Cass Business School
CITY UNIVERSITY LONDON

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(Re)Defining and Managing Diversification

STUDY IT: www.symmys.com **(white papers and code)**

DO IT: Advanced Risk and Portfolio Management® Bootcamp
www.symmys.com/arpm-bootcamp

Standard approach: Modern Portfolio Theory

$$\boldsymbol{w}^* \equiv \underset{\boldsymbol{w} \in \mathcal{C}}{\operatorname{argmax}} (\mathbb{E}\{R\} - \lambda \mathbb{V}\{R\}).$$

Optimal Portfolio = optimal mean-variance weights under constraints

Standard approach: Modern Portfolio Theory

$$R = \sum_{n=1}^{\bar{n}} w_n R_n.$$

Portfolio Return = weighted average of asset returns

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$\mathbb{E}\{R\} = \boldsymbol{w}' \boldsymbol{\mu}_R$

$\mathbb{V}\{R\} = \boldsymbol{w}' \boldsymbol{\Sigma}_R \boldsymbol{w}$

$\bar{n} \times 1$ vector of expected asset returns

$\bar{n} \times \bar{n}$ matrix of asset covariances

Trend 1: From asset-based allocation to factor-based allocation

$$\max(\mathbb{E}\{R\} - \lambda \mathbb{V}\{R\}).$$

Optimal Portfolio = optimal mean-variance

Trend 1: From asset-based allocation to factor-based allocation

$$
\begin{aligned}
R &= \sum_{k=1}^{\bar{k}} b_k F_k + U, \\
&= \sum_{k=0}^{\bar{k}} b_k F_k, \\
&= \sum_k b_k F_k,
\end{aligned}
$$

Portfolio Return = Linear factor model (factors: momentum, value, …,PCA,… strategies,…)

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Portfolio Return = Linear factor model (factors: momentum, value, …,PCA,… strategies,…)

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\boldsymbol{b}^* \equiv \underset{\boldsymbol{b} \in \mathcal{C}}{\operatorname{argmax}}(\mathbb{E}\{R\} - \lambda \mathbb{V}\{R\}).
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Optimal Portfolio = optimal mean-variance exposures under constraints

Trend 1: From asset-based allocation to factor-based allocation

$$R = \sum_{k=1}^{\bar{k}} b_k F_k + U,$$
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$$\mathbb{E}\{R\} = \boldsymbol{b}' \boldsymbol{\mu}_F \qquad \mathbb{V}\{R\} = \boldsymbol{b}' \boldsymbol{\Sigma}_F \boldsymbol{b}$$

$\bar{k} \times 1$ vector of factor premia

$\bar{k} \times \bar{k}$ matrix of factor covariances

Trend 2: from mean-variance to risk parity, or diversification management

$$R = \sum_{n=1}^{\bar{n}} w_n R_n.$$

Portfolio Return = weighted average of asset returns

$$m_n \equiv \frac{1}{\mathbb{S}d\{R\}} \frac{\partial \mathbb{S}d\{R\}}{\partial w_n} = \frac{w_n [\boldsymbol{\Sigma}_R \boldsymbol{w}]_n}{\boldsymbol{w}' \boldsymbol{\Sigma}_R \boldsymbol{w}},$$

"contributions" to risk

Trend 2: from mean-variance to risk parity, or diversification management

$R = \sum_{n=1}^{\bar{n}} w_n R_n.$ Portfolio Return = weighted average of asset returns

$m_1 = m_2 = \cdots$ Optimal Portfolio = equal "contributions" to risk

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"contributions" to risk

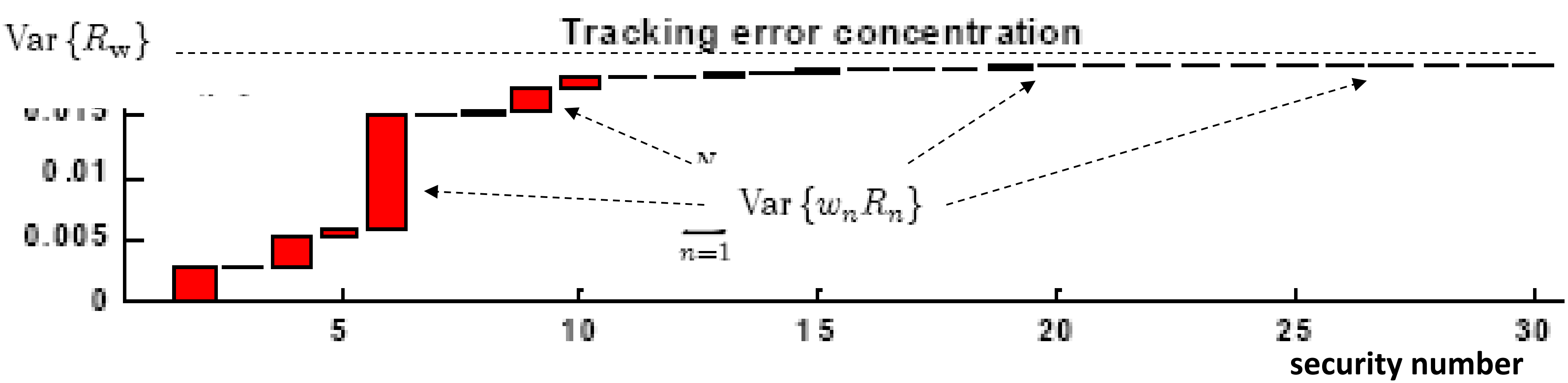
$\bar{n} \times \bar{n}$ matrix of asset covariances

$$R = \sum_{n=1}^{\bar{n}} w_n R_n.$$

Portfolio Return = weighted average of asset returns

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}. \qquad \mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}$$

