Mixed oligopoly and collusion

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Abstract

We introduce a firm with a partial ownership by the public sector in a dynamic model of collusion between private firms. We show that increasing the public ownership of the non-colluding firm may help collusion between the private firms.

JEL codes: L13

Keywords: mixed oligopolies; collusion.

1. Introduction

This article considers the case of a mixed oligopoly (that is, an industry where private firms co-exist with a public or semi-public firm) in a dynamic model of collusion. Starting from the observation that colluding firms often operate in industries where public firms act as well, we investigate the impact of a public firm on the conditions for the sustainability of collusion between the private firms. While the literature on collusion is rich, the role played by a public firm in this context has not been investigated yet by scholars or practitioners.

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1 The vast literature on mixed oligopolies dates back to Merril and Schneider (1966). More recent contributions are Matsumura (1998) and Matsumura and Ogawa (2012).

2 For example, a recent antitrust case handled by the Italian Antitrust Authority (ref. I743-AGCM) involved four independent private firms carrying passengers to and from Sardinia, and competing against a public-owned company that did not participate the collusive agreement.
We develop a model of differentiated products and collusion in a dynamic game. There are two private firms that may collude, and there is another firm that cannot collude. The non-colluding firm may have various degrees of public ownership. We ask whether increasing the public ownership of the non-colluding firm helps or harms collusion between private firms. We obtain that, unless products are perfect substitute, increasing the degree of public ownership of the non-colluding firm may help collusion between the private firms, and this always occurs when products are weak substitute. This happens because the more the non-colluding firm is “public”, the more it wants to expand the industry output in order to increase the welfare, thus making deviation from the cartel less profitable and punishment (in case of deviation) harsher. Further, we show that with a semi-public non-colluding firm the relationship between product substitutability and collusion sustainability is “wave-shape”: negative-positive-negative.

2. The model

Consider a three quantity-setting firms model. Firm 1 and Firm 2 try to set up a cartel, while Firm 3 cannot participate. Marginal costs are constant and normalized to zero. We extend Escrihuela-Villar (2008) by assuming that the non-colluding firm may be a public or semi-public firm. The demand function for product $j = \{1, 2, 3\}$ is:

$$p_j = 1 - q_j - \gamma \sum_{i \neq j} q_i,$$

where $p_j$ and $q_j$ is the price and the quantity for product $j$, respectively, and $\gamma \in [0, 1]$ represents the degree of product substitutability: when $\gamma$ is low (high) the goods are weak (strong) substitutes. Allegedly, the welfare function (consumer surplus plus total profits) is (see Hackner, 2000):

$$W = \sum_{j=1}^{3} q_j - \frac{1}{2} \sum_{j=1}^{3} q_j^2 - \gamma \sum_{j \neq z} q_j q_z.$$  

Firm $k = \{1, 2\}$ maximizes $\pi_k = p_k q_k$; Firm 3 maximizes: $\Gamma = \theta W + (1 - \theta) p_3 q_3$, where $\theta$ measures the degree of public ownership of Firm 3: when $\theta = 1$, Firm 3 is a pure public firm, when $\theta = 0$ it is a pure private firm, when $\theta \in (0, 1)$ it is a semi-public firm. As there are both cartel and non-cartel firms, as

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3 All results can be generalized to the case of increasing marginal costs. See footnote 10.
4 Further, we introduce produce differentiation.
5 See Matsumura (1998). To guarantee interior solutions, when $\gamma \leq 1/2$ we assume that $\theta \leq 0.993$. Therefore, in what follows we consider $\theta \in [0, 1]$ if $\gamma \geq 1/2$, and $\theta \in [0, 0.993]$ if $\gamma \leq 1/2$. 
usual in this literature we assume that in each period the cartel behaves as a Stackelberg leader whereas the non-colluding firm is the follower.\textsuperscript{6}

