Modeling Mortality Trend under Modern Solvency Regimes

Matthias Börger
Daniel Fleischer
Nikita Kuksin

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Introduction

What is longevity risk?

- Longevity risk is the risk of underestimating future mortality improvements
  - Trend risk
  - Mortality risk has a trend risk and a catastrophe risk component
  - Systematic and non-hedgeable risk
  - Explicitly accounted for under Solvency II and the Swiss Solvency Test (SST)
General concept for Solvency Capital Requirement (SCR) under Solvency II
- SCR = 99.5% Value-at-Risk (VaR) of Available Capital over 1 year
- „Capital necessary to cover losses over next year with at least 99.5% probability“
- Overall risk is typically split into several modules, individual SCRs are finally aggregated

Stochastic mortality model is required for mortality/longevity trend risk under Solvency II

In a 1-year setting, longevity/mortality trend risk consists of two components:
- Low/high realized mortality in the one year
- Decrease/increase in expected future mortality, i.e. changes in the long-term mortality trend
Goal: Specification and calibration of a mortality model with the following properties

- Simultaneous modeling of mortality and longevity risk
  - Exploiting of diversification effects
- Full age range
  - 20 to 105 in our case
- Consideration of several populations at the same time
  - Males and females in the same country
  - Populations from different countries
- Quantification of risk over limited time horizons
  - One-year view of Solvency II and the SST particularly relevant
- Plausible tail scenarios
  - 99.5% VaR
- Conservative calibration
We model the logit of mortality rates

\[
\text{logit}(q_{x,t}) = \alpha_x + \kappa_t^{(1)} (x - x_{\text{center}}) + \kappa_t^{(2)} (x_{\text{young}} - x)^+ + \kappa_t^{(3)} (x - x_{\text{old}})^+ + \gamma_{t-x}
\]

- \(x_{\text{center}} = 60, \ x_{\text{young}} = 55, \ x_{\text{old}} = 85\)
- \(\kappa_t^{(1)}\) describes the general level of mortality, \(\kappa_t^{(2)}\) is the slope of the mortality curve, \(\kappa_t^{(3)}\) and \(\kappa_t^{(4)}\) describe additional effects in young and old age mortality, respectively.

Model estimation via Generalized Linear Model Theory

- Logit is canonical link function for Binomial distribution
- Number of deaths is binomially distributed given initial exposures
Multi-Population Setting

- Important note: Even if one is only interested in a single population considering several populations is worthwhile
  - Trend uncertainty can be significantly reduced
  - We generally observe smaller SCR's in the multi-population model compared to the single population model

- There is clearly a common trend in $\kappa^{(1)}_t$
- A model for several populations must account for that
- We apply cointegration and an error correction model for deviations from the common trend
Model Simulation

Projection of $\kappa_{t,\text{total}}^{(1)}$ for the total population

- Linear trends with breaks in the historical data
  - Commonly used random walk with drift does not allow for such trend breaks
  - Trend breaks are particularly important under one-year view (change of best estimate trend)

- Idea: Each year, fit regression line to historical data and forecast future best estimate mortality as
  $\kappa_{t+1,\text{total}}^{(1)} = l_t(t + 1) + \varepsilon_t^{(1)}(\sigma^{(1)} + \sigma^{(1)})$
  - $\sigma^{(1)}$ is a volatility add-on, volatility $\sigma^{(1)}$ may be weighted to stress most recent past
  - Implicit „re-calibration“ of the model with respect to the long-term trend
  - To stress most recent mortality experience, the regression line is fitted with weights
    $w_s = \left(1 + \frac{1}{h}\right)^{v-t}$
- Weighting parameter \( h \) has massive impact
- Plausible one-year and run-off scenarios
- Each run-off scenario is a combination of one-year scenarios
- Disentangling of one-year noise and long-term trend uncertainty
- Possibly more plausible confidence bounds than for a random walk with drift
Model Simulation (ctd.)

Projection of $\kappa_{t,p}^{(i)}$ for individual populations

- For each individual population we project as
  \[ \kappa_{t,p}^{(i)} = \kappa_{t,\text{total}}^{(i)} + a_p + b_p (\kappa_{t-1,p}^{(i)} - \kappa_{t-1,\text{total}}^{(i)}) + \varepsilon_{t,p} \]
  - $b_p$ denotes the "mean reversion speed" (absolute value should be smaller than 1)
  - $a_p/(1-b_p)$ is the long-term difference between the total population and population $p$

- Different approaches of calibrating the long-term difference
  - Fitting of an AR(1) process to historical differences
  - Weighted/unweighted average of historical differences
Model Simulation (ctd.)

