \mathbf{\Phi_2}(\Phi^{-1}(u), \Phi^{-1}(v); \rho),$$

where $\mathbf{\Phi_2}$ is the bivariate distribution of two standard normal distributed random variables with correlation ρ, Φ is the $N(0,1)$ cdf and Φ^{-1} is the inverse of Φ. However, the *CoVaR* in Adrian and Brunnermeier (2011) can be given in explicit form, that is

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(\mathbf{\Phi}\Big(\rho\,\mathbf{\Phi^{-1}}(v_t) + \sqrt{1-\rho^2}\,\mathbf{\Phi^{-1}}(\beta)\Big)\Big).$$

Unfortunately, the *CoVaR* under the definition in Girardi and Ergün (2013) can not be given in explicit form. In this respect, *CoVaR* is obtained numerically by first solving the Normal copula density for the conditional quantile u , that is

$$\int_0^{u_t}\int_0^{v_t} \frac{1}{\sqrt{1-\rho^2}} \exp\Big(-\frac{\rho^2(s^2+t^2) - 2\,\rho\,s\,t}{2(1-\rho^2)}\Big)\,ds\,dt = \alpha^2.$$

Note that from *VaR* definition it holds $v_t = F_{R_{i,t}}(VaR^i_{\alpha,t}) = F_{R_{i,t}}(F^{-1}_{R_{i,t}}(\alpha)) = \alpha$. Since we work on a common significance level for *VaR* and *CoVaR* measures it also holds $\alpha = \beta$. In this regard, *CoVaR* is given by applying the probability integral transform on conditional quantile u_t, i.e., $CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}(u_t)$. In practice, this method for *CoVaR* computation is similar to Girardi and Ergün (2013) with the only difference that we work with a copula density function instead of a probability density function (*pdf*).

A.2 The Student-t Copula

The Student-t copula similar to Normal copula does not have a closed-form density. However, the *CoVaR* based on Adrian and Brunnermeier (2011) definition can be given in explicit form as

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(t_\nu\Big(\rho\,t_\nu^{-1}(v_t) + \sqrt{(1-\rho^2)(\nu+1)^{-1}(\nu + t_\nu^{-1}(v_t)^2)}\,t^{-1}_{\nu+1}(\beta)\Big)\Big).$$

In contrast, the *CoVaR* give the definition in Girardi and Ergün (2013) can be obtained numerically by solving the following numerical integral

$$\int_0^{u_t}\int_0^{v_t} \frac{1}{\sqrt{1-\rho^2}} \frac{\Gamma(\frac{\nu+2}{2})\Gamma(\frac{\nu}{2})}{\Gamma\Big(\frac{\nu+2}{2}\Big)^2} \frac{(1+\frac{s^2-2\,s\,t\,\rho+t^2}{\nu(1-\rho^2)})^{-\frac{\nu+1}{2}}}{(1+\frac{s^2}{\nu})^{-\frac{\nu+1}{2}}(1+\frac{t^2}{\nu})^{-\frac{\nu+1}{2}}}\,ds\,dt = \alpha\cdot\beta.$$

After solving for u_t and applying the probability integral transform, the *CoVaR* is given by

$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}(u_t)$. Similar to the Normal copula, we work on a common significance level for both risk metrics, i.e., $\alpha = \beta$, while $v_t = F_{R_{i,t}}(VaR^i_{\alpha,t}) = F_{R_{i,t}}(F^{-1}_{R_{i,t}}(\alpha)) = \alpha$, it holds from the definition of *VaR*.

B *CoVaR* derivation for Archimedean copulas

B.1 Clayton Copula

The Clayton copula is a member of the Archimedean copula family with dependence parameter $\theta \in (0, \infty)$ and generator function $\varphi = \frac{(u^{-\theta}-1)}{\theta}$. The perfect dependence is observed at $\theta \to \infty$ whereas $\theta \to 0$ implies independence. Clayton copula allows for modelling positive dependence and asymmetric (lower only) tail dependence. The distribution function is given by

$$C(u, v\,; \theta) = (u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}}.$$

Following the notation introduced in section 3.2, an explicit expression for $CoVaR^{=}_{\alpha,\beta,t}$ for the Clayton copula can be derived, that is

$$\frac{\partial C(u, v)}{\partial v} = \Big(1 + u^{\theta}(v^{-\theta} - 1)\Big)^{\frac{-(1+\theta)}{\theta}} = \beta. \tag{21}$$

Solving for u and applying the probability integral transform, $CoVaR^{=}_{\alpha,\beta,t}$ is obtained as follows

$$u^{=} \equiv u = \Big(1 + v^{-\theta} \cdot (\beta^{-\frac{\theta}{1+\theta}} - 1)\Big)^{-\frac{1}{\theta}},$$