if correlations = 0

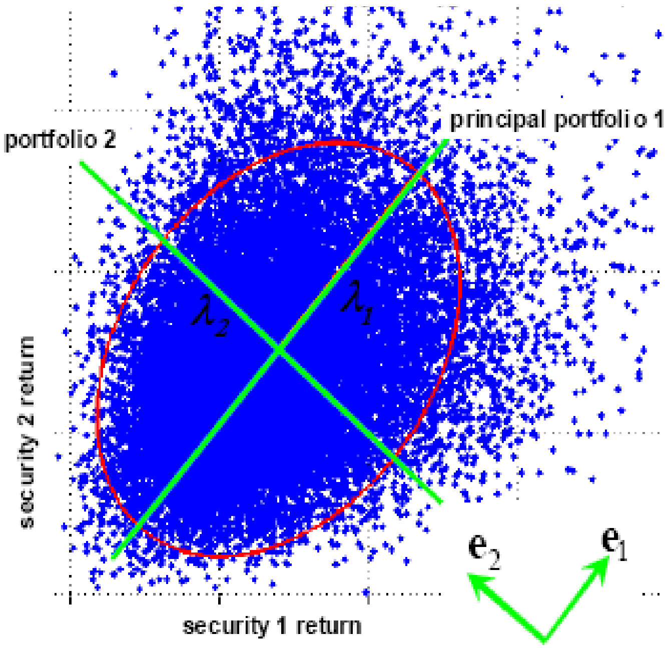


$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

$$\mathrm{Var}\{R_{\mathbf{w}}\} \neq \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}$$

if correlations ≠ 0

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$



$$\boldsymbol{\Sigma} \equiv \mathrm{Cov}\{\mathbf{R}\}$$

$$\downarrow$$

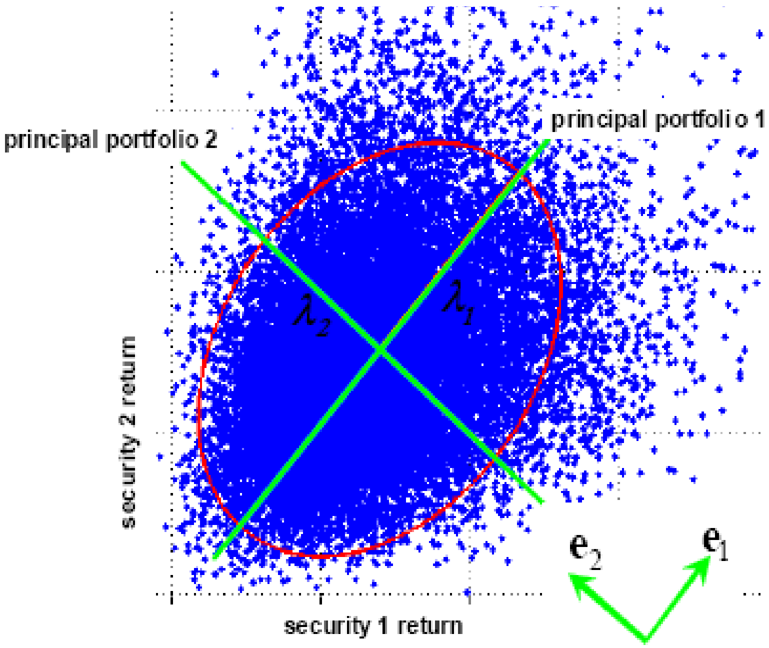
$$\boldsymbol{\Sigma} \equiv \mathbf{E}\,\boldsymbol{\Lambda}\,\mathbf{E}'$$

PCA

$$\mathbf{E} \equiv (\mathbf{e}_1, \ldots, \mathbf{e}_N)$$ eigenvectors

$$\boldsymbol{\Lambda} \equiv \mathrm{diag}\left(\lambda_1^2, \ldots, \lambda_N^2\right)$$ eigenvalues

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$



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$\Updownarrow$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \{\mathbf{e}'\mathbf{\Sigma}\mathbf{e}\}$$

uncorrelated, maximum variance portfolios

$$\mathbf{e}'\mathbf{\Sigma}\mathbf{e}_j \equiv 0, \text{ for all existing } \mathbf{e}_j$$

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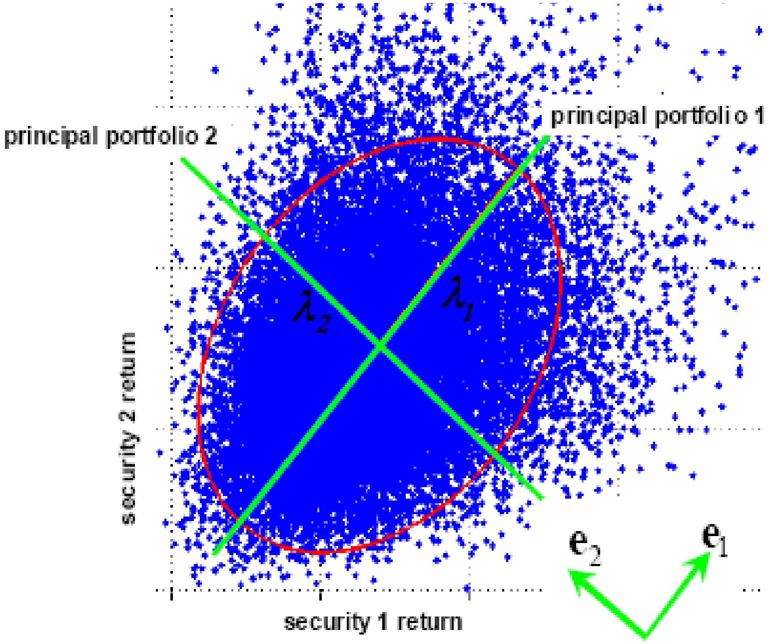
$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$



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eigenvalues

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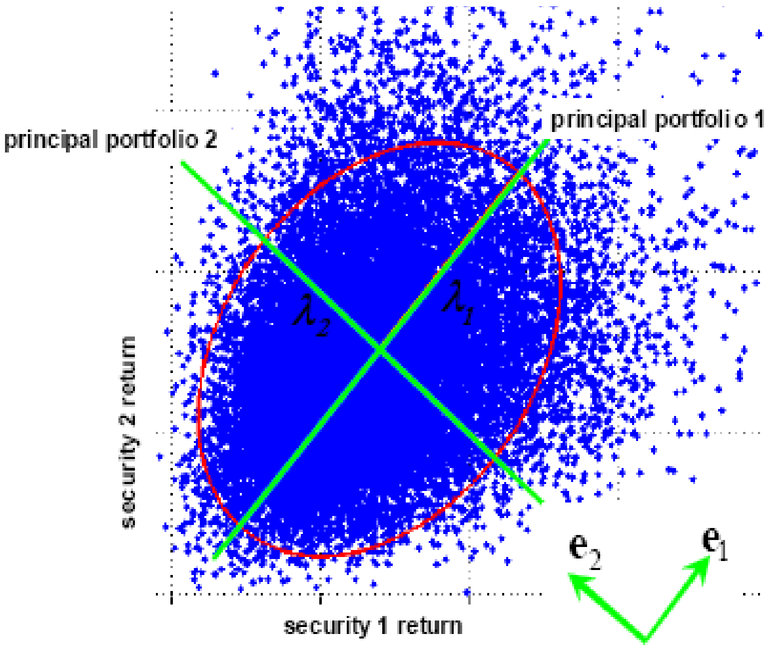
variances of uncorrelated, maximum variance portfolios

$$\lambda_n^2 \equiv \mathrm{Var}\{\mathbf{e}_n'\mathbf{R}\}$$

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$



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$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \{\mathbf{e}'\boldsymbol{\Sigma}\mathbf{e}\}$$ principal portfolios