Denote by $\delta$ the common market discount factor. We consider a grim-trigger strategy in case of a deviation (Friedman, 1971).\textsuperscript{7} Collusion is sustainable as a sub-game-perfect-equilibrium if and only if the discounted value of the profits when remaining into the cartel exceeds the discounted value of cheating. The incentive-compatibility-constraint can be written as (Deneckere, 1983):

$$\delta \geq \delta^* = \frac{\pi^D_k - \pi^C_k}{\pi^D_k - \pi^P_k}$$

where $\pi^D_k$, $\pi^C_k$ and $\pi^P_k$ indicates deviation, collusive and punishment profits of Firm $k = \{1, 2\}$, respectively. The greater is $\delta^*$ the smaller is the set of market discount factors supporting collusion (i.e. collusion is more difficult to sustain), and vice-versa.

3. The impact of public-ownership on collusion sustainability

Suppose Firm 1 and 2 form a cartel, and each produces $q^C_k$. Maximizing $\Gamma$ with respect to $q_3$ given $q^C_k$, Firm 3’s best-reply is: $q_3(q^C_k) = \frac{1-2q^C_k}{2-\theta}$. Using $q_3(q^C_k)$, the optimal quantity of a cartel’s member is: $q^C_k = \frac{2-\gamma-\theta}{2[2(1+\gamma-\gamma^2)-\theta(1+\gamma)]}$. Collusive profits are:

$$\pi^C_k = \frac{(2-\gamma-\theta)^2}{4(2-\theta)[2(1+\gamma-\gamma^2)-\theta(1+\gamma)]}$$

\textsuperscript{6} See Escrihuela-Villar (2008) for details on this assumption and later Section 4.
\textsuperscript{7} More sophisticated punishment mechanisms may exist. In particular, penal codes as in Abreu (1986) may be preferred. However, the two-phase penal code of Abreu (1986) cannot be easily transferred to the present model, as its optimality is shown under the assumption of simultaneous moves.
Suppose now that a cartel’s member cheats. After deviation, the cartel breaks up, and the private firms lose the leadership.\(^8\) Therefore, we have a static Cournot game with firms having different objective functions. The equilibrium quantity of Firm \(k = \{1, 2\}\) is: \(q_*^k = \frac{2 - \gamma - \theta}{(2 - \theta)(2 + \gamma) - 2\gamma^2}\), yielding:

\[
\pi_*^k = \frac{(2 - \gamma - \theta)^2}{[2(2 + \gamma - \gamma^2) - \theta(2 + \gamma)]^2},
\] (3)

Finally, consider deviation profits. The one-period gain from deviation is: \(\pi_*^D = \{1 - q_*^k - \gamma[q_*^k(q_*^C + q_*^N)]q_*^D\}.\) Maximizing with respect to \(q_*^D\) yields

\[
q_*^D = \frac{(2 - \gamma - \theta)[(2 + \gamma)(2 - \theta) - 2\gamma^2]}{4(2 - \theta)[2(1 + \gamma - \gamma^2) - \theta(1 + \gamma)]}
\]

and:

\[
\pi_*^D = \frac{(2 - \gamma - \theta)^2[2(2 + \gamma - \gamma^2) - \theta(2 + \gamma)]^2}{16(2 - \theta)^2(2 - \theta + 2\gamma - \gamma\theta - 2\gamma^2)^2}.
\] (4)

Using (2), (3) and (4) into (1), we have:

\[
\delta^*(\theta, \gamma) = \frac{[2(2 + \gamma - \gamma^2) - \theta(2 + \gamma)]^2}{\theta^2(8 + \gamma + \gamma^2) - 4\theta(8 + 8\gamma - 3\gamma^2 - \gamma^3) + 4(8 + 8\gamma - 7\gamma^2 - 2\gamma^3 + \gamma^4)}.
\] (5)

The next proposition illustrates the impact of public ownership of the non-colluding firm on collusion sustainability between private firms:

**Proposition 1.** \(\frac{\partial \delta^*}{\partial \theta} \geq 0\) if \(\theta \leq (\geq) \hat{\theta} = 2 - 2\gamma\).

