A structural approach to pricing credit derivatives with counterparty adjustments

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Definition of a standard CDS and notations

**Definition of a CDS** - the protection buyer (PB) agrees to pay a periodic coupon $c$ to a protection seller (PS) in exchange for a potential cashflow in the event of a default of the reference name (RN) of the swap before the maturity of the contract $T$

$$CL_t = -\mathbb{E} \left[ \sum_{T_i} cD(t, T_i) 1_{\{T_i \leq \tau_{RN}\}} \Delta T \bigg| \mathcal{F}_t \right]$$

$$DL_t = \mathbb{E} \left[ (1 - R_{RN}) D(t, \tau_{RN}) 1_{\{t < \tau_{RN} < T\}} \bigg| \mathcal{F}_t \right]$$

$\tau_{RN}$ — the default time of the RN

$R_{RN}$ — recovery of the RN

$CF(t, T)$ — the sum of all discounted contractual cashflows between $t$ and the maturity $T$ (both $CL$ and $DL$)

$V_t$ — value of the CDS:

$$V_t = \mathbb{E} [CF(t, T) \big| \mathcal{F}_t]$$
**Definition of CVA/DVA**

**Counterparty credit risk:** the risk of a party to a financial contract defaulting prior to the contract’s expiration and not fulfilling all of its obligations.

**Credit Value Adjustment (CVA)** — the additional cost associated with the possibility of the counterparty’s default

\[ CVA = (1 - R_{PS}) \mathbb{E} \left[ \mathbb{1}_{\{ \tau_{PS} < \min \{ T, \tau_{RN} \} \}} D(t, \tau_{PS}) V_{\tau_{PS}}^+ \mid \mathcal{F}_t \right] \]

**Debt Value Adjustment (DVA)** — the additional benefit of one’s own default

\[ DVA = (1 - R_{PB}) \mathbb{E} \left[ \mathbb{1}_{\{ \tau_{PB} < \min \{ T, \tau_{RN} \} \}} D(t, \tau_{PB}) V_{\tau_{PB}}^- \mid \mathcal{F}_t \right] \]

**Bilateral CVA** — combination of the two adjustments

\[ CVA = (1 - R_{PS}) \mathbb{E} \left[ \mathbb{1}_{\{ \tau_{PS} < \min \{ \tau_{PB}, \tau_{RN}, T \} \}} D(t, \tau_{PS}) V_{\tau_{PS}}^+ \mid \mathcal{F}_t \right] \]

\[ DVA = (1 - R_{PB}) \mathbb{E} \left[ \mathbb{1}_{\{ \tau_{PB} < \min \{ \tau_{PS}, \tau_{RN}, T \} \}} D(t, \tau_{PB}) V_{\tau_{PB}}^- \mid \mathcal{F}_t \right] \]
Structural default framework - I

Original framework proposed by Merton:

- the firm’s value $a_t$ is driven by a log-normal diffusion
- the firm borrowed a zero-coupon bond with face value $N$ and maturity $T$
- defaults at time $T$ if its value $a_T$ is less than the bond’s face value $N$

Extensions:

- more complicated forms of debt
- the default event may be triggered continuously up to the debt maturity
- making default barriers stochastic
- incorporating jumps into the firm’s value dynamics
Structural default framework - II

Firm’s asset value dynamics:

\[ da_t = (\varrho_t - \zeta_t - \lambda_t \kappa_t) a_t dt + \sigma_t a_t dW_t + (e^j - 1) dN_t \]

Default barrier — deterministic function of time: \( l_t = l_0 E_t \), where \( E_t = \exp \left( \int_0^t (r_u - \zeta_u - \lambda_u \kappa_u - \frac{1}{2} \sigma_u^2) \, du \right) \) and \( l_0 = RL_0 \)

Simplifying assumption: vol constant in time We obtain:

\[ a_t = l_0 E_t e^{\sigma x_t} \]

where the stochastic factor \( x_t \) has the following dynamics:

\[ dx_t = dW_t + \frac{j}{\sigma} dN_t, \quad x_0 = \frac{1}{\sigma} \ln \left( \frac{a_0}{l_0} \right) \]

We consider the case without jumps.