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\bigg(\Big(1 + \alpha^{-\theta} \cdot (\beta^{-\frac{\theta}{1+\theta}} - 1)\Big)^{-\frac{1}{\theta}}\bigg). \tag{22}$$

Alternatively, using the general expression in equation (11) an explicit expression for $CoVaR_{\alpha,\beta,t}$ for the Clayton copula can be given as follows

$$\frac{C(u, v)}{v} = \beta,$$

$$\Big(u^{-\theta} + v^{-\theta} - 1\Big)^{-\frac{1}{\theta}} = v \cdot \beta.$$

Thus, solving for u and applying the probability integral transform, $CoVaR_{\alpha,\beta,t}$ can be obtained in a closed-form expression, that is

$$u^{\leq} \equiv u = \Big(1 + (v \cdot \beta)^{-\theta} - v^{-\theta}\Big)^{-\frac{1}{\theta}},$$

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\bigg(\Big(1 + (\alpha \cdot \beta)^{-\theta} - \alpha^{-\theta}\Big)^{-\frac{1}{\theta}}\bigg). \tag{23}$$

B.2 Frank Copula

This copula is also a member of the Archimedean copula family with dependence parameter $\theta \in (-\infty, \infty)\backslash\{0\}$ and generator function $\varphi = -\ln\big(\frac{e^{-\delta u}-1}{e^{-\delta}-1}\big)$. Frank copula allows for both positive and negative dependence structures, however, it does not imply tail dependence. The distribution function is given by

$$C(u, v\,; \delta) = -\frac{1}{\delta}\ln\Big(\frac{1}{1-e^{-\delta}}\Big[(1-e^{-\delta}) - (1-e^{-\delta u})(1-e^{-\delta v})\Big]\Big).$$

An analytical expression for $CoVaR^{=}_{\alpha,\beta,t}$ for this copula family can be derived as

$$CoVaR^{=}_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(-\frac{1}{\delta}\ln\Big(1-(1-e^{-\delta})\cdot\Big[1+e^{-\delta\alpha}\cdot(\beta^{-1}-1)\Big]^{-1}\Big)\Big). \tag{24}$$

In contrast, an explicit expression for $CoVaR_{\alpha,\beta,t}$ for the Frank copula is given as follows

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(-\frac{1}{\delta}\ln\Big[1-\frac{(1-e^{-\delta})-(1-e^{-\delta})(e^{-\delta\beta\alpha})}{(1-e^{-\delta\alpha})}\Big]\Big). \tag{25}$$

B.3 Gumbel Copula

The Gumbel copula with dependence parameter $\theta \in [1, \infty]$ and generator function $\varphi(t) = (-\log t)^{\theta}$ belongs also to the Archimedean copula family. Gumbel copula captures only positive dependence while it allows for asymmetric (upper only) tail dependence. For $\theta = 1$, Gumbel copula implies independence while the perfect positive dependence is observed as $\theta \to \infty$. The distribution function is given by

$$C(u, v\,; \theta) = \exp\Big(-\Big((-\log\, u)^{\theta} + (-\log\, v)^{\theta}\Big)^{\frac{1}{\theta}}\Big).$$

Unfortunately, the $\partial/\partial v\, C(u, v)$ of Gumbel copula is not partial invertible in its first argument u and hence we cannot derive an analytical expression for $CoVaR^{=}_{\alpha,\beta,t}$. However, an analytical expression for $CoVaR_{\alpha,\beta,t}$ can be given as follows

$$CoVaR_{\alpha,\beta,t} = F^{-1}_{s,t}\Big(\exp\Big(-\Big[\big(-\log(\alpha\cdot\beta)\big)^{\theta} - \big(-\log\,\alpha\big)^{\theta}\Big]^{\frac{1}{\theta}}\Big)\Big). \tag{26}$$

B.4 BB7 Copula

The BB7 copula, known as Joe-Clayton copula, is a two-parametric Archimedean copula family with $\theta \geq 1$ and $\delta > 0$. This copula family captures positive dependence while it allows also for asymmetric upper and lower tail dependence. In particular, the δ parameter measures lower tail dependence and the θ parameter measures upper tail dependence. Moreover, the Joe copula is the limiting case of BB7 for $\delta \to 0$ whereas for $\theta = 0$ one obtains the Clayton copula. The distribution function for this copula family is given by

$$C(u, v\,; \theta, \delta) = 1 - \Big(1 - \big[(1-(1-u)^{\theta})^{-\delta} + (1-(1-v)^{\theta})^{-\delta} - 1\big]^{-\frac{1}{\delta}}\Big)^{\frac{1}{\theta}}.$$