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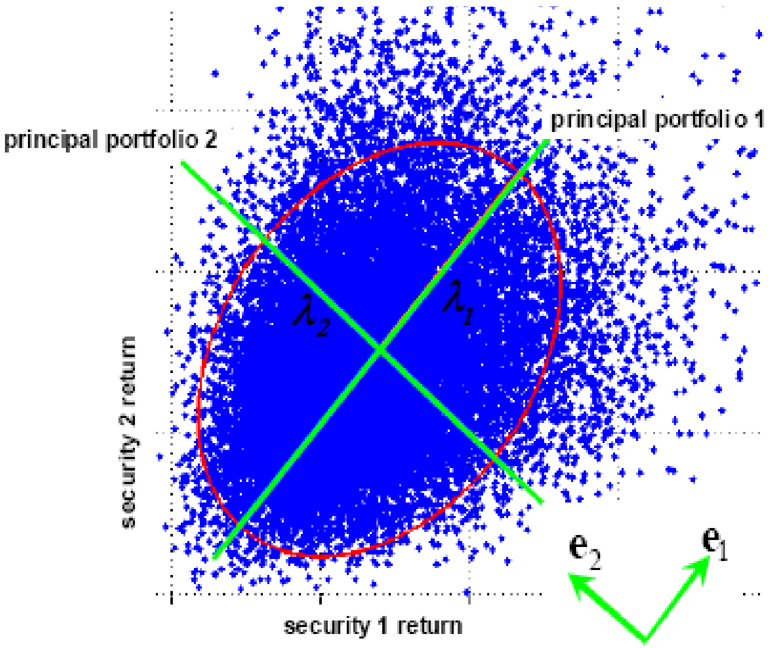
$\Updownarrow$

$$\lambda_n^2 \equiv \mathrm{Var}\{\mathbf{e}_n'\mathbf{R}\}$$ principal variances

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$



$$\boldsymbol{\Sigma} \equiv \mathrm{Cov}\{\mathbf{R}\}$$

$$\downarrow$$

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PCA

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principal portfolios

$$\boldsymbol{\Lambda} \equiv \mathrm{diag}\left(\lambda_1^2, \ldots, \lambda_N^2\right)$$

principal variances

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

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$\widetilde{\mathbf{R}} \equiv \mathbf{E}^{-1}\mathbf{R}$ return of principal portfolios

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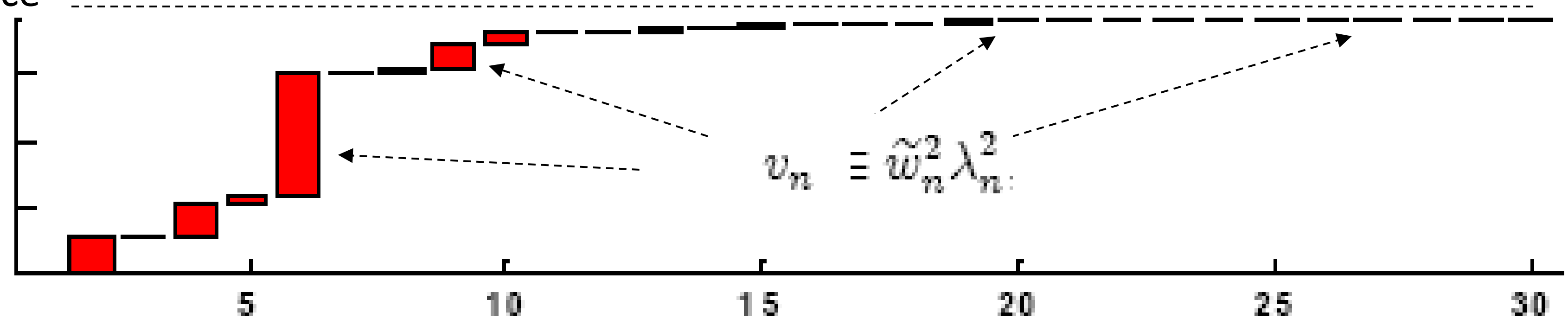
total variance

variance concentration curve



principal portfolio number

$\widetilde{\mathbf{R}} \equiv \mathbf{E}^{-1}\mathbf{R}$ return of principal portfolios

$\widetilde{\mathbf{w}} \equiv \mathbf{E}^{-1}\mathbf{w}$. weights of original portfolio on principal portfolios

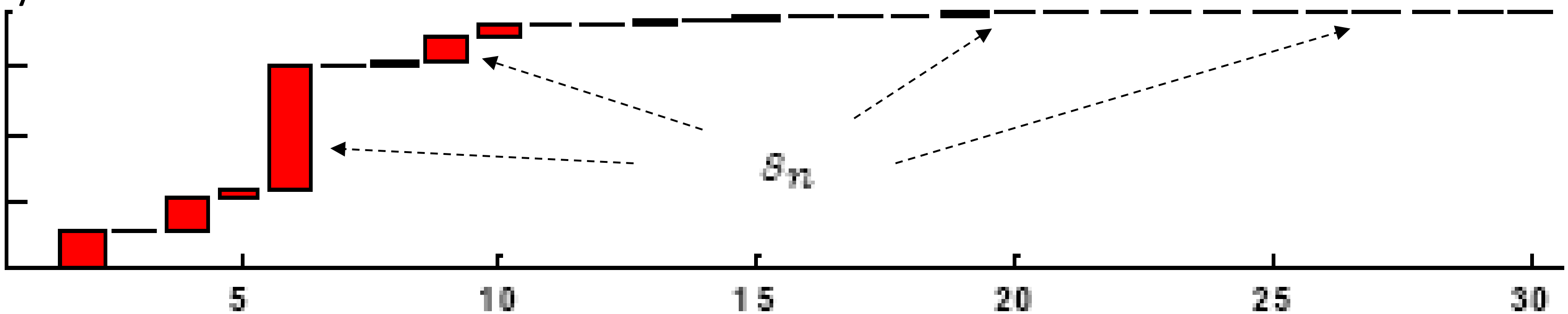
$$R_{\mathbf{w}} \equiv \widetilde{\mathbf{w}}'\widetilde{\mathbf{R}}.$$