- **Projection of** $\kappa_t^{(2)}, \kappa_t^{(3)},$ and $\kappa_t^{(4)}$ **for the individual populations**
  - No substantial trend obvious in the historical data
  - Forecast as correlated 3-dimensional random walk
  - No substantial correlation with $\kappa_t^{(1)}$
  - Volatility add-on $\sigma_t(2)$ for $\kappa_t^{(2)}$
    - The larger the changes in the slope of the mortality, the smaller the correlation between young and old ages
    - Thus the add-on affects diversification between mortality and longevity risk
  - Between populations, increments of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are correlated
    - This also implies slight correlation between the $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$
    - Historical correlations should be checked carefully though and possibly adjusted

- **Projection of** $\gamma_{t-x}$
  - Cohort parameters should stay around zero
  - Forecast as imposed stationary AR(1) process
  - Cohort parameters are rather irrelevant for short-term simulations as under Solvency II
Weighting Parameters and Volatility Add-ons

- Parameters $h, \overline{\sigma}^{(1)},$ and $\overline{\sigma}^{(2)}$ have a massive impact on simulation outcomes and thus SCRs.
- Add-on $\overline{\sigma}^{(1)}$ determines possible severity of short-term events.
- Weighting parameter $h$ determines trend changes over one year and width of confidence bounds.

Calibration is difficult but should be conservative
- Fitting to most severe events/evolutions in the past
  - Example: Rapid increase in Dutch life expectancy gains starting from about 1970
  - Question: At which percentile should such extreme evolutions be observed?

Calibration of $\overline{\sigma}^{(2)}$
- The larger the add-on the smaller the correlation between young and old ages thus limiting diversification
- Choose $\overline{\sigma}^{(2)}$ such that correlation between ages at the boundaries of the age range is (close to) zero for most populations.
Available data contains only little information on tail scenarios which we are interested in. Uncertainty remains whether model outcomes are severe enough. 

→ Incorporate epidemiological/demographic expert opinion

**Specification of mortality/longevity threat scenarios**
- Shock to mortality projection
- Likely effects of finding of a cure for a certain illness
- Scenarios which the statistical model cannot generate, e.g., diverging mortality trends between countries/regions

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**Application of threat scenarios**
- Check of model calibration: Adjustment of weighting parameter or volatility add-ons if the model outcomes should cover the threat scenarios but do not
- Inclusion in SCR computations: SCR as a weighted average of model outcomes and threat scenarios
We have specified and calibrated a mortality model with several appealing properties:

- Full age range
- Variability in simulation outcomes due to 5 stochastic drivers
- Clear interpretation of the model parameters
- Non-trivial correlation structure to allow for simultaneous modeling of mortality and longevity risk
- Stochastic trend modeling spares full re-calibration of the model in each scenario (could be applied in other models as well)
- Plausible outcomes in one-year view and run-off view
- Conservative calibration
- Inclusion of expert opinion
- Multi-population model allowing for diversification effects
Matthias Boerger

Institute of Insurance, Ulm University & Institute for Finance and Actuarial Sciences (ifa), Ulm
Helmholtzstraße 22, 89081 Ulm, Germany
Phone: +49 731 50-31257, Fax: +49 731 50-31239
Email: m.boerger@ifa-ulm.de
Weighting Parameters and Volatility Add-ons

- Parameters $h, \overline{\sigma}^{(1)},$ and $\overline{\sigma}^{(2)}$ have a massive impact on simulation outcomes and thus SCRs.
- Most importantly, $\overline{\sigma}^{(1)}$ determines extreme short-term events, $h$ extreme long-term evolutions.
- Calibration is difficult but should be conservative.
- Fitting to most severe events/evolutions in the past:
  - Short-term: Drop in life expectancy in Russia at the beginning of the 1990’s.
  - Long-term: Rapid increase in Dutch life expectancy gains starting from about 1970.
Weighting Parameters and Volatility Add-ons (ctd.)

- **Calibration of $\sigma^{(1)}$** in the multi-population setting:
  - Add „non-Western“ countries with data available in the HMD (e.g. Russia)
  - About 50 years of historical data
  - Choose $\sigma^{(1)}$ such that most severe event (drop in life expectancy in the 1990’s) is seen e.g. at the 98th percentile

- **Calibration of $h$** in the multi-population setting:
  - Trend break around 1970 is the most significant long-term change in the available data
  - Several years of data required to observe a trend change
  - Choose $h$ such that trend change is observed, e.g., at the 90th or 95th percentile (measure of conservatism)

- **Note:** The parameters interact with each other even though $h$ depends on $\sigma^{(1)}$ only weakly

- **Calibration of $\sigma^{(2)}$**
  - The larger the add-on the smaller the correlation between young and old ages
  - Thus, diversification between mortality and longevity risk is typically reduced
  - Choose $\sigma^{(2)}$ such that correlation between ages at the boundaries of the age range is (close to) zero for most populations