For the multi-dimensional case, the Brownian motions are correlated:

\[ d\langle W_t^i, W_t^j \rangle = \rho_{ij} dt. \]
Structural default framework


- **Extensions to two dimensions**: He et al. [1998], Lipton [2001], Zhou [2001a], Patras [2006], Valuzis [2008]

- **Applications to CVA/DVA computation**: Lipton and Sepp [2009], Blanchet-Scaillet and Patras [2011]
One-dimensional case

Process $y_t$ associated with the RN of the CDS

Green’s function:

$$G(\tau, y_0, y') = \frac{1}{\sqrt{2\pi \tau}} \left( e^{-\frac{(y'-y_0)^2}{2\tau}} - e^{-\frac{(y'+y_0)^2}{2\tau}} \right)$$

Survival probability:

$$Q(t, T, y_0) = \int_0^\infty G(\tau, y_0, y') dy' = 2N \left( \frac{y_0}{\sqrt{T-t}} \right) - 1$$

Price of a standard CDS:

$$V(t, T, y_0) = CL(t, T, y_0) + DL(t, T, y_0)$$

$$= -(c + \varrho (1 - R_{RN})) A(t, T, y_0)$$

$$+ (1 - R_{RN}) \left[ 1 - e^{-\varrho (T-t) Q(t, T, y_0)} \right].$$

where $A(t, T, y_0)$ denotes the annuity leg

$$A(t, T, y_0) = \int_t^T D(t, t')Q(t, t', y_0) dt'$$
Pricing problem and domain

General pricing equation in the positive quadrant:

\[ V_t + \frac{1}{2} V_{xx} + \frac{1}{2} V_{yy} + \rho_{xy} V_{xy} - \varrho V = 0 \]

Changes of function/variables:

\[ U(t, T, x, y) = e^{\varrho(T-t)} V(t, T, x, y) \]

\[
\begin{align*}
\alpha(x, y) &= x \\
\beta(x, y) &= -\frac{1}{\bar{\rho}_{xy}} (\rho_{xy}x - y)
\end{align*}
\]

\[
\begin{align*}
r &= \sqrt{\alpha^2 + \beta^2} \\
\varphi &= \varpi + \arctan\left(\frac{-\alpha}{\beta}\right)
\end{align*}
\]

Final form of the pricing equation:

\[ U_t + \frac{1}{2} \left( U_{rr} + \frac{1}{r} U_r + \frac{1}{r^2} U_{\varphi\varphi} \right) = 0 \]
Green’s function - eigenvalues expansion method

Green’s function solves the forward equation:

\[ G_\tau - \frac{1}{2} \left( G_{r'r'} + \frac{1}{r'} G_{r'} + \frac{1}{r'2} G_{\varphi'\varphi'} \right) = 0 \]

Initial condition:

\[ G(0, r', \varphi') = \frac{1}{r_0} \delta(r' - r_0)\delta(\varphi' - \varphi_0), \]

Boundary conditions:

\[ G(\tau, r', 0) = 0, \quad G(\tau, r', \bar{\omega}) = 0, \quad G(\tau, 0, \varphi') = 0, \quad G(\tau, r', \varphi') \xrightarrow{r' \to \infty} 0. \]

Solution obtained through the eigenvalues expansion method:

\[ G(\tau, r_0, r', \varphi_0, \varphi') = \frac{2e^{-\frac{r'^2 + r_0^2}{2\tau}}}{\bar{\omega}\tau} \sum_{n=1}^{\infty} I_{\nu_n} \left( \frac{r'r_0}{\bar{\omega}\tau} \right) \sin (\nu_n \varphi') \sin (\nu_n \varphi_0). \]

where \( \nu_n = \frac{n\pi}{\bar{\omega}} \).
**Figure:** Green’s function ($x_0 = 0.1$, $y_0 = 0.1$, $\sigma_x = 10\%$, $\sigma_y = 10\%$, $\rho_{xy} = 70\%$, $T = 1$ year).
Joint survival probability

\[ Q(t, T, x, y) \] — the joint survival probability of issuers \( x \) and \( y \) to a fixed maturity \( T \)

\[
Q_t + \frac{1}{2} Q_{xx} + \frac{1}{2} Q_{yy} + \rho_{xy} Q_{xy} = 0,
\]

with final condition \( Q(T, T, x, y) = 1 \) and boundary conditions \( Q(t, T, x, 0) = 0 \) and \( Q(t, T, 0, y) = 0 \)