$v_n \equiv \widetilde{w}_n^2 \lambda_n^2$ variance concentration curve

contribution to original portfolio variance from n-th principal portfolio:

$$\mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} v_n$$

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$



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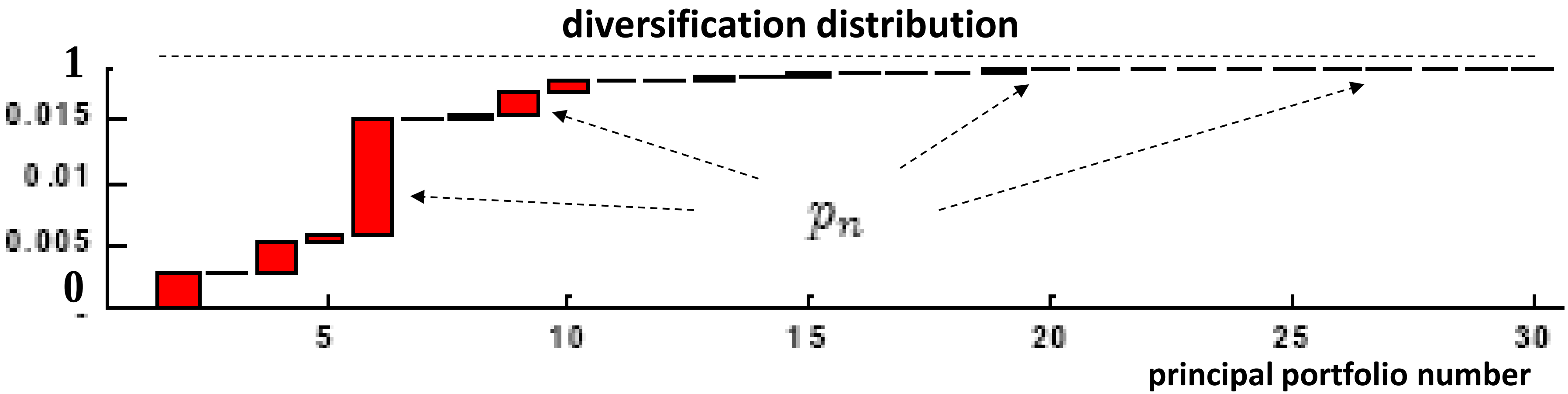
$\Updownarrow$

$s_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Sd}\{R_{\mathbf{w}}\}}$ volatility concentration curve

contribution to original portfolio volatility from n-th principal portfolio: "hot spots"

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$



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$\Updownarrow$

$p_n \equiv \frac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}}$ diversification distribution

contribution to original portfolio r-square from n-th principal portfolio

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⇕

$s_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Sd}\{R_{\mathbf{w}}\}}$ volatility concentration curve

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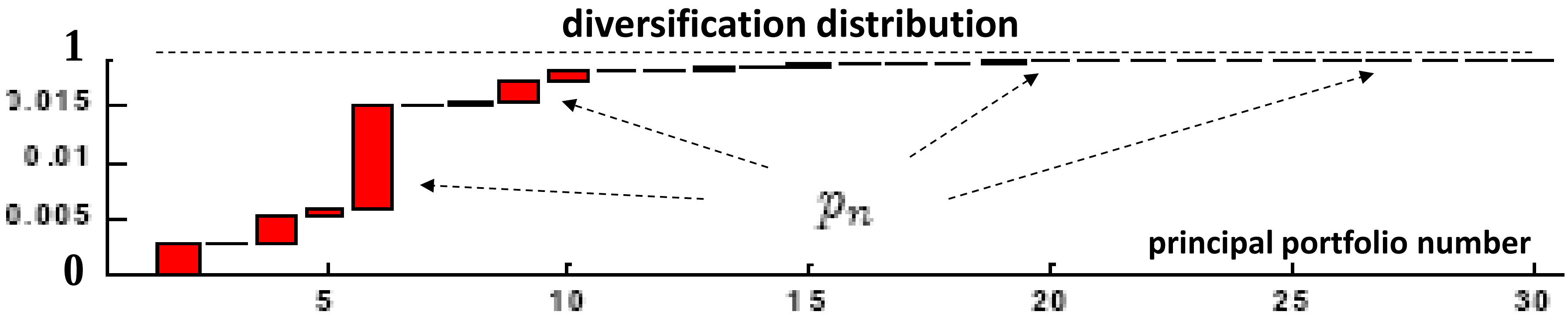

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$p_n \equiv \dfrac{\tilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}}$ diversification distribution: "probability mass"

effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

$\Updownarrow$

diversification

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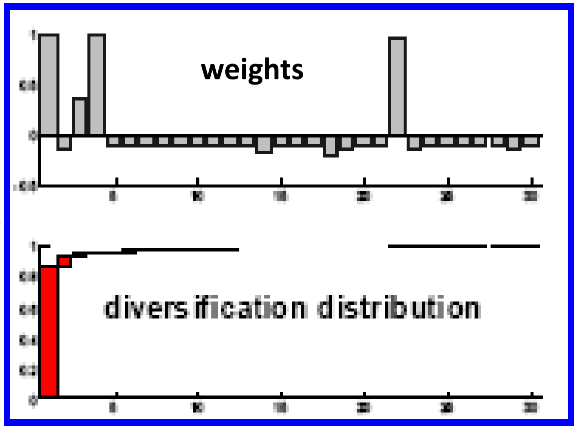
full concentration $\mathcal{N}_{Ent} \approx 1$



$$p_n \equiv \frac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathrm{w}}\}}$$ diversification distribution: “probability mass”

effective number of bets

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full concentration $\mathcal{N}_{Ent} \approx 1$