Analytical expressions for $CoVaR^{=}_{\alpha,\beta,t}$ and $CoVaR_{\alpha,\beta,t}$ can be obtain from the general solutions in equations (10) and (14) respectively, with

$$\varphi(v\,;\theta,\delta) = [1-(1-v)^{-\theta}]^{-\delta} - 1,$$

$$\varphi^{-1}(v\,;\theta,\delta) = 1 - [1-(1+v)^{-1/\delta}]^{1/\theta},$$

$$\varphi^{'}(v\,;\theta,\delta) = -[1-(1-v)^{\theta}]^{-\delta-1}\delta[-(1-v)^{\theta}\theta/(-1+v)].$$

C Dynamic Copula *CoVaR*

For the Clayton and Gumbel copulas the following parametric representation is proposed

$$\theta_t = \Lambda_1\Big(\omega + \beta\cdot\theta_{t-1} + \alpha\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|\Big),$$

where $\Lambda_1(x)$ is an appropriate transformation to ensure the parameter always remains in its domain: $\exp(x)$ for Clayton copula and $(\exp(x)+1)$ for the Gumbel. On the other hand, the parameter δ of Frank copula is defined in $[-\infty,\infty]\backslash\{0\}$ at all times. Thus, we employ the following evolution equation for this particular copula family

$$\delta_t = \omega + \beta\cdot\delta_{t-1} + \alpha\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|,$$

where the evolution of δ_t is constrained to ensure that remains in its domain. For the two-parametric Archimedean BB7 copula a similar parametric representation for each tail dependence coefficient is considered. The BB7 copula is constructed by taking a particular Laplace transformation of Clayton's copula. The BB7 copula distribution is given by

$$C(u,v\,;\theta,\delta) = 1 - \Big(1 - [(1-(1-u)^{\theta})^{-\delta} + (1-(1-v)^{\theta})^{-\delta} - 1]^{-\frac{1}{\delta}}\Big)^{\frac{1}{\theta}},$$

where $\theta = 1/\log_2(2-\tau^U)$, $\delta = -1/\log_2(\tau^L)$ and $\tau^U,\tau^L \in (0,1)$. Therefore, the following evolution equations can be considered for the BB7 copula

$$\tau_t^U = \Lambda_2\Big(\omega_U + \beta_U\cdot\tau_{t-1}^U + \alpha_U\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|\Big),$$

$$\tau_t^L = \Lambda_2\Big(\omega_L + \beta_L\cdot\tau_{t-1}^L + \alpha_L\cdot\frac{1}{10}\sum_{j=1}^{10}|u_{t-j}\cdot v_{t-j}|\Big),$$

where $\Lambda_2(x) \equiv (1+\exp(-x))^{-1}$ is the logistic transformation, used to keep τ^U and τ^L in $(0,1)$ at all times.

D List of European financial institutions

Bank	Datastream tickers	Country	Weight (%)	MCap (€ Bil.)
HSBC	HSBA	Great Britain	20.44	142.65
BCO SANTANDER	SCH	Spain	7.33	51.17
UBS	UBSN	Swiss	6.60	46.08
BNP PARIBAS	BNP	France	6.17	43.09
BARCLAYS	BARC	Great Britain	5.37	37.50
BCO BILBAO VIZCAYA ARGENTARIA	BBVA	Spain	4.95	34.56
STANDARD CHARTERED	STAN	Great Britain	4.64	32.38
DEUTSCHE BANK	DBK	Germany	4.43	30.92
LLOYDS BANKING GRP	LLOY	Great Britain	4.34	30.28
CREDIT SUISSE GRP	CSGN	Swiss	4.32	30.12
NORDEA BANK	NDA	Sweden	3.14	21.89
GRP SOCIETE GENERALE	SGE	France	3.00	20.92
UNICREDIT	UCG	Italy	2.82	19.69
INTESA SANPAOLO	ISP	Italy	2.44	17.00
SWEDBANK	SWED	Sweden	2.28	15.88
SVENSKA HANDELSBANKEN A	SVK	Sweden	2.11	14.69
SKANDINAVISKA ENSKILDA BK A	SEA	Sweden	1.59	11.07
DNB	DNB	Norway	1.49	10.38
DANSKE BANK	DAB	Denmark	1.21	8.41
CREDIT AGRICOLE	CRDA	France	1.02	7.15
ROYAL BANK OF SCOTLAND GRP	RBS	Great Britain	1.00	6.95
COMMERZBANK	CBK	Germany	0.89	6.21
KBC GRP	KB	Belgium	0.88	6.13
ERSTE GROUP BANK	ERS	Austria	0.86	6.00
BCO POPULAR ESPANOL	POP	Spain	0.56	3.93
BCO SABADELL	BSAB	Spain	0.55	3.85
NATIXIS	KNF	France	0.40	2.77
BANK OF IRELAND	BKIR	Ireland	0.37	2.55
POHJOLA BANK	POH	Finland	0.33	2.27
MEDIOBANCA	MB	Italy	0.32	2.23
JYSKE BANK	JYS	Denmark	0.29	2.03
BCO POPOLARE	BP	Italy	0.24	1.65
BCA POPOLARE EMILIA ROMAGNA	BPE	Italy	0.23	1.63
BCA MONTE DEI PASCHI DI SIENA	BMPS	Italy	0.20	1.43
BCO ESPIRITO SANTO	BES	Portugal	0.20	1.38
BCO COMERCIAL PORTUGUES	BCP	Portugal	0.19	1.31
NATIONAL BANK OF GREECE	ETE	Greece	0.18	1.29
BCA POPOLARE DI SONDRIO	BPSO	Italy	0.17	1.21
SYDBANK	SYD	Denmark	0.17	1.17
BCA POPOLARE DI MILANO	PMI	Italy	0.15	1.07
BANQUE CANTONALE VAUDOISE	BCV	Swiss	0.15	1.05
VALIANT	VATN	Swiss	0.15	1.03