Solution:

\[
Q(\tau, r_0, \varphi_0) = \sum_{k=0}^{\infty} \frac{4 \sin (\nu_{2k+1} \varphi_0)}{(2k + 1) \pi} \left( \frac{r_0^2}{2 \tau} \right)^{\nu_{2k+1}/2} \frac{\Gamma \left( 1 + \frac{\nu_{2k+1}}{2} \right)}{\Gamma \left( 1 + \nu_{2k+1} \right)} \text{E} \left( \frac{\nu_{2k+1}}{2}, 1 + \nu_{2k+1}, -\frac{r_0^2}{2 \tau} \right)
\]

\[
= \frac{2 r_0 e^{-\frac{r_0^2}{4 \tau}} \sqrt{2 \pi \tau}}{\sqrt{2 \pi T}} \sum_{k=0}^{\infty} \frac{\sin (\nu_{2k+1} \varphi_0)}{2k + 1} \left[ I_{\nu_{2k+1}/2} -1 \left( \frac{r_0^2}{4 \tau} \right) + I_{\nu_{2k+1}/2} +1 \left( \frac{r_0^2}{4 \tau} \right) \right]
\]
Application to CVA computation

Pricing equation:

$$V_t + \frac{1}{2} V_{xx} + \frac{1}{2} V_{yy} + \rho_{xy} V_{xy} - \rho V = 0,$$

with the final condition $V(T, T, x, y) = 0$

- If the RN of the CDS contract defaults first: since the PS has not defaulted it will be able to honour the payment: $V(t, T, x, 0) = 0$.
- If the PS defaults first: it will no longer be able to honour its payments and hence the shortfall for the PB will be a fraction of the outstanding PV of the CDS:

$$V(t, T, 0, y) = (1 - R_{PS}) V^{CDS}(t, T, y)^+.$$

- If the PS is risk free there is no shortfall: $V(t, T, \infty, y) = 0$.
- If the CDS reference name is virtually risk-free we do not care what happens to the PS: $V(t, T, x, \infty) = 0$.

$$V^{CVA}(t, T, r_0, \varphi_0) = -\frac{1 - R_{PS}}{2} \int_{t}^{T} \int_{0}^{\infty} D(t, t') G_{\varphi}(t' - t, r, \varpi) V^{CDS}(t', T, \bar{\rho}_{xy} r') \frac{1}{r} dr dt'.$$
Pricing problem and domain

Pricing equation in the positive octant

\[ V_t + \frac{1}{2} V_{xx} + \frac{1}{2} V_{yy} + \frac{1}{2} V_{zz} + \rho_{xy} V_{xy} + \rho_{xz} V_{xz} + \rho_{yz} V_{yz} - \varrho V = 0 \]

Changes of variables:

\[
\begin{cases}
\alpha(x, y, z) = x \\
\beta(x, y, z) = \frac{1}{\rho_{xy}} (-\rho_{xy} x + y) \\
\gamma(x, y, z) = \frac{1}{\rho_{xy} \chi} \left[ (\rho_{xy} \rho_{yz} - \rho_{xz}) x + (\rho_{xy} \rho_{xz} - \rho_{yz}) y + \rho_{xyz}^2 z \right],
\end{cases}
\]

\[
\begin{cases}
\alpha = r \sin \theta \sin \varphi \\
\beta = r \sin \theta \cos \varphi \\
\gamma = r \cos \theta
\end{cases} \quad \begin{cases}
\varrho = \sqrt{\alpha^2 + \beta^2 + \gamma^2} \\
\theta = \arccos \left( \frac{\gamma}{\varrho} \right) \\
\varphi = \arctan \left( \frac{\alpha}{\beta} \right)
\end{cases}
\]
Final form of the pricing equation

\[ U_t + \frac{1}{2} \left[ \frac{1}{r} \frac{\partial}{\partial r^2} (rU) + \frac{1}{r^2} \left( \frac{1}{\sin^2 \theta} U_{\varphi \varphi} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta U_{\theta}) \right) \right] = 0 \]