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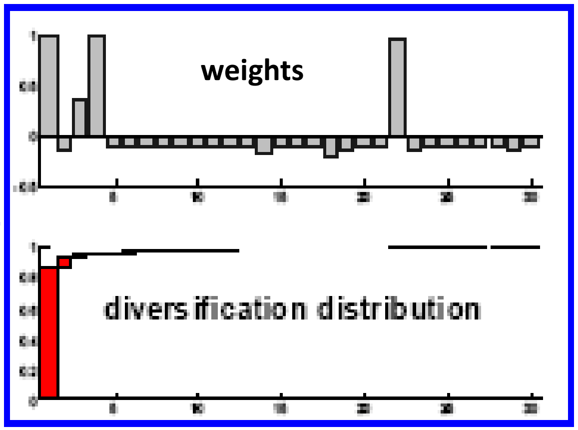

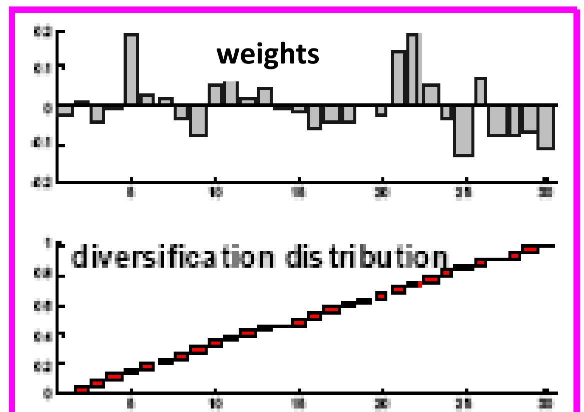

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full concentration $\mathcal{N}_{Ent} \approx 1$

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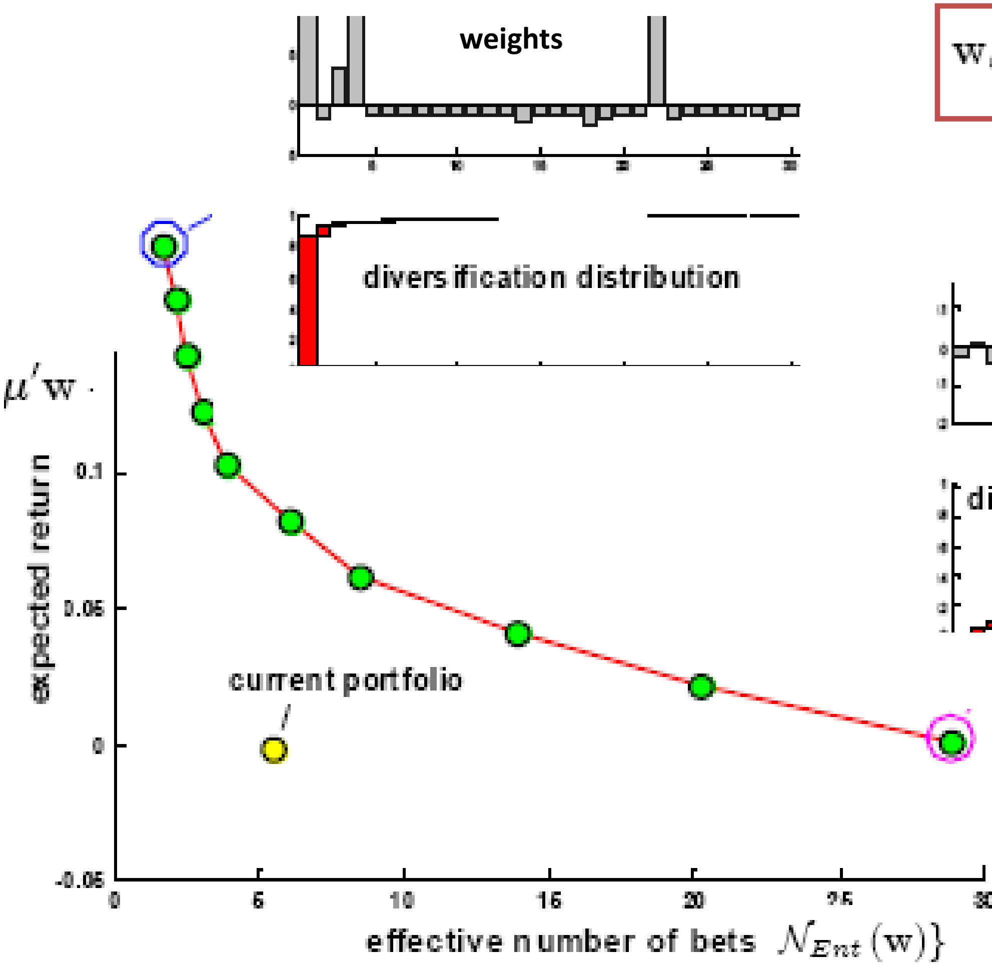
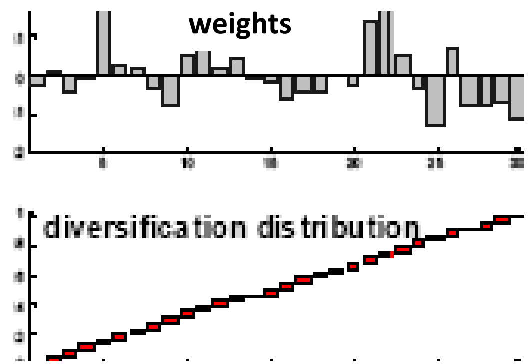
mean-diversification frontier

$$\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w}\in\mathcal{C}}{\operatorname{argmax}} \left\{\varphi \mu' \mathbf{w} + (1-\varphi)\,\mathcal{N}_{Ent}(\mathbf{w})\right\}$$





Assets		Factors
Modern Portf. Theory	→	Factor-Based Allocation

	Assets
Premia	Modern Portf. Theory
↓ Parity	{ 1) Marg. Contribs 2) Effective Num. Bets

	Assets	Factors
Premia	Modern Portf. Theory	Factor-Based Allocation
Parity	1) Marg. Contribs 2) Effective Num. Bets	**???**

	Assets	Factors
Premia	Modern Portf. Theory	Factor-Based Allocation
Parity	{ 1) Marg. Contribs 2) Effective Num. Bets	**Factor-based Risk Parity**

	Assets	Factors
Premia	Modern Portf. Theory	Factor-Based Allocation
Parity	1) Marg. Contribs 2) Effective Num. Bets	1) Marg. Contribs 2) Factors on Demand + Minimal Torsion Bets