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\(^8\) Here we refer to Barcena-Ruiz and Garzòn (2010, Propositions 2-3-4) that show that in case of mixed oligopoly with equal marginal costs there is always an equilibrium where all firms produce simultaneously. However, in mixed oligopoly literature it is often considered the case of sequential timing where (non-colluding) private firms move first (Pal, 1998). Therefore, we have also considered the case of a sequential play during punishment. Proposition 1 (see later) is completely unaffected, whereas Proposition 2 slightly modifies (see footnote 10).
Proposition 1 indicates that increasing the public ownership of the non-colluding firm may increase or decrease collusion sustainability between private firms. The intuition is the following. When considering (2), (3) and (4), it can be observed that they decrease with $\theta$. Indeed, the more the non-colluding firm is “public”, the more it produces in order to increase welfare. This has contrasting effects in terms of collusion sustainability. On one hand, collusion profits are lower, and this reduces collusion sustainability; but, on the other hand, deviation profits and punishment profits are lower too, and this increases collusion sustainability. In our model, when products are weak substitutes or public ownership is small, the cartel is able to internalize most of the non-cartel firm’s reaction: thus, the pro-collusive effect during the punishment and the deviation phase dominates. The opposite holds when products are strong substitutes or public ownership is large. In more general terms, the increase of the industry output caused by the public ownership of a non-cartel firm may help collusion, by making punishment harsher and deviation less profitable.

The next proposition illustrates the impact of product substitutability:

**Proposition 2.** $\frac{\partial \delta^*}{\partial \gamma} \leq 0$ if $\gamma \in [\gamma_1, \gamma_2]$, and $\frac{\partial \delta^*}{\partial \gamma} \geq 0$ if $\gamma \leq \gamma_1$ or $\gamma \geq \gamma_2$, where $\gamma_1 \equiv \frac{2-\theta}{4}$ and $\gamma_2 \equiv \frac{2-\theta}{2}$.

Proposition 2 shows the existence of a “wave-shape” (negative-positive-negative) relationship between collusion sustainability and product substitutability, a result which is new in literature (see Colombo, 2013). Indeed, all profits decrease with product substitutability. The reduction of deviation and punishment profits has a pro-collusive effect, whereas the reduction of collusive profits has an anti-collusive effect. While the

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9 When $\gamma \leq 1/2$, $\delta^*$ strictly decreases with $\theta$; instead, $\delta^*$ strictly increases with $\theta$ only when $\gamma = 1$.

10 With sequential moves during punishment, Proposition 2 becomes: $\frac{\partial \delta^*}{\partial \gamma} \leq (\geq 0$ if $\gamma \leq (\geq \gamma_2)$. On the other hand, with a quadratic cost function $cq^2/2$, the thresholds become: $\hat{\theta} = 2 + c - 2\gamma$, $\gamma_1 = (2 + c - \theta)/4$ and $\gamma_2 = (2 + c - \theta)/2$. Derivations are in the Technical Appendix.

11 The “wave-shape” relationship does not occur only if the non-cartel firm is pure private, as, $\gamma_2$ is at the upper bound: then, $\delta^*$ is inverse U-shape in $\gamma$. 

former dominates for intermediate levels of product substitutability, the latter prevails for low and high levels, thus determining the “wave-shape” relationship.