- \( r > 0 \)
- \( 0 \leq \varphi \leq \varpi \), where \( \varpi = \arccos(-\rho_{xy}) \)
- the possible range of values for \( \theta \) depends on \( \varphi \)

\[ \varphi (\omega) = \arccos \left( \frac{1 - \rho_{xy} \omega}{\sqrt{1 - 2 \rho_{xy} \omega + \omega^2}} \right) \]

\[ \Theta (\omega) = \arccos \left( - \frac{\rho_{yz} - \rho_{xz} \rho_{xy} + \omega (\rho_{xz} - \rho_{yz} \rho_{xy})}{\sqrt{\rho_{xy} \left( \rho_{xz}^2 - 2 \omega (\rho_{xy} - \rho_{xz} \rho_{yz}) + \omega^2 \rho_{yz}^2 \right)}} \right) \]

Figure: Domain after the change in coordinates for \( \rho_{xy} = 20\% \), \( \rho_{xz} = 0\% \), \( \rho_{yz} = 30\% \)
Green’s function

Green’s function satisfies the forward equation:

\[ G_\tau - \frac{1}{2} \left[ \frac{1}{r'} \frac{\partial^2}{\partial r'^2} (r' G) + \frac{1}{r'^2} \left( \frac{1}{\sin^2 \theta'} G_{\varphi \varphi'} + \frac{1}{\sin \theta'} \frac{\partial}{\partial \theta'} (\sin \theta' G_{\theta'}) \right) \right] = 0, \]

with initial condition:

\[ G(0, r', \varphi', \theta') = \frac{1}{r_0^2 \sin \theta_0} \delta (r' - r_0) \delta (\varphi' - \varphi_0) \delta (\theta' - \theta_0), \]

and boundary conditions:

\[ G(\tau, r', 0, \theta') = G(\tau, r', \varphi, \theta') = G(\tau, r', \varphi', 0) = 0, \]
\[ G(\tau, r', \varphi', \Theta(\varphi')) = G(\tau, 0, \varphi', \theta') = 0, \quad G(\tau, r', \varphi', \theta') \xrightarrow{r' \to \infty} 0. \]
Solving the forward equation

- solve using eigenvalues expansion method

- separation of variables:  \( G(\tau, r', \varphi', \theta') = g(\tau, r')\Psi(\varphi', \theta') \).

- Radial part (similar to the two dimensional case):
  \[
g(\tau, r') = \frac{e^{-\frac{r'^2+r_0^2}{2\tau}}}{\tau\sqrt{r'r_0}} I_{\sqrt{\Lambda^2+1/4}} \left( \frac{r'r_0}{\tau} \right).
  \]

- For the angular part

  \[
  \frac{1}{\sin^2 \theta'} \psi_{\varphi'\varphi'} + \frac{1}{\sin \theta'} \frac{\partial}{\partial \theta'} (\sin \theta' \psi_{\theta'}) = -\Lambda^2 \psi,
  \]

  \( \psi(0, \theta') = 0, \quad \psi(\varpi, \theta') = 0, \quad \psi(\varphi', 0) = 0, \quad \psi(\varphi', \Theta(\varphi')) = 0. \)
Solving the angular part of the forward equation

- we solve this using the finite element method (FEM)
- we map the domain onto the \((\varphi, \theta)\) plane

\[ \rho_{xy} = 0.8, \rho_{xz} = 0.5, \rho_{yz} = 0.3 \]

\[ \rho_{xy} = 0.8, \rho_{xz} = 0.05, \rho_{yz} = 0.6 \]

- Variational (weak) formulation:

\[ \int_{\Omega} \frac{1}{\sin \theta'} \Psi_{\varphi'} \Psi'_{\varphi'} d\Omega + \int_{\Omega} \sin \theta' \Psi_{\theta'} \Psi'_{\theta'} d\Omega = \Lambda^2 \int_{\Omega} \Psi \Psi' \sin \theta' d\Omega \]
Construction of the grid

- triangular mesh
- can build both uniform and adaptive meshes
- an iterative algorithm is used
- after each iteration, the Delaunay triangulation method is used

Figure: $\rho_{xy} = 80\%$, $\rho_{xz} = 20\%$, $\rho_{yz} = 50\%$ (mesh uses 1800 points).
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Figure: \( \rho_{xy} = 80\% \), \( \rho_{xz} = 50\% \), \( \rho_{yz} = 50\% \) (mesh uses 1500 points).