	Assets	Factors
Premia	Modern Portf. Theory	Factor-Based Allocation
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i) Factors on Demand $R = \sum_{k=1}^{\bar{k}} b_k F_k + U$, best portfolio-specific linear factor model

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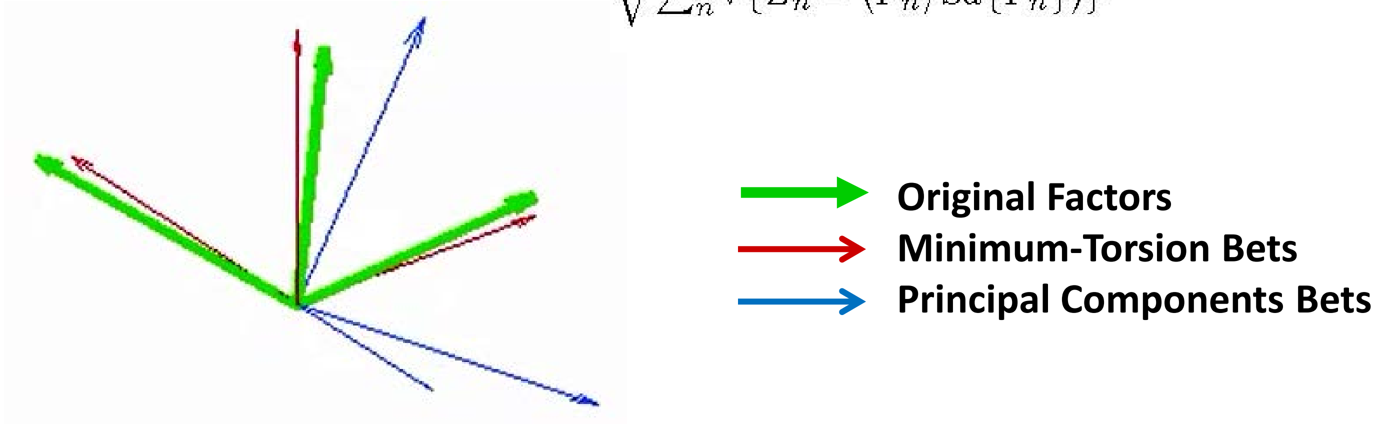
i) Factors on Demand $R = \sum_{k=1}^{\bar{k}} b_k F_k + U$, best portfolio-specific linear factor model

ii) Minimal Torsion Bets: $\mathring{\boldsymbol{F}}_{MT} \equiv \mathring{\boldsymbol{t}}_{MT} \boldsymbol{F}$, uncorrelated factors closest to factors

$$\mathring{\boldsymbol{t}}_{MT} = \underset{\mathbb{C}r\{\boldsymbol{tF}\} = \boldsymbol{i}_{\bar{n}\times\bar{n}}}{\operatorname{argmin}} \; NTE\{\boldsymbol{tF}, \boldsymbol{F}\}$$

$$NTE\{\boldsymbol{tF}, \boldsymbol{F}\} = \sqrt{\sum_n \mathbb{V}\{Z_n - (F_n/\mathbb{S}d\{F_n\})\}}$$



	Assets	Factors
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Parity	1) Marg. Contribs 2) Effective Num. Bets	1) Marg. Contribs 2) Factors on Demand + Minimal Torsion Bets

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ii) Minimal Torsion Bets: $\mathring{\boldsymbol{F}}_{MT} \equiv \mathring{\boldsymbol{t}}_{MT} \boldsymbol{F}$, <u>uncorrelated</u> factors closest to factors

$$\mathring{\boldsymbol{t}}_{MT} = \underset{\mathbb{C}r\{\boldsymbol{t}\boldsymbol{F}\} = \boldsymbol{i}_{\bar{n}\times\bar{n}}}{\operatorname{argmin}} NTE\{\boldsymbol{t}\boldsymbol{F}, \boldsymbol{F}\} \approx dg(\boldsymbol{\sigma}_{F})\, \boldsymbol{c}^{-1}\, dg(\boldsymbol{\sigma}_{F})^{-1}$$

$NTE\{\boldsymbol{t}\boldsymbol{F}, \boldsymbol{F}\} = \sqrt{\sum_n \mathbb{V}\{Z_n - (F_n/\mathbb{S}d\{F_n\})\}}$