Table D.1: This table lists the 42 out of 46 in total banks from 15 European countries belonging to STOXX 600 Banks Index and corresponding Datastream tickers, Market Capitalisation values and relative STOXX 600 Banks Index weights as of June, 2013. Source: STOXX Limited (www.stoxx.com).

E Systemic risk measure comparisons

This Appendix presents a graphical comparison of $CoVaR$ and $CoES$ measures given the alternative $CoVaR$ definitions in Adrian and Brunnermeier (2011) and Girardi and Ergün (2013), respectively. Figure E.1 shows dynamic $CoVaR^{=}_{\alpha,\beta,t}$ and $CoVaR_{\alpha,\beta,t}$ estimates, while Figure E.2 displays dynamic $CoES^{=}_{\alpha,\beta,t}$ and $CoES_{\alpha,\beta,t}$ estimates. All measures are generated by a Frank copula function with skewed-t margins conditional of HSBC returns. It is clear that the systemic risk estimates do not differ significantly from each other. Similar patterns are also observed for the rest of the pairs in our study.

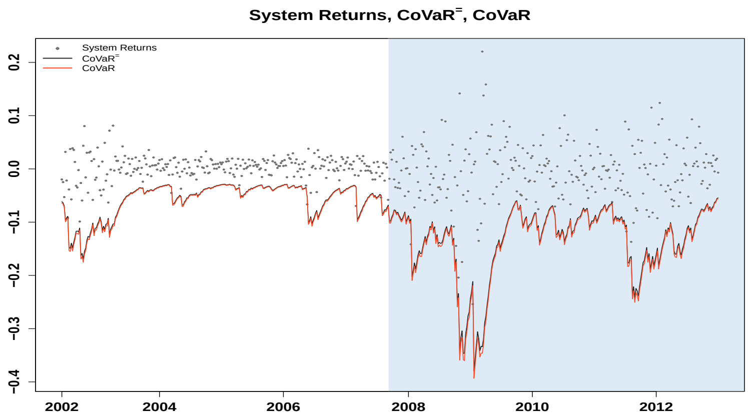

Figure E.1: This figure shows financial system returns (grey points), dynamic $CoVaR^{=}_{\alpha,\beta,t}$ (black line) and $CoVaR_{\alpha,\beta,t}$ (red line) estimates conditional on HSBC returns. Both systemic risk measure estimates are generated by a Frank copula function with skewed-t marginals. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

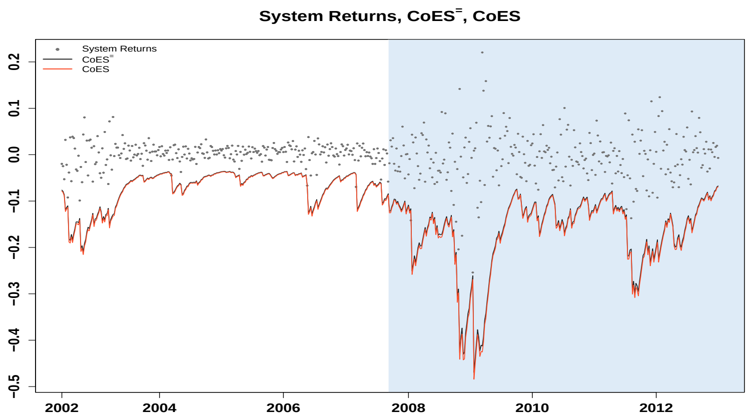

Figure E.2: This figure shows financial system returns (grey points), dynamic $CoES^{=}_{\alpha,\beta,t}$ (black line) and $CoES_{\alpha,\beta,t}$ (red line) estimates conditional on HSBC returns. Both systemic risk measure estimates are generated by a Frank copula function with skewed-t marginals. The light blue shaded area corresponds to Q3 2007 - Q4 2012 crisis period.

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