Figure: \( \rho_{xy} = 20\% \), \( \rho_{xz} = -10\% \), \( \rho_{yz} = -60\% \) (mesh uses 1600 points).
Applying FEM

- dimension of the space in which we search for the solution = number of free points in the mesh \( n \); \( (\Phi_i)_{1 \leq i \leq n} \) basis for this space
- linear basis functions on each triangle
- the variational formulation is approximated by a linear system:

\[
K\Psi = \Lambda^2 M\Psi
\]

where we denote by \( K = (K_{ij})_{1 \leq i, j \leq n} \) the stiffness matrix, and by \( M = (M_{ij})_{1 \leq i, j \leq n} \) the mass matrix:

\[
K_{ij} = \int_{\Omega} (A \nabla \Phi_j) \cdot \nabla \Phi_i d\Omega,
\]

\[
M_{ij} = \int_{\Omega} \sin \theta' \Phi_i \Phi_j d\Omega,
\]

with the matrix \( A \) given by \( A = \begin{pmatrix} 1 & 0 \\ \frac{1}{\sin \theta'} & \sin \theta' \end{pmatrix} \).
Eigenvalues and eigenvectors ($\rho_{xy} = 0\%, \rho_{xz} = 0\%, \rho_{yz} = 0\% - 1500$ point mesh)

(a) Eigenvector 1: $\Lambda_1^2 = 12.0$

(b) Eigenvector 2: $\Lambda_2^2 = 30.2$

(c) Eigenvector 3: $\Lambda_3^2 = 30.2$

(d) Eigenvector 6: $\Lambda_6^2 = 56.8$

(e) Eigenvector 10: $\Lambda_{10}^2 = 92.4$

(f) Eigenvector 20: $\Lambda_{20}^2 = 189.2$
Eigenvalues and eigenvectors ($\rho_{xy} = 20\%, \rho_{xz} = -10\%, \rho_{yz} = -60\% - 1600$ pts mesh)

(a) Eigenvector 1: $\Lambda_1^2 = 21.5$

(b) Eigenvector 2: $\Lambda_2^2 = 42.2$

(c) Eigenvector 3: $\Lambda_3^2 = 63.8$

(d) Eigenvector 5: $\Lambda_5^2 = 96$

(e) Eigenvector 7: $\Lambda_7^2 = 129.5$

(f) Eigenvector 12: $\Lambda_{12}^2 = 200.3$
Green’s function

Eigenfunction expansion for Green’s function:

\[ G(\tau, r', \varphi', \theta') = \sum_{n=1}^{\infty} C_n g_n(\tau, r') \psi_n(\varphi', \theta') \]

Coefficients \( C_n \) - can be computed by imposing the initial condition:

\[ G(0, r', \varphi', \theta') = \frac{1}{r_0^2 \sin \theta_0} \delta(r' - r_0) \delta(\varphi' - \varphi_0) \delta(\theta' - \theta_0), \]

and we obtain \( C_n = \psi_n(\varphi_0, \theta_0) \).

Final formula for Green’s function:

\[ G(\tau, r_0, r', \varphi_0, \varphi', \theta_0, \theta') = \frac{e^{-\frac{r'^2 + r_0^2}{2\tau}}}{\tau \sqrt{r' r_0}} \sum_{n=1}^{\infty} l \sqrt{\Lambda_n^2 + \frac{1}{4}} \left( \frac{r' r_0}{\tau} \right) \psi_n(\varphi_0, \theta_0) \psi_n(\varphi', \theta') \]
Joint survival probability

$Q(t, T, x, y, z)$: the joint survival probability of issuers $x$, $y$ and $z$ to a fixed maturity $T$. Satisfies the backward pricing equation:

$$Q_t + \frac{1}{2} Q_{xx} + \frac{1}{2} Q_{yy} + \frac{1}{2} Q_{zz} + \rho_{xy} Q_{xy} + \rho_{xz} Q_{xz} + \rho_{yz} Q_{yz} = 0,$$

with final condition $Q(T, T, x, y, z) = 1$ and zero boundary conditions.