$\boldsymbol{c}$: **Riccati root of correlation**

	Assets	Factors
Premia	Modern Portf. Theory	Factor-Based Allocation
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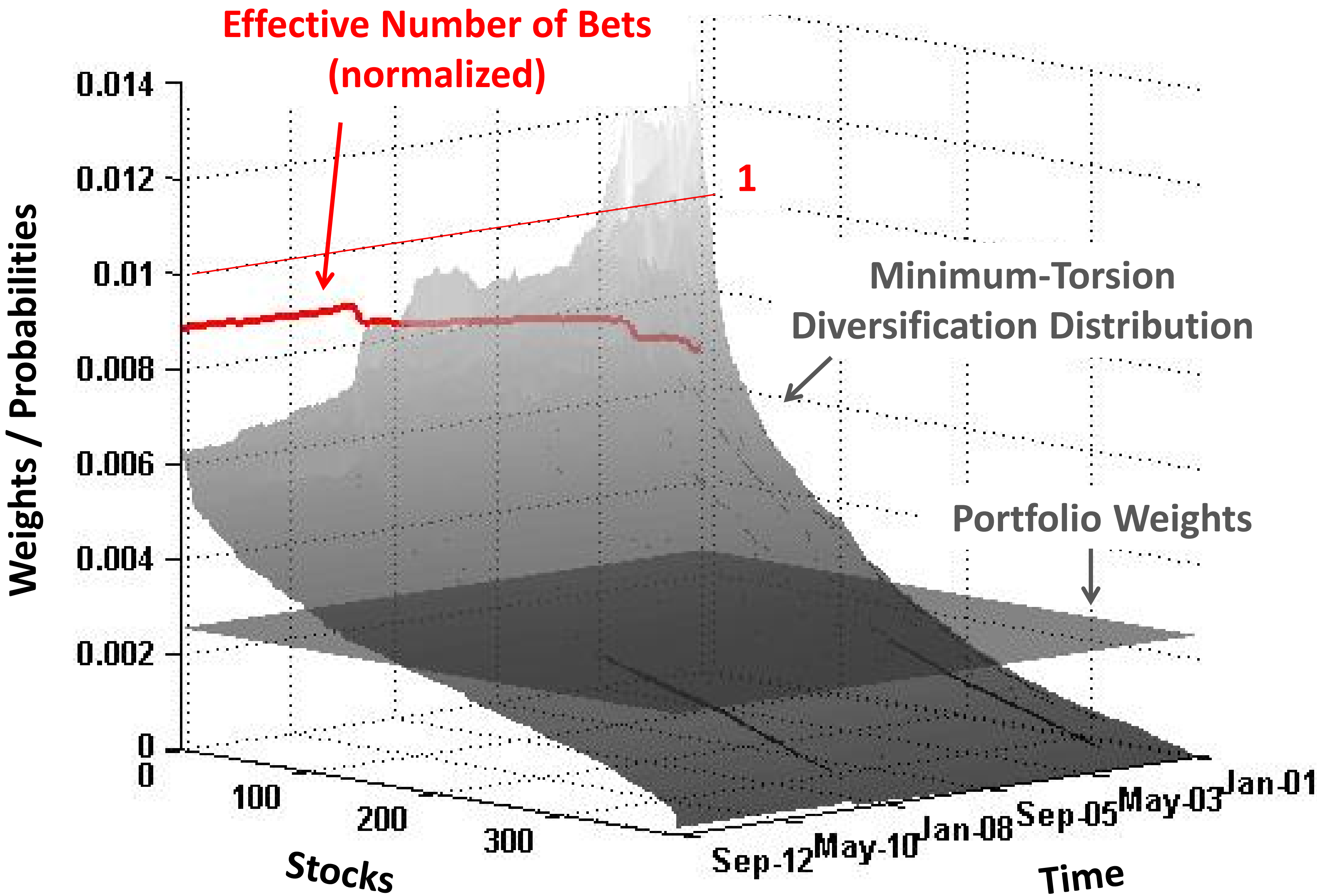
$NTE\{\boldsymbol{tF},\boldsymbol{F}\} = \sqrt{\sum_n \mathbb{V}\{Z_n - (F_n/\mathbb{S}d\{F_n\})\}}$

$\boldsymbol{c}^{-1}$: **Riccati root of correlation**

ii) Diversification Distribution and Effective Number of Minimum Torsion Bets

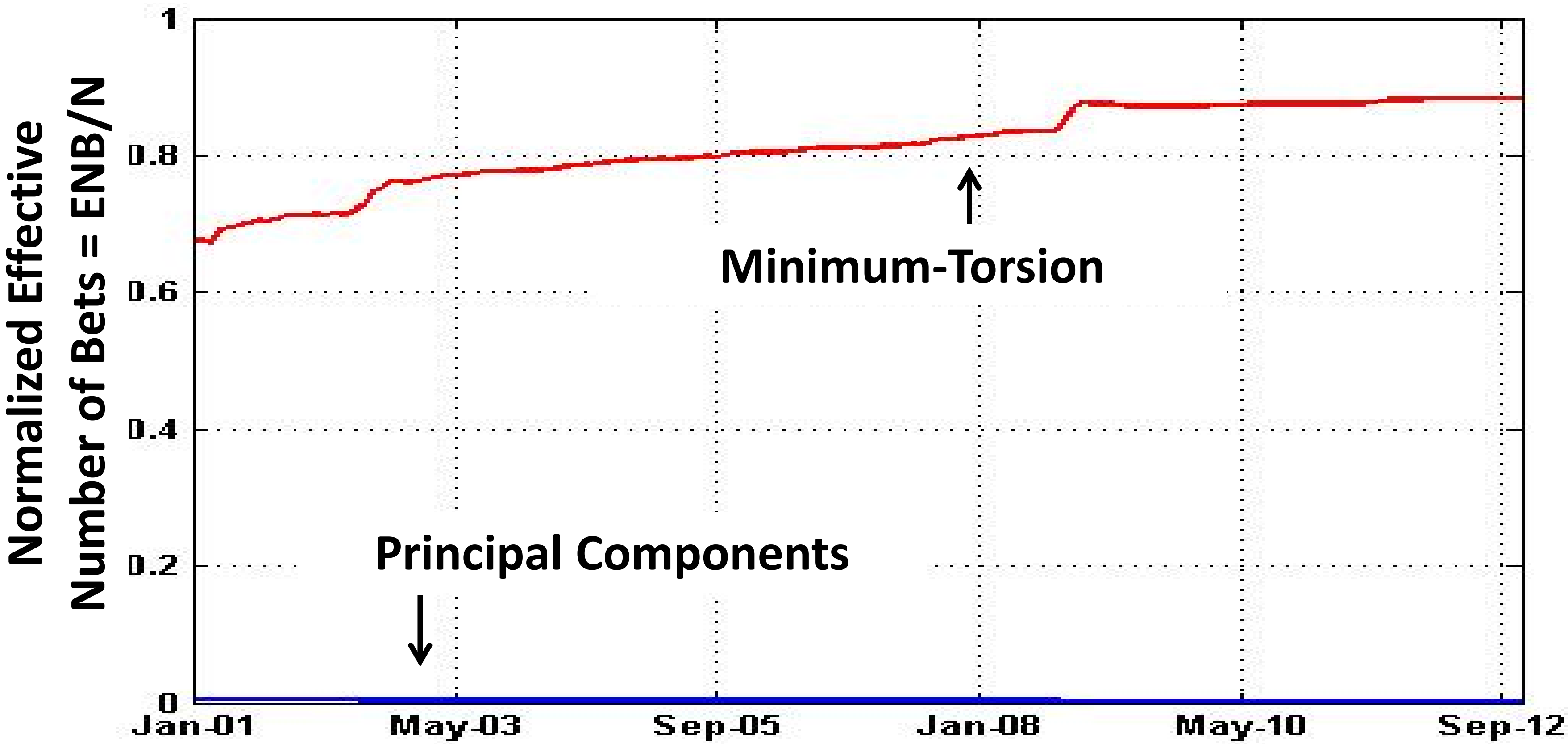
$$\boldsymbol{p}_{MT}(\boldsymbol{b}) = \frac{(\mathring{\boldsymbol{t}}_{MT}'^{-1}\boldsymbol{b}) \circ (\mathring{\boldsymbol{t}}_{MT}\boldsymbol{\Sigma}_F\boldsymbol{b})}{\boldsymbol{b}'\boldsymbol{\Sigma}_F\boldsymbol{b}} \quad \Rightarrow \quad \mathbb{N}_{MT}(\boldsymbol{b}) = e^{-\boldsymbol{p}_{MT}(\boldsymbol{b})'\ln \boldsymbol{p}_{MT}(\boldsymbol{b})}$$