4. Endogeneity of the sequential play

Following the Cournot collusion literature with cartel and non-cartel firms, we assumed a sequential play where the cartel is the leader during collusion. Shaffer (1995) has shown in a private oligopoly that this sequence endogenously emerges if pre-commitment to collude is reversible. In what follows, we show that, under certain conditions, this result holds also in mixed oligopoly. To begin, note that, in case of simultaneous play during collusion, we have:

\[ \pi_{k,\text{sim}}^C = \frac{(1 + \gamma)(2 - \gamma - \theta)^2}{4(2 - \theta + 2\gamma - \gamma^2 - \gamma\theta)^2} \]

and

\[ \Gamma_{C,\text{sim}} = \frac{4 + 3\theta^3(1 + \gamma) - 2\theta^2(6 + 6\gamma - 3\gamma^2) + \theta(10 + 8\gamma - 13\gamma^2 + 3\gamma^2)}{4(2 - \theta + 2\gamma - \gamma^2 - \gamma\theta)^2} \].

With a sequential choice where the cartel is the follower, we have:

\[ \pi_{k,\text{foll}}^C = \frac{[2 + \gamma - 2\gamma^2 - \theta(1 + \gamma - \gamma^2)]^2}{(1 + \gamma)[4 + 4\gamma - 4\gamma^2 - \theta(2 + 2\gamma - \gamma^2)]^2} \]

Comparing \( \pi_k^C \), \( \pi_{k,\text{sim}}^C \) and \( \pi_{k,\text{foll}}^C \), note that the cartel never prefers a simultaneous play, while it may prefer being the leader or the follower. On the other hand, by comparing \( \Gamma(q_k^C) \), \( \Gamma_{C,\text{sim}}^C \) and \( \Gamma_{C,\text{foll}}^C \), there is no a dominated timing. Therefore, the crucial point is whether a mechanism exists such that the cartel is able to impose its preferred timing to the non-colluding firm even when the respective preferred timings are different. As suggested by Shaffer (1995), such a mechanism consists in threatening not to collude, and must satisfy two conditions: the first (effectiveness) requires that the non-colluding firm prefers collusion in the sequential timing rather than simultaneous competition; the second (credibility) requires that colluding firms prefer not to collude rather than collude simultaneously. Suppose that the cartel’s preferred timing consists in being the follower. Such timing can arise only if the cartel is able to impose it to the non-cartel firm, as when the cartel prefers to be the follower, the non-cartel firm never prefers to be the leader. However, when \( \pi_{k,\text{foll}}^C > \pi_k^C \), the effectiveness condition is never satisfied, as \( \Gamma(q_k^C) < \Gamma(q_k^P) \). Therefore, the timing where the cartel is the follower cannot arise, even when it is

\[12\] Details are available in the Technical Appendix

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preferred by the cartel. Next, suppose that the cartel’s preferred timing consists in being the leader. First, a non-empty parameter set exists where both the cartel and the non-cartel firm prefer this timing. In this case, no threat is necessary. Further, suppose that the non-cartel firm does not want to be the follower when the cartel is leader. Does the mechanism proposed by Shaffer (1995) work in this case? The effectiveness and the credibility condition together require \( \Gamma(q^*_k) > \Gamma(q^*_l) \) and \( \pi^*_p > \pi^*_c \). It is easy to see that when \( \gamma \geq 0.44 \) a non-empty parameter set of \( \theta \) always exists such that the conditions above are satisfied. Hence, under appropriate parameter restrictions, the cartel is able to impose the sequential timing where it is the leader by threatening the non-cartel member to play a non-collusive simultaneous game if not permitted to play as a leader in a collusive sequential play.\(^{13}\)

5. Conclusions

This article studies the impact of a public firm on collusion sustainability between two private firms, and shows that increasing the public-ownership of the non-cartel firm may help sustaining collusion between private firms.\(^{14}\)

References


\(^{13}\) Recall that when \( \gamma \leq 0.44 \) the sequential play may emerge even without threat if \( \theta \) is such that \( \Gamma^{C,sim} < \Gamma(q^*_c) \).

\(^{14}\) This result can be easily generalized to \( N \geq 3 \) colluding firms and \( G \geq 1 \) private non-colluding firms in addition to the semi-public firm.