Solution using Green’s function:

$$Q(\tau, r_0, \varphi_0, \theta_0) = \int_0^{\infty} \int_0^{\varphi} \int_0^{\Omega} \Theta(\varphi) G(\tau, r_0, r', \varphi_0, \varphi', \theta_0, \theta') r'^2 \sin \theta' \, d\theta' \, d\varphi' \, dr'$$

$$= \sum_{n=1}^{\infty} \left( \frac{r_0^2}{2\tau} \right)^{\nu_n/2} - \frac{1}{4} \frac{\Gamma\left(\frac{\nu_n}{2} + \frac{5}{4}\right)}{\Gamma(\nu_n + 1)} \mathbf{1}_{F_1}\left(\frac{2\nu_n - 1}{4}, \nu_n + 1, -\frac{r_0^2}{2\tau}\right)$$

$$\times \psi_n(\varphi_0, \theta_0) \int_0^{\Omega} \psi_n(\varphi', \theta') \sin \theta' \, d\varphi' \, d\theta'$$
Pricing problem for CVA/DVA

Pricing equation:

\[
V_t + \frac{1}{2} V_{xx} + \frac{1}{2} V_{yy} + \frac{1}{2} V_{zz} + \rho_{xy} V_{xy} + \rho_{xz} V_{xz} + \rho_{yz} V_{yz} - \varrho V = 0
\]

Final condition: \( V(T, T, x, y, z) = 0 \)

Boundary conditions:

- If the PS defaults:
  \[
  V^{\text{CVA}}(t, T, 0, y, z) = (1 - R_{PS}) V(t, T, y)\]

- If the PB defaults:
  \[
  V^{\text{DVA}}(t, T, x, y, 0) = (1 - R_{PB}) V(t, T, y)\]
Pricing formulas for CVA/DVA

Pricing formula for CVA:

\[
U^{\text{CVA}}(t, T, r_0, \varphi_0, \theta_0) = \frac{1}{2} \int_0^T \int_0^\infty \int_0^\Theta(0) \int_0^\infty U^{\text{CVA}}(t', T, r, 0, \theta) G_{\varphi}(t' - t, r, 0, \theta) \frac{d\theta dr dt'}{\sin \theta},
\]

Pricing formula for DVA:

\[
U^{\text{DVA}}(t, T, r_0, \varphi_0, \theta_0) =
\]

\[
- \frac{1}{2} \int_0^T \int_0^\infty \int_0^\varpi \sin \Theta(\varphi) U^{\text{DVA}}(t', T, r, \varphi, \Theta(\varphi)) G_\theta(t' - t, r, \varphi, \Theta(\varphi)) d\varphi dr dt'
\]

\[
+ \frac{1}{2} \int_0^T \int_0^\infty \int_0^\infty U^{\text{DVA}}(t', T, r, \varphi(\omega), \Theta(\omega)) G_{\varphi}(t' - t, r, \varphi(\omega), \Theta(\omega)) \frac{d\omega dr dt'}{\sin \Theta(\omega)}.
\]
Numerical results for a CDS

- **AIG**: protection seller
- **GE**: reference name of the CDS
- **UNICREDIT**: protection buyer

Calibrated inputs (15th of December 2011):

<table>
<thead>
<tr>
<th>Inputs</th>
<th>AIG</th>
<th>GE</th>
<th>UNICREDIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial value</td>
<td>0.0359</td>
<td>0.3035</td>
<td>0.1199</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.44%</td>
<td>10.45%</td>
<td>6.3%</td>
</tr>
<tr>
<td>Recovery</td>
<td>50%</td>
<td>40%</td>
<td>40%</td>
</tr>
</tbody>
</table>
PS, PB and RN are uncorrelated

(a) Break-even coupon

(b) Change in BEC (compared to the standard CDS with non-risky counterparts)

Figure: Impact of counterparty adjustments on the break-even coupon of a CDS: $\rho_{xy} = 0\%, \rho_{xz} = 0\%, \rho_{yz} = 0\%$. 
PS is highly correlated to the RN

Figure: Impact of counterparty adjustments on the break-even coupon of a CDS: $\rho_{xy} = 80\%$, $\rho_{xz} = 50\%$, $\rho_{yz} = 30\%$. 
PB is highly anti-correlated to the RN

Figure: Impact of counterparty adjustments on the break-even coupon of a CDS: $\rho_{xy} = 20\%, \rho_{xz} = -10\%, \rho_{yz} = -60\%$. 

(a) Break-even coupon

(b) Change in BEC (compared to the standard CDS with non-risky counterparts)
Conclusion

- bilateral CVA/DVA for a standard single name default swap
- 3D extension of the structural default framework
- semi-analytical expression for Green’s function
- application to computing joint survival probabilities and CVA/DVA for a CDS