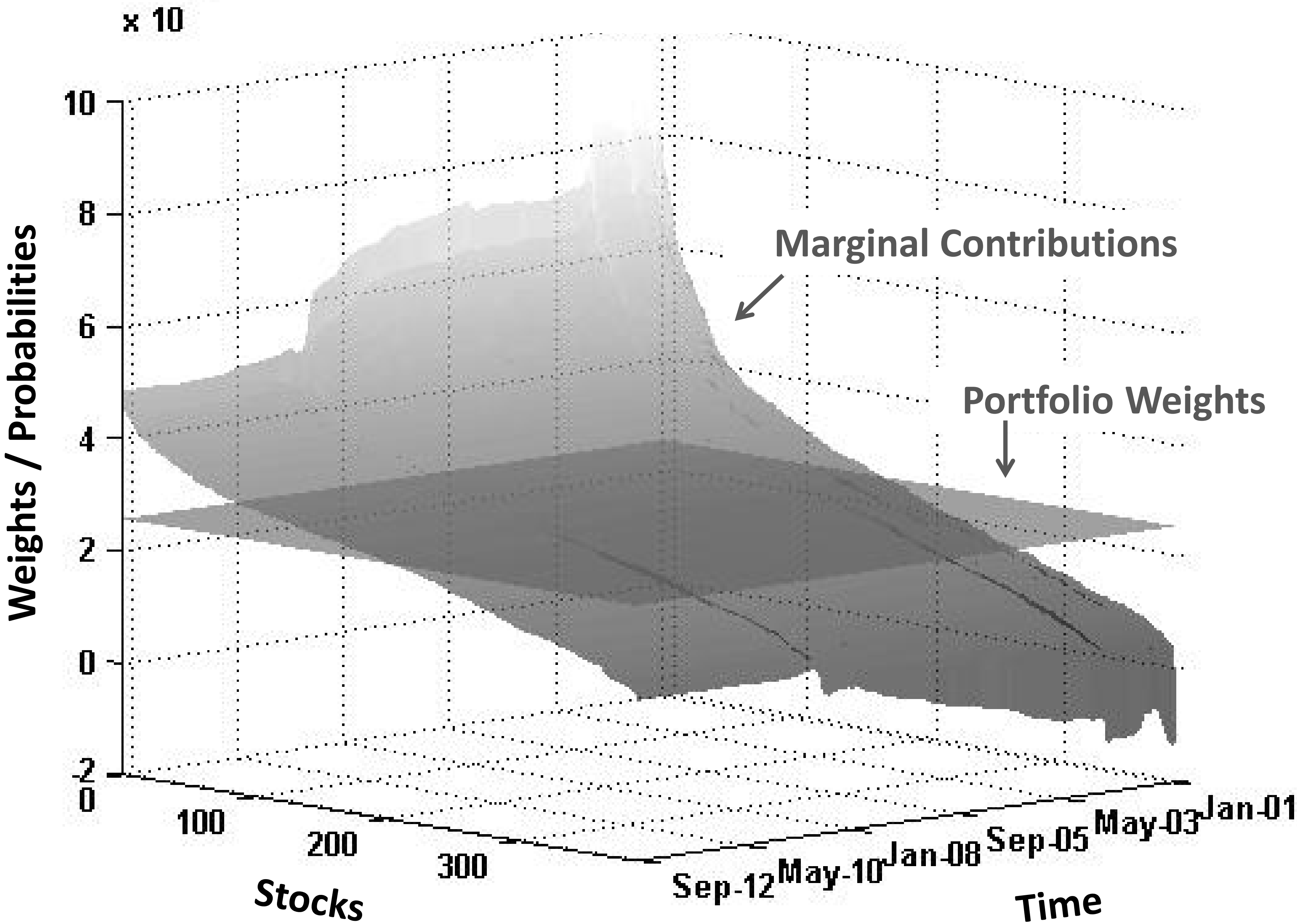


Effective Number of Bets
(normalized)
1
Minimum-Torsion
Diversification Distribution
Portfolio Weights
Weights / Probabilities
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
100
200
300
Stocks
Sep-12
May-10
Jan-08
Sep-05
May-03
Jan-01
Time

The Effective Number of PCA Bets in the S&P500 is close to 1, since the first PCA factor loadings are similar to the weights of the stocks in the S&P500

The Effective Number of Minimal Torsion Bets in the S&P500 yields intuitive results




x 10
-3
10
8
6
4
2
0
-2
Weights / Probabilities
Marginal Contributions
Portfolio Weights
0
100
200
300
Stocks
Sep-12
May-10
Jan-08
Sep-05
May-03
Jan-01
Time

	Assets	Factors
Premia	Modern Portf. Theory	Factor-Based Allocation
Parity	{ 1) Marg. Contribs 2) Effective Num. Bets	{ 1) Marg. Contribs 2) Factors on Demand + Minimal Torsion Bets

	Traditional	**Effective Number of Bets**
Risk contrib.	Marginal Contributions	Diversification Distributions
Expression	$\boldsymbol{m} \equiv \frac{\boldsymbol{b} \circ (\boldsymbol{\Sigma}_F \boldsymbol{b})}{\boldsymbol{b}' \boldsymbol{\Sigma}_F \boldsymbol{b}}$	$\boldsymbol{p} \equiv \frac{(\mathring{\boldsymbol{t}}_{MT}^{\prime -1} \boldsymbol{b}) \circ (\mathring{\boldsymbol{t}}_{MT} \boldsymbol{\Sigma}_F \boldsymbol{b})}{\boldsymbol{b}' \boldsymbol{\Sigma}_F \boldsymbol{b}}$
Meaning	spurious contributions from original factors	proper contributions from Minimum-Torsion Bets
Properties	$\sum_n m_n = 1, \quad m_n \lesseqgtr 0$	$\sum_k p_n = 1, \quad p_n \geq 0$

Effective Number of Bets: http://symmys.com/node/199

Factors on Demand: http://symmys.com/node/164

Minimal Torsion Bets: http://symmys.com/node/599